ALEXANDRE ALMEIDA DEL SAVIO

A COMPONENT METHOD MODEL FOR SEMI-RIGID STEEL JOINTS
INCLUDING BENDING MOMENT-AXIAL FORCE INTERACTION

Ph.D. Thesis

Thesis presented to the Post-graduate Program in Structural Engineering of Department of Civil Engineering, PUC-Rio, as partial fulfillment of the requirements for the Ph.D. Degree in Structural Engineering.

Supervisors: Prof. Sebastião Arthur Lopes de Andrade
Prof. Pedro Colmar Gonçalves da Silva Vellasco
Prof. David Arthur Nethercot

Rio de Janeiro
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Steel structures; Semi-rigid joints; Joint behaviour; Axial versus bending moment interaction; Mechanical model; Component method; Rotational stiffness.
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Abstract


The correct knowledge of the joint moment-rotation characteristic is an essential prerequisite for the use of the so called semi-continuous approach to steel and composite frame design.

Although the axial force transferred from the beam is frequently low, so that its effect on the moment-rotation characteristic may often be neglect, certain circumstances do exist in which axial compression or tension forces will be sufficiently large that it is no longer reasonable to ignore their influence.

The current thesis is centred on the development of a generalised component-based model for semi-rigid beam-to-column joints including the full axial force versus bending moment interaction. The detailed formulation of the proposed analytical model is fully described in this work, as well as all the analytical expressions used to evaluate the model properties. Detailed examples demonstrate how to use this model to predict moment-rotation curves for any axial force level. Numerical results, validated against experimental data, were also performed in order to verify the accuracy and validity of the proposed model. A tri-linear approach to characterise the force-displacement relationship of the joint components is also proposed to model the joint model structural response. Comparisons of the present development to key prior studies of this topic was also made and commented in detail.

A series of parametric and sensitivity studies were executed varying several key parameters that influence on the joint structural behaviour. The axial force-bending moment interaction was also carefully analysed and the axial force effect on the joint response was discussed. The proposed model and associated analytical studies form the basis of important design considerations, involving the presence of the axial force, which are suggested in this work to be included in future improvements of structural design codes.

Finally, in addition to the proposed model and due to the fact of relatively
few experimental results have been reported to investigate the axial force effect, an alternative method is presented herein. This alternative approach extends the range of application of available experimental data to generate moment-rotation characteristics that implicitly make proper allowance for the presence of significant levels of either tension or compression at the adjacent beams. The applicability and validity of the proposed methodology is demonstrated through comparisons against several tests on endplate joints and baseplate arrangements.

**Keywords**

Steel structures; Semi-rigid joints; Joint behaviour; Axial versus bending moment interaction; Mechanical model; Component method; Rotational stiffness.
Resumo


A compreensão correta da curva característica momento-rotação de uma ligação é uma condição essencial para a utilização das chamadas abordagens semi-contínuas para o aço e o projeto de estruturas mistas.

Embora a força axial proveniente da viga seja frequentemente baixa de modo que o seu efeito sobre a curva característica momento-rotação da ligação possa muitas vezes ser negligenciado, existem certas circunstâncias nas quais as forças axiais de compressão ou tração serão suficientemente grandes, não sendo mais possível ignorar sua influência.

Esta tese é centrada no desenvolvimento de um modelo mecânico generalizado, baseado no método das componentes para conexões semi-rígidas do tipo viga-cola incluindo a interação completa entre a força axial e o momento fletor. A formulação detalhada do modelo analítico proposto é descrita totalmente neste trabalho bem como todas as expressões analíticas utilizadas para avaliar as propriedades do modelo mecânico. Exemplos detalhados demonstram como utilizar este modelo para prever curvas momento-rotação para qualquer nível de força axial. Resultados numéricos validados contra dados experimentais também foram realizados a fim de verificar a exatidão e a validade do modelo proposto. Uma abordagem tri-linear para caracterizar a relação força-deslocamento das componentes de uma ligação também é proposta para modelar a resposta estrutural do modelo de conexões. Comparações do atual desenvolvimento com estudos fundamentais realizados anteriormente sobre este tema também foram feitas e comentadas em detalhes.

Uma série de estudos paramétricos e sensitivos foram executados variando os parâmetros principais que influenciam no comportamento estrutural da conexão. A interação força axial-momento fletor também foi cuidadosamente analisada e seu efeito sobre a resposta da ligação foi discutido. O modelo proposto
associado aos estudos analíticos formaram a base para as considerações, que envolvem a presença da força axial, sugeridas neste trabalho para ser incluídas em futuras melhorias de normas de projetos estruturais.

Por fim, além do modelo proposto e devido ao fato de que relativamente poucos resultados experimentais foram relatados investigando o efeito da força axial, um método alternativo é apresentado. Este método estende o leque de aplicações dos dados experimentais disponíveis para gerar curvas características momento-rotação que consideram implicitamente a presença de níveis significativos de tração ou compressão nas vigas adjacentes. A aplicabilidade e validade da metodologia proposta é demonstrada através de comparações com vários ensaios de ligações com placas de extremidade e com placas de base.

**Palavras-Chave**

Estruturas metálicas; Ligações semi-rígidas; Comportamento estrutural de ligações; Interação momento fletor versus força axial; Modelo mecânico; Método das componentes; rigidez rotacional.
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Notation

All symbols used in this thesis are defined as they first appear. For the reader’s convenience, the principal meanings of the commonly used notations are contained in the list below.

**Roman Symbols**

- $a$: modelling parameter
- $a_b$: throat thickness of the beam flange-to-column flange weld
- $a_c$: throat thickness of the column web-to-flange weld
- $a_j$: least-square curve fitting coefficient
- $a_p, a_pf$: throat thickness of the weld between the beam flange and the endplate
- $b_1$: bar 1: rigid bar representing the beam end
- $b_2$: bar 2: rigid bar representing the column flange centreline
- $b_b$: width of the beam cross section
- $b_c$: width of the column cross section
- $b_{eff,c,wc}$: effective width of column web in compression
- $b_{eff,t,wb}$: effective width of beam web in tension
- $b_{eff,t,wc}$: effective width of column web in tension
- $b_j$: least-square curve fitting coefficient
- $bfwc$ (7): beam flange and web in compression
- $b_p$: width of the plate welded to an I or H section
- $bt$ (10): bolts in tension
- $bwt$ (8): beam web in tension
- $c_j$: modelling parameter
- $c_{fb}$ (4): column flange in bending
- $cwc$ (2): column web in compression
cws (1)  column web in shear

cwt (3)  column web in tension

d  lever arm: distance from the loading application centre to the rigid link

d_b  bolt diameter

d_h  bolt head diameter

d_i  system displacements, i=1..4: u_{b1}, \theta_{h1}, u_{b2}, \theta_{h2}

d_n  nut diameter

d_w  washer diameter; width across points of the bolt head or nut

d_{wc}  clear depth of the column web

e  distance from the loading application centre to the beam bottom flange

epb (5)  endplate in bending

e_w  \frac{d_w}{4}

f_{bc,i}^y  yield strength of the joint bolt-row i

f_{cp}^y  joint component yield capacity

f_{cp}^u  joint component ultimate capacity

f_i  force in spring/row i

f_i^y  yield capacity of spring/row i

f_i^u  ultimate capacity of spring/row i

f_{y,bp}  yield strength of the backing plates

f_{y,f}  yield strength of the flange of the I or H section

f_{y,p}  yield strength of the plate welded to the I or H section

f_{u,p}  ultimate strength of the plate welded to the I or H section

f_{y,wc}  yield strength of the beam web

f_{y,wc}  yield strength of the column web

h_b  depth of the beam cross section; beam height

h_c  depth of the column cross section; column height
\( h_{ep} \)  endplate height
\( h_p \)  depth of the plate
\( h_r \)  distance of bolt-row \( r \) from the compressive centre
\( h_i \)  lever arm
\( k \)  non-dimensional stiffness parameter
\( k_1 \)  stiffness coefficient of the column web panel in shear
\( k_2 \)  stiffness coefficient of the column web in compression
\( k_3 \)  stiffness coefficient of the column web in tension
\( k_4 \)  stiffness coefficient of the column flange in bending
\( k_5 \)  stiffness coefficient of the endplate in bending
\( k_7 \)  stiffness coefficient of the beam flange and web in compression
\( k_8 \)  stiffness coefficient of the beam web in tension
\( k_{10} \)  stiffness coefficient of the bolts in tension
\( k_b \)  factor that depends on the frame type
\( k_{bbf} \)  elastic stiffness of the bottom flange of the beam
\( k_{br1} \)  elastic stiffness of bolt-row 1
\( k_{br2} \)  elastic stiffness of bolt-row 2
\( k_{br3} \)  elastic stiffness of bolt-row 3
\( k_{btf} \)  elastic stiffness of the top flange of the beam
\( k_{cp}^r \)  joint component elastic stiffness
\( k_{cp}^p \)  joint component plastic stiffness
\( k_{cp}^u \)  joint component reduced strain hardening stiffness
\( k_{eff,r} \)  effective stiffness coefficient of bolt-row \( r \)
\( k_{eq} \)  equivalent stiffness coefficient
\( k_{i,r} \)  stiffness coefficient representing basic component \( i \) in bolt-row \( r \)
\( k_{k,bbf} \)  elastic stiffness of the compressive rigid link referred to the bottom flange of the beam
\( k_{kq} \) elastic stiffness of the compressive rigid link referred to the top flange of the beam

\( k_t \) elastic stiffness of the tensile rigid link referred to the lever arm

\( k_{tl1} \) elastic stiffness of tensile rigid link 1 referred to bolt-row 1

\( k_{t2} \) elastic stiffness of tensile rigid link 2 referred to bolt-row 2

\( k_{t3} \) elastic stiffness of tensile rigid link 3 referred to bolt-row 3

\( k_{wc} \) reduction factor that accounts for the influence of the vertical normal stress

\( l_{eff} \) effective length

\( l_{ep} \) length of the endplate over the beam flange

\( l_i \) distance from joint spring/row \( i \) to the beam bottom flange centre

\( m \) number of knots (junction of multi-part curve)

\( \bar{m} \) non-dimensional moment resistance parameter

\( n \) shape factor

\( n_r \) total number of bolt-rows in tension

\( nbr \) number of bolt-rows

\( nc \) row/spring component number

\( ns \) system spring/row number

\( n_w \) number of washers

\( r_a \) radius of the fillet of the angle legs

\( r_c \) radius of the fillet of the web-to-flange connection of the column

\( r_i \) effective stiffness of model spring/row \( i \)

\( r_i^e \) elastic effective stiffness of spring/row \( i \)

\( r_i^p \) plastic effective stiffness of spring/row \( i \)

\( r_i^{sh} \) reduced strain hardening effective stiffness of spring/row \( i \)

\( s \) length that depends on if the column section is rolled or welded

\( s_p \) length obtained by dispersion at 45° of the compressive action through the endplate thickness

\( t_a \) angle thickness
\( t_{bp} \) thickness of the backing plates
\( t_{ep} \) thickness of the endplate
\( t_f \) thickness of the flange of an I or H section
\( t_{fb} \) thickness of the beam flange
\( t_{fc} \) thickness of the column flange
\( t_h \) thickness of the bolt head
\( t_n \) thickness of the nut
\( t_p \) thickness of the plate (under the bolt or the nut)
\( t_w \) thickness of the web of an I or H section
\( t_{wb} \) thickness of the beam web
\( t_{wc} \) thickness of the column web
\( t_{wh} \) thickness of the washer
\( u_{bi} \) first bar displacement
\( u_{b2} \) second bar displacement
\( u_i \) absolute displacement of spring/row \( i \) (first bar)
\( ul_i \) absolute displacement of spring/row \( i \) (second bar)
\( z \) lever arm
\( z_{eq} \) equivalent lever arm

**Capital letter**
\( A_s \) tensile stress area of the bolt
\( A_{ic} \) shear area of the column
\( C \) constant that controls the curve slope
\( C_1; C_2; C_3 \) curve-fitting constants
\( C_i \) spring/row \( i \) vertical coordinates
\( CF_M \) correction factor for the moment axis
\( CF_\phi \) correction factor for the rotation axis
\( E \) elastic modulus of structural steel
\( F \) internal loading vector
\( F_{bbf} \) row compressive yield capacity (beam bottom flange)
$F_{c,wc,Rd}$: design resistance of the column web in compression

$F_{c,wc,Rd,br}$: design buckling resistance of the column web in compression

$F_{c,wc,Rd,cr}$: design crushing resistance of the column web in compression

$F_{c,ws,Rd}$: design resistance of the column web in shear

$F_{link}$: rigid link tensile capacity that joins the second bar to the supports

$F_{Rd,\min}$: smallest design resistance of the basic components

$F_{t,Rd}$: design tension resistance of a bolt

$F_{T,Rd}$: design tension resistance of a T-stub flange

$F_{1,wc,Rd}$: design resistance of the column web in tension

$F_{w,Rd}$: effective tension resistance of bolt-row $r$

$I_b$: second moment of the area of the supported beam section

$K$: model stiffness matrix; parameter that depends on the geometrical and mechanical properties of the connection details

$K_b$: ratio of the relative rigidity of all beams at the top of the storey

$K_c$: ratio of the relative rigidity of all columns at the top of the storey

$K_i$: initial stiffness

$K_{ij}$: terms of the system stiffness matrix, $i=1..4$ and $j=1..4$

$K_p$: strain hardening stiffness

$L_b$: span of the supported beam; bolt elongation length taken equal to the grip length (total thickness of material and washers) plus half the sum of the height bolt head and the height of the nut.

$M$: bending moment applied to the joint

$M - \phi$: bending moment versus rotation curve

$M_{\chi\phi(0)}$: moment-rotation curve disregarding the axial force effect

$M_{\chi\phi(N_j)}$: moment-rotation curve considering the axial force $N_j$

$M(\theta)$: moment-rotation relationship

$M^f$: bending moment referred to a 0.05-radian joint final rotation

$M^u$: bending moment that leads the joint to the failure

$M^y$: bending moment that leads the joint to the yield
$M_0$ initial moment; reference moment

$M_{0,p}$ bending moment on the reference $M - \phi$ curve disregarding the axial force at point $p$

$M_{b1,Ed}$ joint internal bending moments

$M_{b2,Ed}$ joint internal bending moments

$M_{br,i}^u$ bending moment that leads to the failure of the joint spring/row $i$, located between the first and second bars

$M_{br,i}^y$ bending moment that leads to the yield of the joint spring/row $i$, located between the first and second bars

$M_{c,Rd}$ design moment resistance of the beam cross-section

$M_d$ design bending moment

$M_{fr,i}^u$ bending moment that leads to the failure of the joint spring/row $i$, located between the second bar and supports

$M_{fr,i}^y$ bending moment that leads to the yield of the joint spring/row $i$, located between the second bar and supports

$M_{int}$ design bending moment considering the axial force $N_i$

$M_j$ upper bound moment in the $j$-th part of the curve

$M_{j,\lim}$ limit bending moment of spring/row $j$, located between the first and second bars

$M_{j,Ed}$ design moment action

$M_{j,Rd}$ design moment resistance of the joint, the design plastic moment resistance of the connected member

$M_{\max}$ design bending moment disregarding the axial force

$M_{N=0}$ bending moment referred to $Mx\phi(0)$ curve

$M_{N,p}$ bending moment on the reference $M - \phi$ curve considering the axial force at point $p$

$M_p$ plastic moment; bending moment evaluated for the target $M - \phi$ curve at point $p$

$M_{pl,Rd}$ design plastic moment resistance of the connected member
$M_u$  ultimate moment; idealised elastic-plastic mechanism moment

$N$ shape parameter obtained through the least square method

$N_u$ axial load that leads the joint to the failure

$N_y$ axial load that leads the joint to the yield

$N_{b1,Ed}$ joint internal normal forces

$N_{b2,Ed}$ joint internal normal forces

$N_i$ axial force present in interaction $i$

$N_{pl}$ beam’s axial plastic capacity

$P$ axial load applied to the joint

$R(\Delta)$ load-deformation relationship

$R_0$ reference load

$S_j$ secant stiffness

$S_{j,ini}$ initial rotational stiffness of the joint

$V_{b1,Ed}$ joint internal shear forces

$V_{b2,Ed}$ joint internal shear forces

$V_{wp,Rd}$ design shear force of the column web in shear

**Greek Symbols**

$\alpha_{1,2,3,4}$ coefficients of Eq. 3.41

$\beta$ transformation parameter which account for the possible influence of the web panel in shear

$\gamma_{M0}$ partial safety factor for resistance of cross-section whatever the class is

$\gamma_{M1}$ partial safety factor for resistance of members to instability assessed by member checks

$\gamma_{M2}$ partial safety factor for resistance of cross-sections in tension to fracture

$\delta u_{b1}$ first bar virtual displacement

$\delta u_{b2}$ second bar virtual displacement

$\delta U$ internal virtual work
\( \delta W \)  
external virtual work

\( \delta \theta_{b1} \)  
first bar virtual rotation

\( \delta \theta_{b2} \)  
second bar virtual rotation

\( \delta \Delta \)  
virtual displacement field

\( \delta \Delta_i \)  
virtual displacement of spring \( i \)

\( \eta_{1,2,3,4} \)  
coefficients of Eq. 3.41

\( \theta \)  
joint rotation

\( \theta^p \)  
joint rotation capacity necessary to develop the joint plastic bending moment

\( \theta^y \)  
joint rotation capacity necessary to develop the joint yield bending moment

\( \theta^f \)  
joint final rotation (assumed to be equal to 0.05 radians)

\( \theta_0 \)  
reference rotation

\( \theta_{b1} \)  
first bar rotation

\( \theta_{b2} \)  
second bar rotation

\( \kappa \)  
stiffness coefficient (Eq. 3.18)

\( \lambda \)  
stiffness coefficient (Eq. 3.18)

\( \bar{\lambda}_p \)  
plate slenderness

\( \mu \)  
stiffness ratio \( S_{j,ini}/S_j \) that accounts for the joint non-linear behaviour

\( \mu^p \)  
plastic stiffness strain hardening coefficient

\( \mu^u \)  
ultimate stiffness strain hardening coefficient

\( \xi \)  
stiffness coefficient (Eq. 3.23)

\( \rho \)  
reduction factor for plate buckling; stiffness coefficient (Eq. 3.18)

\( \nu \)  
stiffness coefficient (Eq. 3.26)

\( \phi_{0,p} \)  
rotation on the reference \( M - \phi \) curve disregarding the axial force at point \( p \)

\( \phi_{Cd} \)  
design rotation capacity

\( \phi_d \)  
design rotation

\( \phi_{Ed} \)  
rotation between connected members of the joint
\( \phi_{\text{int}} \) design rotation considering the axial force \( N_i \)

\( \phi_{\text{max}} \) design rotation disregarding the axial force

\( \phi_{\Phi=0} \) rotation referred to \( M_{\Phi}(0) \) curve

\( \phi_{\Phi,p} \) rotation on the reference \( M-\phi \) curve considering the axial force at point \( p \)

\( \phi_p \) rotation evaluated for the target \( M-\phi \) curve at point \( p \)

\( \varphi \) stiffness coefficient (Eq. 3.27)

\( \chi_1 \) stiffness coefficient (Eq. 3.23)

\( \chi_2 \) stiffness coefficient (Eq. 3.20)

\( \psi \) stiffness coefficient (Eq. 3.20); coefficient that depends on the connection type

\( \omega \) reduction factor to allow for the possible effects of interaction with shear in the column web panel

\( \omega_1 \) stiffness coefficient (Eq. 3.23)

\( \omega_2 \) stiffness coefficient (Eq. 3.20)

**Capital letter**

\( \Delta \) relative displacement field

\( \Delta_i \) spring/row \( i \) relative displacement

\( \Delta_{br,i} \) spring/row \( i \) relative displacement located between the first and second bars

\( \Delta_{fr,i} \) spring/row \( i \) relative displacement located between the second bar and the supports

\( \Delta' \) relative displacement that leads to the yield of the model spring/row \( i \)

\( \Delta'' \) relative displacement that leads to the failure of the model spring/row \( i \)

\( Z \) stiffness coefficient (Eq. 3.25)

\( X \) stiffness coefficient (Eq. 3.22)

\( \Omega \) stiffness coefficient (Eq. 3.22)