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Apêndice A

Extensividade da Entropia

Considere a expressão para a entropia de informação de Shannon dada pela expressão (3.3). Para 2 sistemas A e B independentes, a entropia do sistema combinado é:

$$S(p_A * p_B) = - \sum_{j=1}^m \sum_{i=1}^n p_i p_j \ln(p_i p_j) \quad (\text{A.1})$$

$$S(p_A * p_B) = - \sum_{j=1}^m \sum_{i=1}^n p_i p_j \ln p_i - \sum_{j=1}^m \sum_{i=1}^n p_i p_j \ln p_j \quad (\text{A.2})$$

$$S(p_A * p_B) = \left(- \sum_{j=1}^m p_j \right) \left(\sum_{i=1}^n p_i \ln p_i \right) - \left(\sum_{i=1}^n p_i \right) \left(\sum_{j=1}^m p_j \ln p_j \right) \quad (\text{A.3})$$

$$S(p_A * p_B) = - \sum_{i=1}^n p_i \ln p_i - \sum_{j=1}^m p_j \ln p_j \quad (\text{A.4})$$

$$S(p_A * p_B) = S[p_A] + S[p_B] \quad (\text{A.5})$$

ou seja, a soma das entropias individuais, garantindo que ela é uma grandeza extensiva.

Para o caso da estatística generalizada, a entropia é dada pela equação (3.11):

$$S_q[p] = \frac{1 - \sum_{i=1}^W p_i^q}{q-1} \quad (\text{A.6})$$

Considerando o caso particular em que $p_i = W^{-1}$, com W o número total de estados (distribuição uniforme), temos que:

$$\sum_{i=1}^W p_i^q = \sum_{i=1}^W W^{-q} = W W^{-q} = W^{1-q} \quad (\text{A.7})$$

A entropia do sistema é dada então, por:

$$S_q[p] = \frac{1 - W^{(1-q)}}{q-1} \quad (\text{A.8})$$

Dados dois sistemas independentes A e B, a entropia do sistema combinado é:

$$S_q[p_A * p_B] = \frac{1 - \sum_{i=1}^{W_A} \sum_{j=1}^{W_B} (p_i^q p_j^q)}{q-1} \quad (\text{A.9})$$

Para o caso de distribuição uniforme, usando (A.7):

$$S_q[p_A * p_B] = \frac{1 - (W_A^{(1-q)})(W_B^{(1-q)})}{q-1} \quad (\text{A.10})$$

Escrevendo a entropia total como a soma de entropias individuais mais outros termos, teremos:

$$S_q[p] = \left(\frac{1}{q-1} \right) \left[(1 - W_A^{(1-q)}) + (1 - W_B^{(1-q)}) + (-1 + W_A^{(1-q)} + W_B^{(1-q)} - W_A^{(1-q)}W_B^{(1-q)}) \right] \quad (\text{A.11})$$

$$S_q[p] = \left(\frac{1}{q-1} \right) \left[(1 - W_A^{(1-q)}) + (1 - W_B^{(1-q)}) - (1 - W_A^{(1-q)})(1 - W_B^{(1-q)}) \right] \quad (\text{A.12})$$

Usando (A.8):

$$S_q[p] = S_q[p_A] + S_q[p_B] - \left[\frac{1}{(q-1)} \right] (1 - W_A^{(1-q)})(1 - W_B^{(1-q)}) \quad (\text{A.13})$$

$$S_q[p] = S_q[p_A] + S_q[p_B] - (q-1)S_q[p_A]S_q[p_B] \quad (\text{A.14})$$

Assim, a entropia generalizada é uma grandeza não-extensiva.

Apêndice B

Algoritmo para misturar dados em série.

```
program Misturar

implicit real(a-h,o-z)
parameter n=10
real XX(n)
integer indx(n)

open (unit=3,file='data1s.dat',status= 'unknown')
open (unit=5,file='data1.dat',status= 'old')

do 20 i=1,n
  read(5,*)x
  XX(i)=x
20  continue

call indexx(n,XX,indx)

do 30 i=1,n
  write(3,*)indx(i),XX(i)
  write(*,*)i,indx(i),XX(i)
30  continue

stop
end

SUBROUTINE indexx(n,arr,indx)
INTEGER n,indx(n),M,NSTACK
REAL arr(n)
PARAMETER (M=7,NSTACK=50)
INTEGER i,indxt,ir,itemp,j,jstack,k,l,istack(NSTACK)
REAL a
do 11 j=1,n
```

```
    indx(j)=j
11  continue
    jstack=0
    l=1
    ir=n
1  if(ir-l.lt.M)then
    do 13 j=l+1,ir
        indxt=indx(j)
        a=arr(indxt)
        do 12 i=j-1,l,-1
            if(arr(indx(i)).le.a)goto 2
            indx(i+1)=indx(i)
12  continue
        i=l-1
2    indx(i+1)=indxt
13  continue
    if(jstack.eq.0)return
    ir=istack(jstack)
    l=istack(jstack-1)
    jstack=jstack-2
else
    k=(l+ir)/2
    itemp=indx(k)
    indx(k)=indx(l+1)
    indx(l+1)=itemp
    if(arr(indx(l)).gt.arr(indx(ir)))then
        itemp=indx(l)
        indx(l)=indx(ir)
        indx(ir)=itemp
    endif
    if(arr(indx(l+1)).gt.arr(indx(ir)))then
        itemp=indx(l+1)
        indx(l+1)=indx(ir)
        indx(ir)=itemp
    endif
    if(arr(indx(l)).gt.arr(indx(l+1)))then
        itemp=indx(l)
```

```
        indx(l)=indx(l+1)
        indx(l+1)=itemp
    endif
    i=l+1
    j=ir
    indxt=indx(l+1)
    a=arr(indxt)
3    continue
    i=i+1
    if(arr(indx(i)).lt.a)goto 3
4    continue
    j=j-1
    if(arr(indx(j)).gt.a)goto 4
    if(j.lt.i)goto 5
    itemp=indx(i)
    indx(i)=indx(j)
    indx(j)=itemp
    goto 3
5    indx(l+1)=indx(j)
    indx(j)=indxt
    jstack=jstack+2
    if(jstack.gt.NSTACK)pause 'NSTACK too small in indexx'
    if(ir-i+1.ge.j-l)then
        istack(jstack)=ir
        istack(jstack-1)=i
        ir=j-1
    else
        istack(jstack)=j-1
        istack(jstack-1)=l
        l=i
    endif
endif
goto 1
END
```