6. Referências Bibliográficas

ALMEIDA, C. **Affine Processes, Arbitrage-Free Term Structures of Legendre Polynomials, and Option Pricing**. International Journal of Theoretical and Applied Finance, Vol.8, 2, 2005.

ANG, A.; DONG, S; PIAZZESI, M. **No-Arbitrage Taylor Rules**. Working Paper, Columbia University, 2005.

ANG, A.; PIAZZESI, M. **A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macro and Latent Variables**. Journal of Monetary Economics (50), 745-787, 2003.

ANGELETOS, G; WERNING, I. **Crises and Prices: Information Aggregation, Multiplicity and Volatility**. American Economic Review, 96, 1720-1736, 2006.

BLOOMFIELD, R; O'HARA, M. **Market Transparency: Who wins and Who Loses?**. The Review of Financial Studies, 12, 5-35, 1999.

BLOOMFIELD, R; O'HARA, M. **Can Transparent Markets Survive?**. Journal of Financial Economics, 55, 425-459, 2000.

BONOMO, M., LOWENKRON, A., **A Term Structure Model for Defaultable Bonds with Macro and Latent Variables**. Working Paper. 2008.

BROOKS, R; YAN, D. **London Inter-Bank Offer Rate (LIBOR) versus Treasury Rate: Evidence from the Parsimonious Term Structure Model**. Journal of Fixed Income, 9 (1), 71-93, 1999.

CAMPBELL, J. E SHILLER, R. **Yield Spreads and Interest Rate Movements: A Bird's Eye View**. Review of Economic Studies, (58), pp. 495-514. 1991.

PUC-Rio - Certificação Digital Nº 0410928/CA

CHERNOV, M; MUELLER, P. **The Term Structure of Inflation Expectations**. Working Paper, London Business School, 2008.

COCHRANE, J. E PIAZESSI, M. **Bond Risk Premia**. American Economic Review, (95), pp. 138-160. 2005.

DAI, Q. E SINGLETON, K. **Expectation Puzzles, Time-varying Risk Premia and Affine Models of the Term Structure**. Journal of Financial Economics, (63), pp. 415-441. 2002.

DASGUPTA, A. **Coordination and Delay in Global Games**. Forthcoming in the Journal of Economic Theory. 2006.

DATTELS, P. **The Microstructure of Government Securities Markets**. IMF Working Paper, 95/117. 1995.

DUFFE, G. **Term Premia and Interest Rate Forecasts in Affine Models**. Journal of Finance, (57), pp.405-443. 2002.

FAMA, E. E BLISS, R. **The Information in Long-Maturity Forward Rates**. American Economic Review, (77), pp. 680-92. 1987.

FLEMING, M; REMOLONA, E. **Price Formation and Liquidity in the US Treasury Market: The Response to Public Information**. Journal of Finance, 54, 1901-1915. 1999.

GARCIA, M E OLVIARES, G. **O Prêmio de Risco da Taxa de Câmbio no Brasil**. In: XXI Encontro Brasileiro de Econometria, Belém. Anais. Rio de Janeiro: Sociedade Brasileira de Econometria, 1999, v1, pp. 464-481. 1999.

GRAVELLE, T. **The Microstructure of Multiple-Dealer Equity and Government Securities: How They Differ**. Bank of Canada. Working Paper 2002-9. 2002.

HO, T.S.Y; STOLL, H.R. **The Dynamics of Dealer Market Under Competition**. The Journal of Finance, 38, 1053-1074. 1983.

PUC-Rio - Certificação Digital Nº 0410928/CA

KYLE, A. **Continuous Auctions and Insider Trading**. Econometrica, 53, 1315-1335. 1985.

LEACH, J; MADHAVAN, A. **Price Experimentation and Security Market Structure**. Review of Financial Studies, 6, 373-404. 1993.

LIMA, A. E ISSLER, J.V. **A Hipótese de Expectativas na Estrutura a Termo de Juros no Brasil: Uma Aplicação de Modelos de Valor Presente**. Revista Brasileira de Economia, 57(4), pp:873-898. 2003.

LITTERMAN, R., SCHEINKMAN, J**. Common Factors Affecting Bond Return**s. Journal of Fixed Income 1, 51-61. 1991.

LYONS, R. **Optimal Transparency in a Dealer Market with an Application to Foreign Exchange**. Journal of Financial Intermediation, 5, 225-254. 1996.

LYONS, R. **A Simultaneous Trade Model of the Foreign Exchange Hot Potato**. Journal of International Economics, 42, 275-298. 1997.

MATSUMURA, M., MOREIRA, A.R.B. **Identification of Affine Term Structure Models with Observed Factors: Economic Shocks on Brazilian Yields.** Discussion Paper IPEA. 2007.

MINELLI, E; POLEMARCHAKIS, H. **Information at Equilibrium.** Economic Theory, 21, 573-584. 2003.

NAYK, N; NEUBERGER, A; VISWANATHAN, S**. Trade Disclosure Regulation in Markets with Negotiated Trades**. Review of Financial Studies, 12, 4, 873-900. 1999.

NELSON, C.R; SIEGEL, A.F. **Parsimonious Model of Yield Curves**. Journal of Business, (60), pp: 473-489. 1987.

PUC-Rio - Certificação Digital Nº 0410928/CA

SHOUSHA, S. **Macroeconomic Dynamics and the Term Structure of Interest Rates in Emerging Markets: The Brazilian Case**. Department of Economics, PUC-Rio. 2006.

STAMBAUGH, R. **The Information in Forward Rates: Implications for Models of Term Structure**. Journal of Financial Economics, (22), pp 3-25. 1988.

SUBRAHMANYAM, A. **Risk Aversion, Market Liquidity and Price Efficiency**. Review of Financial Studies, 4, 417-441. 1991.

SVENSSON, L.E.O. **Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994**. Working Paper, NBER. 1994.

VISWANATHAN, S; WANG, J. **Inter-Dealer Trading in Financial Markets**. Journal of Business, 77, 4. 2004.

VOGLER, K. **Risk Allocation and Inter-Dealer Trading**. European Economic Review, 41, 1615-1634. 1997.

PUC-Rio - Certificação Digital Nº 0410928/CA

7. Apêndice

7.1. Apêndice do Capítulo 3

7.1.1.Modelo de Fama-Bliss (1987)

Pelo modelo de Fama e Bliss (1987) cada excesso de retorno é regredido sobre o spread do forward de mesma duração.

$$rx_{t+3}^{n} = \alpha + \beta\left(f_t^n - y_t^1\right) + \varepsilon_{t+3}^{n} \qquad \text{A. 1}$$

As tabela A1 e A2 apresentam os resultados. Os ajustes medidos pelo R², ainda que significativos, foram inferiores aos obtidos pelos modelos irrestrito e restrito discutidos anteriormente. Para nenhum excesso de retorno das maturidades estudadas o modelo de Fama e Bliss (1987) apresentou resultados superiores aos obtidos pelo modelo de restrito de fator único de Cochrane e Piazzesi (2005).

Tabela A 1: Modelo de Fama e Bliss – Curva DI

	α	β	R2	R2 adj.	X^2 NW 9L
6 meses	0,57	63,80	0,247	0,236	5,00
SE - HH 6L	0,61	28,87			0,03
9 meses	0,76	85,15	0,196	0,184	4,65
SE - HH 6L	1,61	41,04			0,03
1 ano	0,75	108,17	0,165	0,152	4,69
SE - HH 6L	2,75	52,40			0,03
1 ano 3m	0,62	134,12	0,149	0,136	4,94
SE - HH 6L	3,92	63,08			0,03
1 ano 6m	0,40	163,27	0,143	0,130	5,31
SE - HH 6L	5,0682	73,241			0,02

Tabela A 2: Modelo de Fama e Bliss – Curva CC

	α	β	R2	R2 adj.	X^2 NW 9L
6 meses	1,60	-21,37	0,009	-0,008	0,85
SE - HH 6L	1,40	9,21			0,36
9 meses	3,94	-29,26	0,010	-0,006	0,70
SE - HH 6L	2,19	24,17			0,40
1 ano	6,12	-28,09	0,006	-0,010	0,41
SE - HH 6L	2,92	25,20			0,52
1 ano 3m	8,25	-28,25	0,005	-0,012	0,31
SE - HH 6L	3,62	11,92			0,58
1 ano 6m	10,44	-31,45	0,004	-0,012	0,32
SE - HH 6L	4,2846	28,189			0,57

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

7.1.2. Modelo de Três Fatores Clássicos: Nível, Inclinação e Curvatura

Para relacionar o fator de previsão de retorno a modelos de curva de juros representamos o mesmo como função de taxas yield ao invés da representação usual por taxas forwards. Os coeficientes equivalentes das taxas yields são dados pela transformação:

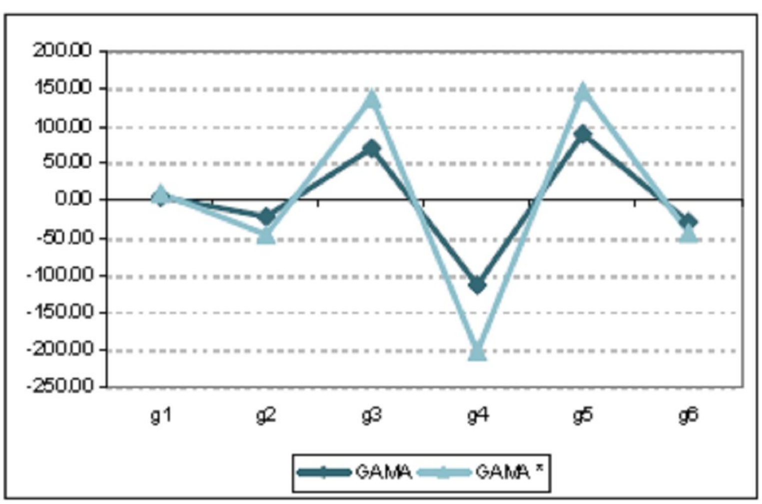
$$\gamma^T f_t = \gamma^0 + \gamma^1 y_t^1 + \gamma^2 \left(\frac{(1+y_t^2)^{6/12}}{(1+y_t^1)^{3/12}} - 1 \right) + \ldots \qquad \text{A. 2}$$

$$\ldots + \gamma^2 \left(\frac{(1+y_t^6)^{18/12}}{(1+y_t^5)^{15/12}} - 1 \right)$$

$$\gamma^{*T} y_t = \gamma^{*0} + \gamma^{*1} y_t^1 + \gamma^{*2} y_t^2 + \ldots + \gamma^{*6} y_t^6$$

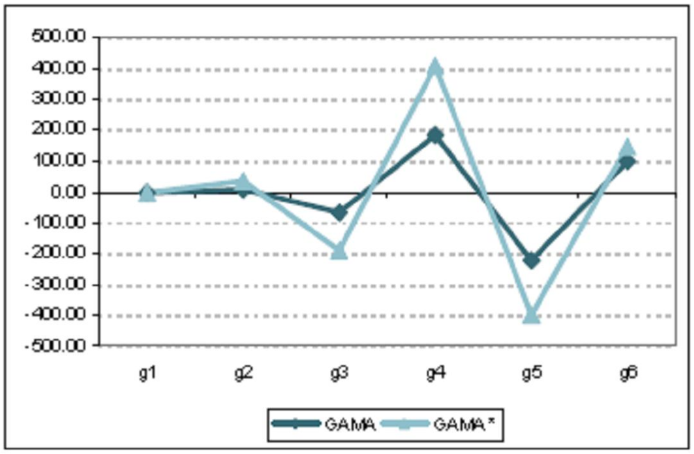
As figuras A1 e A2 apresentam os coeficientes do modelo restrito estimado anteriormente e os coeficientes equivalentes das taxas yield. Os coeficientes equivalentes dos 4º e 5º vértices apresentam valores bem mais significativos do que os obtidos pelos gamas do fator de previsão de retorno do modelo restrito. Ou seja, como discutido anteriormente, as taxas de 1 ano e 1 ano e 3 meses, exatamente a de vértices relativamente mais líquidos, são importantes para a determinação do excesso de retorno. De forma geral, contudo, não podemos descartar de início que os coeficientes das taxas yield sejam diferentes o suficiente a ponto de rejeitá-los como previsores equivalentes do excesso de retorno.

Figura A. 1: Coeficientes Equivalentes Taxas Yield - Curva DI



PUC-Rio - Certificação Digital Nº 0410928/CA

Figura A. 2: Coeficientes Equivalentes Taxas Yield - Curva Cupom Cambial



Como de costume, os três primeiros fatores de uma decomposição em componentes principais das taxas yield explicam quase que integralmente a sua variância. Na curva de DI, na ordem: 96.72, 3.15 e 0.13. Na curva de Cupom Cambial: 98.13, 1.81 e 0.05. Mesma decomposição em componentes principais do fator de previsão de retorno $\gamma^T f_t$ mostra que o primeiro fator, ou nível, também responde por parcela igualmente considerável da variância, sendo na curva de DI 99.58 e na de Cupom Cambial 99.73. Ou seja, o nível das taxas yield e do fator de previsão de retorno são determinantes para a variância de ambas as taxas.

Portanto, se aparentemente os fatores tradicionais da curva de juros nos parecem possivelmente tão bons candidatos a previsores dos retornos excessivos como o fator de previsão de retorno, como podemos comparar as duas abordagens? Se os três primeiros fatores do fator de previsão de retorno respondem pela quase totalidade de sua variância, e são fortemente correlacionados com as três primeiras taxas yield que também respondem pela maior parte da variância das mesmas, então talvez estes fatores pudessem explicar de forma equivalente os retornos excessivos. Por exemplo, como que nível, inclinação e curvatura da curva de juros seriam capazes de prever os retornos excessivos dos mesmos vértices fixos? E se a inclusão dos forwards f(4), f(5) e f(6), presentes no fator de previsão de retorno, é relevante para a previsibilidade do excesso de retorno?

Calculamos os fatores nível, inclinação e curvatura com base no mesmo modelo parcimonioso de Nelson e Siegel. A tabela A3 apresenta os resultados das regressões do excesso de retorno médio sobre diferentes combinações destes três fatores. No caso mais geral, em que incluímos os três e uma constante, o ajuste medido pelo R^2 foi de 35,4% na curva de DI e 12,8% na de Cupom Cambial. Em ambos os mercados, substancialmente abaixo dos ajustes

Tabela A 3: Excesso de Retorno Médio sobre Fatores da Curva de Juros

	C	Nível	Inclinação	Curvatura	R^2	R^2 adj.	X^2 NW(9L)
DI Curve							
N, I & C	-21,18	1,27	-0,81	0,42	0,354	0,323	33,99
SE HH 6L	5,79	0,26	0,20	0,14			
N, I	-17,77	1,02	-0,73	-	0,281	0,258	29,03
SE HH 6L	4,46	0,18	0,20	-			
N	-7,36	0,42	-	-	0,232	0,221	10,68
SE HH 6L	4,12	0,13	-	-			
I	0,80	-	0,40	-	0,154	0,141	6,93
SE HH 6L	2,49	-	0,16	-			
DDI Curve							
N, I & C	-2,43	0,55	-0,05	0,16	0,128	0,084	54,06
SE HH 6L	-5,80	-0,39	-0,06	-0,02			
N, I	-4,75	0,59	0,11	-	0,074	0,043	23,32
SE HH 6L	-6,22	-0,40	-0,01	-			
N	-3,51	0,57	-	-	0,052	0,037	2,22
SE HH 6L	-6,48	-0,40	-	-			
I	4,23	-	0,10	-	0,020	0,003	10,44
SE HH 6L	3,40	-	0,01	-			

obtidos com todos os forwards no fator de previsão de retorno. Esta evidencia reforça a idéia de que os forwards além de 1 ano talvez sejam importantes para a previsibilidade do prêmio de risco à posteriori.

7.2. Artigos Originais dos Capítulos 3 e 4.

Os capítulos 3 e 4 baseiam-se em dois artigos que foram originalmente escritos em inglês. Ou invés de referir cada um ao conjunto de tabelas, gráficos equações e derivações que compõe cada um destes trabalhos, apresenta-se aqui os próprios artigos.

Os artigos apresentados neste apêndice são: "*Currency Risk in Affine Term Strcuture Models*" e *"Trade Disclosure and Strategic Behavior in a Three Stage Inter-Dealer Model".*

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

Currency Risk in an Affine Term Structure Model for Brazil

Sylvio Klein Trompowsky Heck* PUC-Rio

Diogo Abry Guillén† PUC-Rio

January 22, 2009

Abstract

This paper estimates a term structure model of interest rates with two forward looking currency variables for Brazil. We use both expected currency devaluation and currency risk premium as macro factors to model the short rate dynamics and the market price of risk that determinates the entire term structure. We follow Ang and Piazzesi (2003) and impose no-arbitrage restrictions in a Vector Autoregression to study the joint dynamics of yields and essentially forward looking currency factors. The forecasting power of the model improves with the macro factors when compared with a pure latent model. An impulse response analysis shows that the composition of the forward premium impacts the yield curve. Currency devaluations that are of compensated by equivalent increase in the currency risk premium term such that the forward premium remains constant contribute to higher yields in the entire curve. Variance decompositions reveal that currency factors can explain up to 51% of the variation in medium term yields.

Keywords: Forward Premium; Currency Risk Premium; Affine Term Stucture Models; Estimation

1 Introduction

This paper estimates a term structure model of interest rates with two forward looking currency variables for Brazil. We use both expected currency devaluation and currency risk premium instead of the usual inflation and output as macro factors to model the short rate dynamics and the market price of risk that determinates the entire term structure. The variables considered are essentially forward looking and can be easily inverted from widely traded non-deliverable forward contracts and currency expectation surveys collected among

*PhD Candidate, Economics Department PUC-Rio. sheck@econ.puc-rio.br

†PhD Student, Princeton University.

market participants. The currency risk premium will be defined as the excess return between the forward premium and the expected currency devaluation. A positive risk premium confirms the forward premium puzzle documented in several studies, as in Frankel and Engel (1984), Bekaert and Hodrick (1993) and Backus, Gregory and Telmer (1993), just to new a few.

The forward rates and market expectations reveal most of the information available in the economy, as well as the relevant expectations used to determinate the interest rate path. We view the short rate equation as a simplified uncovered interest parity (UIP), where the one month nominal rate reacts to one year ahead expected currency devaluation and currency risk premium. Results show that those forward looking factors can explain much of the short rate trajectory and improve a lot the forecasts both in-sample and out-of-sample for every maturity in the curve when compared with a model with only unobserved factors.

There are many advantages in using the proposed simplified UIP instead of a Taylor rule. First, we handle daily data obtained in widely traded financial instruments. Avoiding the use of output variables we drop a major estimation limitation of traditional models related to the frequency of the data. Such limitation may not be critical if there is no intention to use and evaluate the model regularly, but is certainly a serious issue if you want to price and follow movements in the curve frequently. Second, the UIP here is essentially forward-looking, such as the nature of the term structure that we want to evaluate. Modeling the short term rate as simple function of expected exchange rate movements and currency risk premium allows us to study the impact of those in the entire curve.

This work is related to the no-arbitrage macro literature beginning with Ang and Piazzesi (2003). We follow the literature and impose no-arbitrage restrictions in a Vector Autoregression to describe the joint dynamics of swap Pre x DI yields in Brazil and the currency variables. The option for a traditional macro-finance framework when we use financial data brings concerns about the consistency of the model. As in Ang e Piazzesi (2003), we prefer to see the currency variables as macro factors and for simplification assume that interest rates have no impact on our currency factors. The limitation to consider the contemporaneous correlations between macro and latent factors is a common issue in most studies in this literature. Complete macro models including Taylor rules, Phillips and IS curves in a no-arbitrage setup have been formulated to capture the cross effects among all the variabels included. The estimation of many parameters in a highly non-linear system is a serious issue in structural models. Rudebusch and Wu (2004), Bekaert, Cho and Moreno (2005), Hordahl, Tristani and Vestin (2006) and Hordahl and Tristani (2007) are example of studies that use structural macro frameworks in arbitrage free models. We discuss later in this article an alternative specification to account for the joint dynamics of interest rates and currency expectations in a consistent framework, as proposed by Chernov and Mueller (2008), but prefer to leave this extensions for a future research.

The model we propose here shows that shocks to expected devaluation and currency risk premium impact the entire term structure. Currency spot move-

PUC-Rio - Certificação Digital Nº 0410928/CA

ments that do not change the forward premium, such that all the variation is assumed to be compensated by an equivalent movement in the currency risk term, have a positive impact on the curve. We view this result as an increase in currency risk perception affecting the curve term premium.In the Brazilian case, Shousha (2006) demonstrates that output and inflation do not appear to explain much of the variance of yields in the curve. He shows, on the other side, that nominal spot currency capture as much as 41% of this variance.

This paper is divided as follows. Section 2 describes the data. Section 3 proposes an affine representation for the UIP used to model the short rate. Section 4 presents the general affine term structure model. Section 5 estimates the model using Chen and Scott (1993) while Section 6 presents the results and evaluate the dynamics of the model. Section 7 concludes.

2 Data

2.1 Yields

We use one month swap PrexDI as the short rate and 3, 4, 6, 12, 18, 24 and 30 months equivalent swaps for the entire term structure. Swap PrexDI is an interest rate swap contract where one party borrows in a fixed rate and pays a cash flow of overnight interbank deposit rates. The rate is quoted like zero coupon bound and the curve is liquid for several different maturities. Contracts are traded over the counter in local markets and usually registered at CETIP (Central de Custodia e de Liquidação Financeira de Títulos). Yields are continuously compounded in 252 business days basis. All data compiled in this paper is from Bloomberg. Figure 1 plots the yields from 24-apr-2004 to 01-feb-2008 and tables 1 and 2 show the central moments and autocorrelation.

$$Figure 1$$

$$Table 1$$

$$Table 2$$

Yields of different maturities have a similar pattern, but in periods of higher uncertainty longer maturities stress more, as expected. Longer maturities yields present more pronounced skewness and kurtosis, the opposite of what is documented for United States. Another interesting aspect is related to the first two moments of brazilian yields. Mean and standard deviation decreases as maturities rises, also contrary to what is usually observed in developed countries.

2.2 Forward Premium, Expected Devaluation and Currency Risk Premium

We will use currency variables to model the short rate dynamics and the market price of risk that will determinate the term structure of interest rates. We take one year Non-deliverable Forward (NDF) Brazilian Reais contracts traded in US

PUC-Rio - Certificação Digital Nº 0410928/CA

markets. NDFs are over the counter forward contracts in which counterparties settle the difference in a specific future date between the spot price and the contracted price. There is no physical delivery. By absence of arbitrage, the forward currency rate should equal the interest rate differential between deposits in two currencies, plus a premium for credit and currency risks between issuers and currencies. Furthermore, that interest rate differential should reflect the expected currency devaluation of the higher yield deposit, otherwise one betting against that currency movement would be willing to buy the currency. We assume that this parity holds and use currency expectations to decompose the forward premium into expected devaluation and currency risk. In logs we can write:

$$fp_t^{t+k} = E_t\left(\Delta e\right) + crp_t^{t+k} \tag{1}$$

The Central Bank of Brazil (BCB) evaluates weekly currency expectation surveys collected among market participants. We use those to construct a daily series of expected currency rate exactly one year ahead for each day in our sample. Figure 2 plots the series in the forward premium equation above.

$Figure 2$

From Figure 2, we see that currency risk premium and expected currency devaluation from Central Bank surveys have opposite behavior, risk premium rises when there is appreciation expectation.

3 Short Rate Dynamics

We assume that movements in short rate r_t can be explained by a modified interest rate parity condition where the 1 month nominal rate reacts to one year ahead expected currency devaluation and currency risk premium. Both variables keep a significant correlation with the broad macroeconomic environment and reflect the attractivity of capital flows that move the entire term structure:

$$r_t = a_0 + a_1 E_t\left(\Delta e\right) + a_2 crp_t^{t+k} + e_t \tag{2}$$

This setup provides, in our view, two main advantages when compared to usual Taylor rules for the short rate equations. First, markets provide daily liquid instruments that trade all the variables evolved. This allows us to estimate our model in a daily basis, while inflation indices and output measures are at best available monthly. Second, our variables capture a great deal of future expectation and so we can drop lagged arguments in the right hand side, the way forward looking versions of Taylor rule usually handle the problem. Such simplification eliminates a considerable number of variables that need to be estimated in an already highly non-linear model.

The failure of traditional uncovered parity conditions is usually explained by a risk premium term that capture differences mainly between issuers' credit risk. For the purpose of this article we let the residual e_t in (2) be treated as a latent factor X_t^u:

$$e_t = a_5 X_t^u \tag{3}$$

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

We can combine equations (2) and (3) to write an extended short rate equation:

$$r_t = \delta_0 + \delta_1 E_t\left(\Delta e\right) + \delta_2 crp_t^{t+k} + \delta_3 X_t^u \tag{4}$$

3.1 OLS Estimation of the Short Rate Equation

Independence assumption of the macro factor and the latent factor in (4) allows us to estimate parity equation coefficients as in (2). Finally, in Table 3 we report the results. Expected currency devaluation and currency risk premium are highly significant to explain one-month Brazilian yield. Residuals correlation suggest that usual "level" factor should reappear in our setup.

$Table 3$

The estimates for all coefficients are highly significant and have an intuitive interpretation. The higher is the expected currency devaluation, the higher will be the Brazilian 1-month yield. The same intuition can be used to explain the positive sign of the coefficient of currency risk.

4 Term Structure Model

4.1 State Dynamics

We assume that both macro and latent factor $X_t = (macro; latent)$ follow a first order Gaussian VAR:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \tag{5}$$

To impose independence between macro and latent factor we write Φ and as diagonal $5x5$ matrices. As noted by And and Piazzesi (2003), this independence assumption will be a drawback of the model. In our formulation this simplification means that interest rates have no impact on the currency variables we use. One way to consider contemporaneous correlations between macro and latent factors could be done by freeing up the companion matrix Φ. Another way would be considering a complete model to accomodate the joint dynamics of both interest rates and the currency factors. Chernov and Mueller (2008) use survey forecasts of inflation and construct private sector expectations of inflation that enter a modified no-arbitrage term structure model. A variation of their model using currency expectation instead of inflation could be proposed. For the sake of tractability we let those extensions for a future research.

4.2 No-Arbitrage Restrictions

To impose no-arbitrage restriction we assume the existence of an equivalent martingale measure Q under which a zero-coupon bond S_t can be written as $S_t = E_t^Q\left(\exp(-r_t)S_{t+1}\right)$.Under Q the current price of a zero-coupon will be discounted present value using a risk-free rate. To recover from the equivalent measure Q to real data generating measure we take the Radon-Nikodym derivate ξ_{t+1} such that:

$$E_t^Q\left(S_{t+1}\right) = E_t\left(\xi_{t+1}S_{t+1}\right)/\xi_t$$

Assuming that ξ_{t+1} follows a log-normal process we model as in Ang and Piazessi (2003) the nominal pricing kernel that prices all nominal assets in the economy as a function of the macro variables:

$$m_{t+1} = \exp\left(-r_t\right)\exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right) \quad (6)$$

where r_t follows the short rate equation $r_t = \delta_0 + \delta' X_t$ and the market price of risk λ_t can be parameterized as an affine process:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \quad (7)$$

Shocks to the macro variables in X_t affect the term structure through both the short term rate r_t and market price of risk λ_t. The return R_{t+1} of any nominal bond will be:

$$E_t\left(m_{t+1}R_{t+1}\right) = 1$$

Particularly, bond prices will be recursively determined by:

$$p_t^{n+1} = E_t\left(m_{t+1}p_{t+1}^n\right)$$

Duffie and Kahn (1996) show that bond prices are exponential affine function of the state variable X_t and can be written in the form:

$$p_t^n = \exp\left(A_n + B_n' X_t\right)$$

where $\overline{A}_n$ and $\overline{B}_n$ follow the difference equations:

$$\begin{aligned} A_{n+1} &= A_n + B_n'\left(\mu - \Sigma\lambda_0\right) + \frac{1}{2}B_n'\Sigma\Sigma' B_n - \delta_0 \\ B_{n+1}' &= B_n\left(\Phi - \Sigma\lambda_1\right) - \delta' \end{aligned}$$

5 Estimations

We follow Chen and Scott (1993) and assume that as many yields as unobservable factors are measured without error. In particular, we let 2, 6 and 24 months be considered without error. To handle the non-linearity of the system and help achieve convergence we also follow the literature and adopt a two-step procedure. In the first step we estimate the macro coefficients in both the short rate dynamics (4) and the state dynamics VAR. In the second step we estimate all other parameters holding the ones collected in the first step fixed. We assume independence between the macro variables and the non-observed factors such that Φ, Σ and λ_1 will be block diagonal.

PUC-Rio - Certificação Digital Nº 0410928/CA

For a given parameter $\theta = (\mu, \phi, \Omega, \delta_0, \delta_1, \lambda_0, \lambda_1)$ we invert from the yields observed without error to solve for X_t^u, and then from the yields measured with error to collect e_t. Finally, to proceed with the maximum likelihood estimation of the remaining parameters in θ we define f_x and f_e as the normal density functions of the state variables X_t^u and e_t respectively. The joint likelihood $L(\theta)$ of the observed data on zero coupon yields and the macroeconomics variables observed is given by:

$$L(\theta) = \prod_{t=2}^{T} f\left(y_t, X_t^o | y_{t-1}, X_{t-1}^o\right)$$

Applying logs in both sides:

$$\begin{aligned}
\log\{L(\theta)\} &= \sum_{t=2}^{T} f\left(y_t, X_t^o | y_{t-1}, X_{t-1}^o\right) \\
&= \sum_{t=2}^{T} -\log|\det(J)| + \log f_x\left(X_t^o, X_t^u | X_{t-1}^o, X_{t-1}^u\right) + \log\left(f_e(e)\right) \\
&= -(T-1)\log|\det(J)| - \frac{(T-1)}{2}\log\left(\det\left(\Sigma\Sigma'\right)\right) \\
&\quad -\frac{1}{2}\sum_{t=2}^{T}\left(X_t - \mu - \phi X_{t-1}\right)'\left(\Sigma\Sigma'\right)\left(X_t - \mu - \phi X_{t-1}\right) \\
&\quad -\frac{(T-1)}{2}\log\sum_{i=1}^{4}\sigma_i^2 - \frac{1}{2}\sum_{t=2}^{T}\sum_{i=1}^{4}\frac{e_i^2}{\sigma_i^2}
\end{aligned}$$

where σ_i^2 is the variance of the ith measurement error and the Jacobian term is given by $J = \begin{pmatrix} I & 0 & 0 \\ B^o & B^u & B^e \end{pmatrix}$

6 Results

Table 4 presents the results of the macro model. The coefficients of latent factors in (4) can be seen at the top of the table. We investigate the correlation between the three unobserved factors and yield transformations identified with usual level, slope and curvature of the yield curve as proposed by Litterman and Scheinkman (1991). The first unobserved does not seems to be linked to a level factor. The correlation is below 10%, contrary to usual values for the US curve of above 90%. The first factor, however, seems to be more correlated to a slope factor (71%). The second latent factor, on the other side, has a 63% correlation with the level transformation. The third unobserved series has a negative 92% correlation with the curvature transformation.

$Table 4$

Figure 3 shows the normalized weights B_n estimated for different maturities n. These weights B_n represent the contribution of the various factors to yield

PUC-Rio - Certificação Digital Nº 0410928/CA

curve. As can be seen, the currency factors contribute with a great extent of the short end of the curve. Those factors decay as we move across the curve, increasing the contribution of the second latent factor.

$Figure3$

6.1 Impulse Response

To investigate the effects of each factor in the curve we perform impulse responses in each variable in the complete model Vector Autoregression. The magnitude of the currency coefficients in the short end of the curve suggests that shocks to those variables should affect shorter yields greater than longer yields. Figures 4 and 5 show the impulse responses of one standard deviation on expected currency devaluation and currency risk premium. The standard deviation of expected currency devaluation is 4.31%, slightly higher than the 3.84% observed for the currency risk series. Shocks of those magnitudes induce significant effects on the curve. Shorter 1 month yields increase by 3% after shocks in the first macro variable that decay slowly trough time. After 90 days the curve is still over 2% above its level before the shock. Six month and one year yields do also react quite strong to shocks on the expected currency devaluation, but less than what is observed for the shorter yield. The dynamic after one standard deviation shock on the currency risk premium reveals a similar pattern.

$figure4$

$figure5$

An interesting way to investigate the effects of shocks on the currency variables is to simulate a simultaneous but oppossite shock on both. We can simulate a change in the composition of the forward premium that takes place when a currency spot devaluation, for example, occurs while the expected currency 1 year ahead remains steady. If that happens by definition the currency risk premium would have to increase by the same extent to compensate the lower expected currency devaluation. Figure 2 shows tha this is something quite common in the sample considered, as agents takes longer to change their forecasts and keep on trading currency NDFs on the same levels. Figure 6 shows the impulse response of 100 basis point decrease on the expected currency devaluation that is followed by and equivalent increase in the currency risk premium. Results suggest that this would have a positive impact on the curve. Yields would increase due to this increase in risk perception, up to 7 bps in the shorter yield after 120 days.

$figure6$

6.2 Variance Decomposition

Table 5 reports the proportion of the forecast variance attributable to each factor. As documented by Shousha (2006), the currency is able to explain a

PUC-Rio - Certificação Digital Nº 0410928/CA

considerable extent of the variance of several maturities. We confirm those findings here, but do also offer a deeper understanding of what can be explained by two currency factors. The proportion that can be credited to expected devaluation is greater for the 3 month yield (33,6%) and looses significance as we move to the end of the curve. The contribution of the currency risk premium is similar, reaching 18% of the variance of the 4 month yield. Together those variables explain up to 51% of the variance of the 3 and 4 month yields. The results do not change much as we compare 1 and 60 days forecast out-of-sample.

$table5$

6.3 Forecast

We compare the predictive power of the macro model here proposed and a pure latent model. The macro model reveals better results both in and out-of-sample. We considered as in-sample the model estimated for the entire 972 days in our data. As out-of-sample we consider the 60 days after the end of our sample, for which we do also have the realized data.

Figure 7 shows the fit of estimated curves for two selected dates. Figures 8 and 9 compare the forecasted one year yield for approximately 1 year at the end of our sample and for 60 days out-of-sample. It can be easily seen that the latent model fails to capture the upward sloping at the end of the sample and across the out-of-sample interval. The macro model presents a better fit and successfully capture the upward trajectory of the one year yield after the end of the sample. We test different sub-samples in our data and perform several forecasts. The ability of the model to outperform the latent model is confirmed, although the macro model fails to capture some others downward and upward trends in the sample. Therefore, we still rely on the macro model here as a better predictor when compared with the canonical model, but would be careful when considering its ability to capture shifts in rates trajectory.

$Figure7$

$Figure8$

$Figure9$

To compare the forecasts of each model we calculate the Mean Squared Error (MSE). The mean squared error of each model is constructed as the average of the mean squared error of each yield[1]. Table 6 presents the results. The macro model by far reveals much better results.

$Table6$

[1] Three maturities have zero in-sample mean squared error by construction (Chen-Scott)

PUC-Rio - Certificação Digital Nº 0410928/CA

7 Conclusion

This paper proposes an affine term structure model with observable currency factors for Brazil. The option for currency variables instead of traditional output and inflation is due to the incapacity of those to explain a significant proportion of the variance of yields in the curve. The currency appears as a natural candidate because it is widely traded and reveals much of the mood with the general macro environment. We focus on two currency factors: the expected currency devaluation and the currency risk premium inverted from the first and the NDF contracts traded in US over the counter markets. We follow Ang and Piazzesi (2003) and use these currency variables to estimate a daily model with additional 3 unobserved latent factors.

Results show that shocks to both currency factors contribute to significant changes in the curve. The composition of the forward premium is also relevant to the shape of the curve. Yields appear to be higher as currency risk premium increase after an equivalent reduction in the expected currency devaluation followed by a spot devaluation. A significant proportion of the variance of the yields in the curve can be attributed to the currency factors. The expected devaluation can explain as much as 33,6% of the 3 month yield variance, while the currency risk explain 18% of the 4 month yield. Together both factors account for up to 51% of the variance of 3 and 4 months yields.

References

[1] Ang, A., Piazzesi, M., 2003. A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macro and Latent Variables. Journal of Monetary Economics (50), 745-787.

[2] Backus, D. K., Gregory, A., Telmer, C.I. 1993. Accounting for Forward Rates in Markets for Foreign Currency. Journal of Finance, 48, 1887-1908.

[3] Bekaert, G., Cho, S., Moreno, A. 2005. New-Keynesian Macroeconomics and the Term Structure. Working Paper, 11340, NBER.

[4] Bekaert, G., Hodrick, R. 1992. On Biases in the Measurement of Foreign Exchange Risk Premiums. Journal of International Money and Finance, 12, 115-138.

[5] Bonomo, M., Lowenkron, A., 2008. A Term Structure Model for Defaultable Bonds with Macro and Latent Variables. Working Paper.

[6] Chernov, M, Mueller, P. 2008. The Term Structure of Inflation Expectations. Working Paper, Columbia University.

[7] Frankel, J., Engel, C. 1984. Do Asset Demand Functions Optimize over the Mean and Variance of Real Returns? A Six Currency Test. Journal of International Economics, 17, 309-323.

PUC-Rio - Certificação Digital Nº 0410928/CA

[8] Han, B, Hammond, P. 2003. Affine Models of the Joint Dynamics of Exchange Rates and Interest Rates. Working Paper, University of Calgary.

[9] Hordahl, P., Tristani, O. 2007. Inflation Risk in the Term Strcuture of Interest Rates. Working Paper, 734, ECB.

[10] Hordahl, P., Tristani, O., Vestin, D. 2006. A Joint Econometric Model of Macroeconomic and Term Structure Dynamics. Journal of Econometrics, 131, 405-444.

[11] Litterman, R., Scheinkman, J., 1991. Common Factors Affecting Bond Returns. Journal of Fixed Income, 1, 51-61.

[12] Matsumura, M., Moreira, A.R.B, 2007. Identification of Affine Term Structure Models with Observed Factors: Economic Shocks on Brazilian Yields. Discussion Paper, IPEA.

[13] Rudebusch, G.D., Wu, T. 2004. A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy.Working Paper, FED San Francisco.

[14] Shousha, S., 2006. Macroeconomic Dynamics and the Term Structure of Interest Rates in Emerging Markets: The Brazilian Case. Department of Economics, PUC-Rio.

PUC-Rio - Certificação Digital Nº 0410928/CA

Table1: Central Moments

	Mean	Standard-Deviation	Skewness	Kurtosis
BRZ 1-month yield	15.44	2.89	-0.05	-1.29
BRZ 2-month yield	15.40	2.91	-0.04	-1.31
BRZ 3-month yield	15.37	2.93	-0.04	-1.34
BRZ 4-month yield	15.34	2.93	-0.06	-1.38
BRZ 6-month yield	15.30	2.93	-0.08	-1.43
BRZ 1-year yield	15.18	2.84	-0.18	-1.52
BRZ 1.5-year yield	15.13	2.75	-0.23	-1.47
BRZ 2-year yield	15.08	2.69	-0.22	-1.37
BRZ 2.5-year yield	15.05	2.68	-0.17	-1.21
Expec. Currency Devaluation 12m	0.00	4.31	0.23	-0.15
Currency Risk 12m	0.00	3.84	1.34	2.12
Lat1	9.16	25.99	-0.32	-0.45
Lat2	39.04	9.33	0.65	0.05
Lat3	-109.13	57.39	-0.10	-0.24

PUC-Rio - Certificação Digital Nº 0410928/CA

Table2: Autocorrelation

	Autocorrelation (lag1)	Autocorrelation (lag5)	Autocorrelation (lag10)	Autocorrelation (lag50)	Autocorrelation (lag100)
BRZ 1-month yield	1.00	1.00	1.00	0.97	0.87
BRZ 2-month yield	1.00	1.00	1.00	0.97	0.88
BRZ 3-month yield	1.00	1.00	1.00	0.97	0.88
BRZ 4-month yield	1.00	1.00	1.00	0.97	0.89
BRZ 6-month yield	1.00	1.00	1.00	0.97	0.90
BRZ 1-year yield	1.00	1.00	0.99	0.97	0.92
BRZ 1.5-year yield	1.00	0.99	0.99	0.96	0.91
BRZ 2-year yield	1.00	0.99	0.98	0.95	0.89
BRZ 2.5-year yield	1.00	0.99	0.98	0.94	0.88
Expec. Currency Devaluation 12m	0.98	0.89	0.83	0.61	0.43
Currency Risk 12m	0.97	0.86	0.76	0.42	0.22
Lat1	0.98	0.93	0.89	0.76	0.48
Lat2	1.00	0.98	0.95	0.84	0.62
Lat3	0.99	0.98	0.96	0.75	0.40

Table3: Short Rate Dynamics - OLS

Dependent variable: BRZ 1-month yield Sample: 04/24/2004 - -2/29/2008 (972 observations)	
Constant	15.44 (402.74)
Expec. Currency Devaluation 12m	0.72 (68.53)
Currency Risk 12m	0.44 (36.89)
Adjusted R-squared	0.83
F-statistic	2348.05

in parenthesis, t-statistic

PUC-Rio - Certificação Digital Nº 0410928/CA

Table4: Complete Model Estimates

Short Rate Equation: Three Latent Factor

$\delta_3 =$ 0,0807 $\delta 4 =$ 0,0063 $\delta 5 =$ -0,0001

State Dynamics

	Φ				
	Exp.Curr	Curr.Risk	Lat1	Lat2	Lat3
Exp.Curr	1,0000	-0,0096	0,0000	0,0000	0,0000
Curr.Risk	0,0013	0,9858	0,0000	0,0000	0,0000
Lat1	0,0000	0,0000	0,9634	0,0180	0,0125
Lat2	0,0000	0,0000	0,0057	0,9988	-0,0022
Lat3	0,0000	0,0000	0,0100	0,0099	0,9973

Price of Risk

	λ_0	λ_1				
		Exp.Curr	Curr.Risk	Lat1	Lat2	Lat3
Exp.Curr	-0,0006	0,1808	0,0001	0,0000	0,0000	0,0000
Curr.Risk	-0,0016	0,5934	0,0002	0,0000	0,0000	0,0000
Lat1	0,0045	0,0000	0,0000	0,0055	-0,0034	-0,0002
Lat2	0,0029	0,0000	0,0000	-0,0098	0,0009	-0,0010
Lat3	-0,0011	0,0000	0,0000	-0,0025	0,0000	0,0001

Table 5: Variance Decomposition (h-steps ahead forecasts)

		Yields (months to maturity)								
	h	**1**	**2**	**3**	**4**	**6**	**12**	**18**	**24**	**30**
Expec. Currency Dev. 12m	1	27,73%	32,79%	33,64%	32,70%	29,22%	18,07%	10,29%	5,91%	3,61%
	60	28,49%	33,35%	33,99%	32,91%	29,30%	18,07%	10,29%	5,91%	3,61%
Currency Risk 12m	1	9,40%	14,38%	16,99%	17,98%	17,63%	11,97%	7,02%	4,09%	2,52%
	60	9,58%	14,50%	17,03%	17,96%	17,53%	11,88%	6,97%	4,06%	2,50%
Lat1	1	58,97%	41,45%	31,17%	25,61%	20,85%	18,83%	18,76%	18,09%	16,98%
	60	57,78%	40,21%	30,03%	24,58%	19,94%	17,96%	17,90%	17,25%	16,18%
Lat2	1	1,37%	4,02%	7,11%	10,36%	17,29%	39,74%	58,37%	70,23%	76,80%
	60	1,43%	4,14%	7,27%	10,55%	17,54%	40,21%	59,05%	71,03%	77,62%
Lat3	1	2,54%	7,36%	11,09%	13,34%	15,01%	11,40%	5,56%	1,68%	0,10%
	60	2,71%	7,80%	11,68%	13,99%	15,68%	11,88%	5,79%	1,75%	0,10%

Table 6: Forecast Comparison

RMSE Criteria		
	In-Sample	60-step ahead
Macro Model	0,0560	0,2174
Latent Model	1,149	4,920
MAD Criteria		
	In-Sample	60-step ahead
Macro Model	0,1782	0,3414
Latent Model	0,453	1,118

Figure 1: Brazilian PrexDI Swap Rates

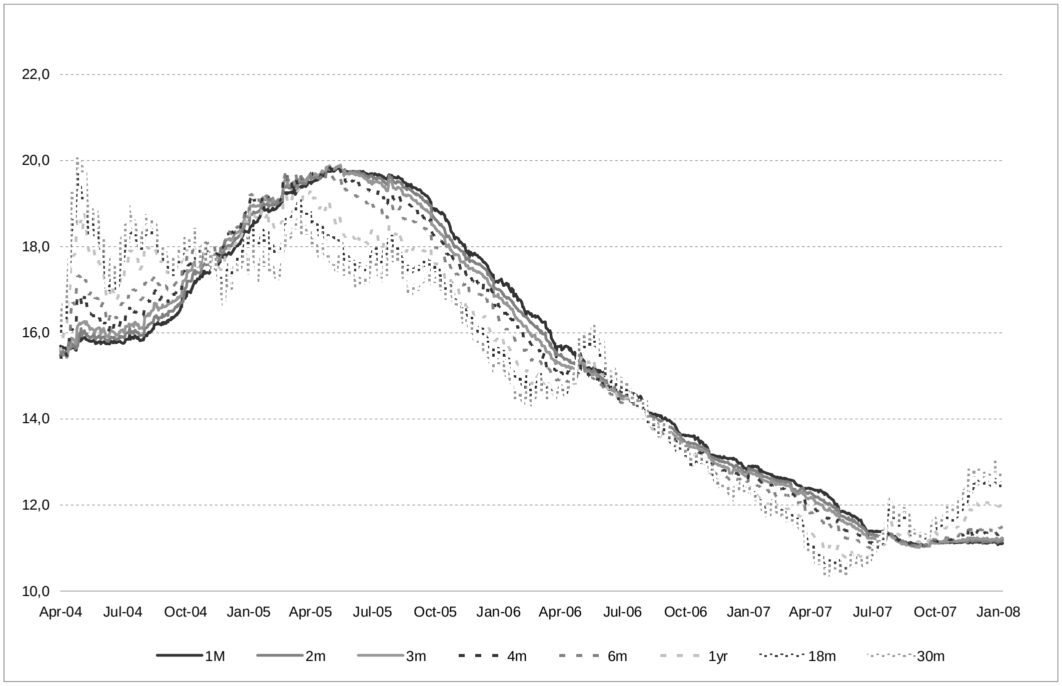
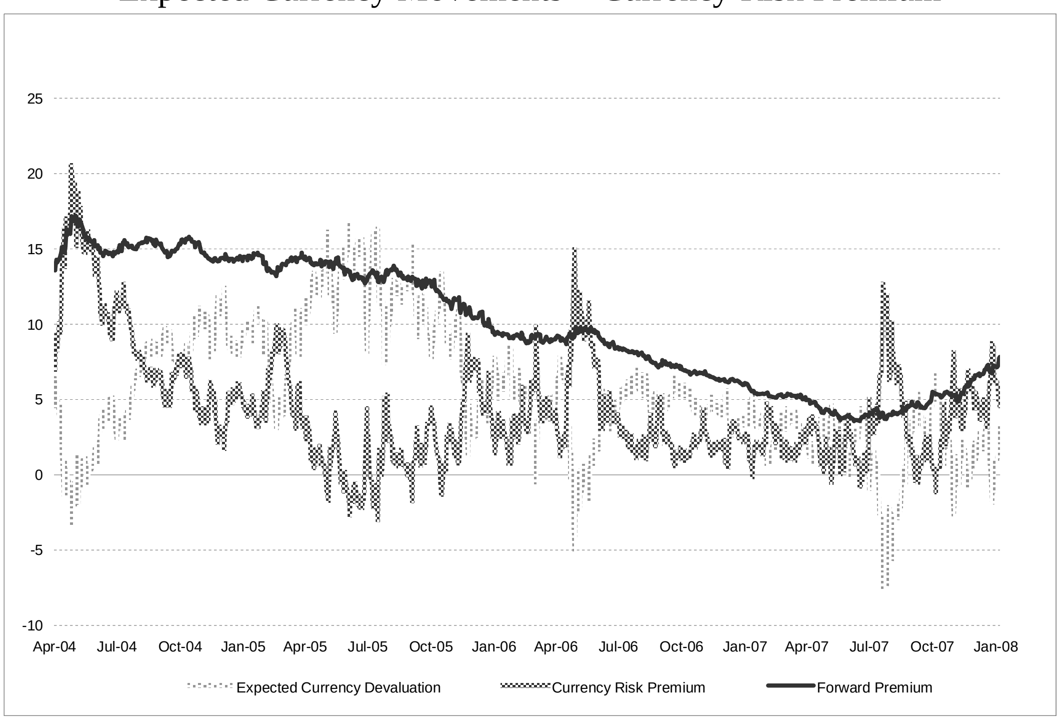
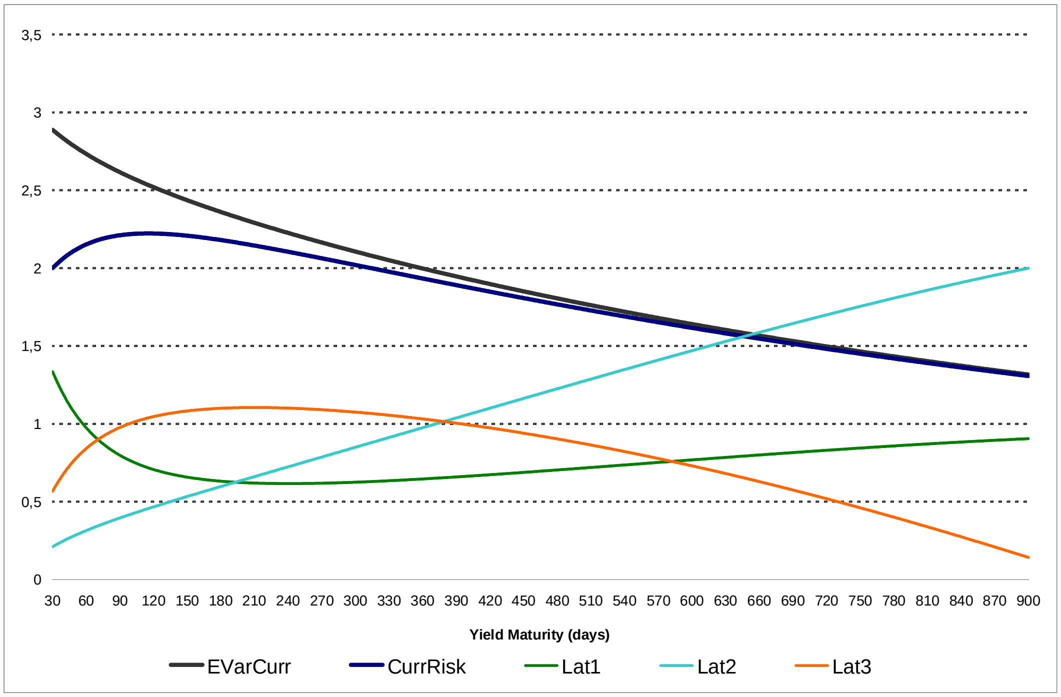


Figure2: Forward Premium
Expected Currency Movements + Currency Risk Premium



PUC-Rio - Certificação Digital Nº 0410928/CA

Figure3: State Dynamics Equation Normalized Coefficients
(from 30 to 900 days)



PUC-Rio - Certificação Digital Nº 0410928/CA

Figure 4: Impulse Response
+1dp E(e)

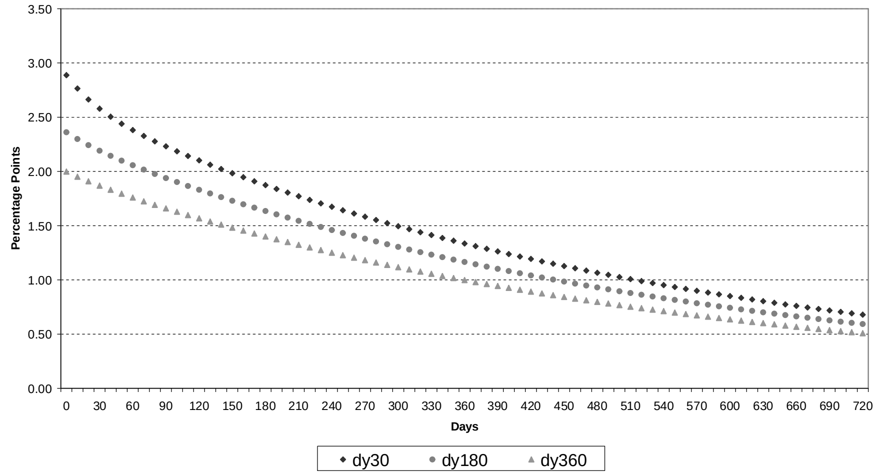


Figure 5: Impulse Response
+1dp CRP

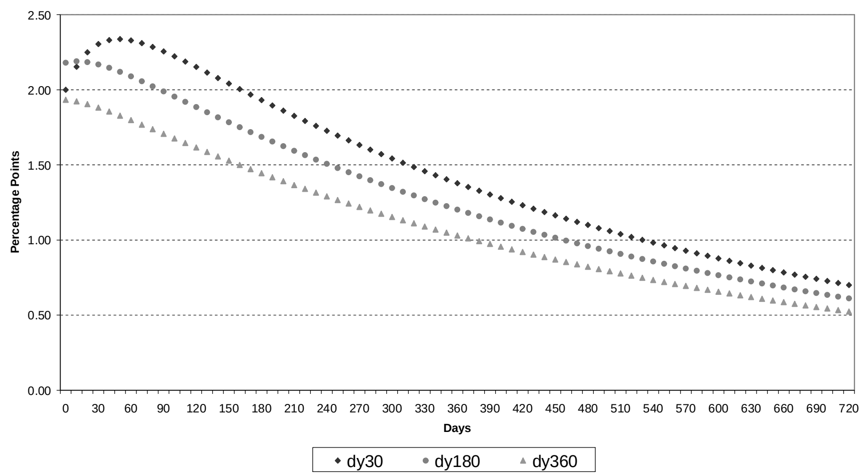
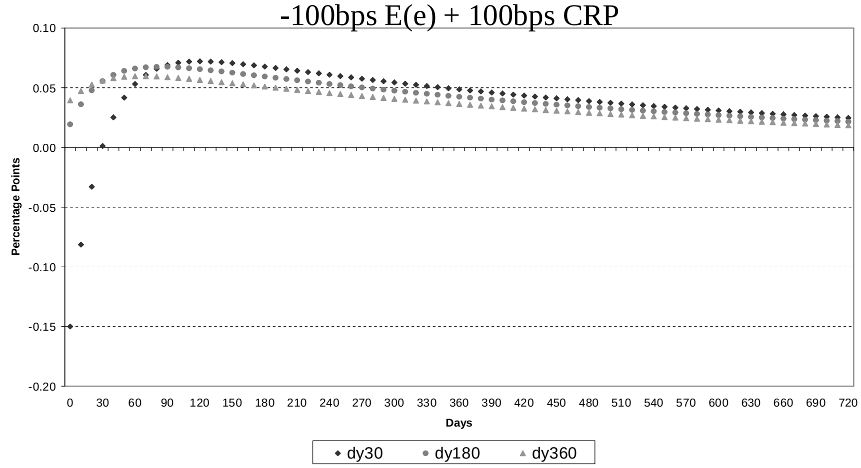
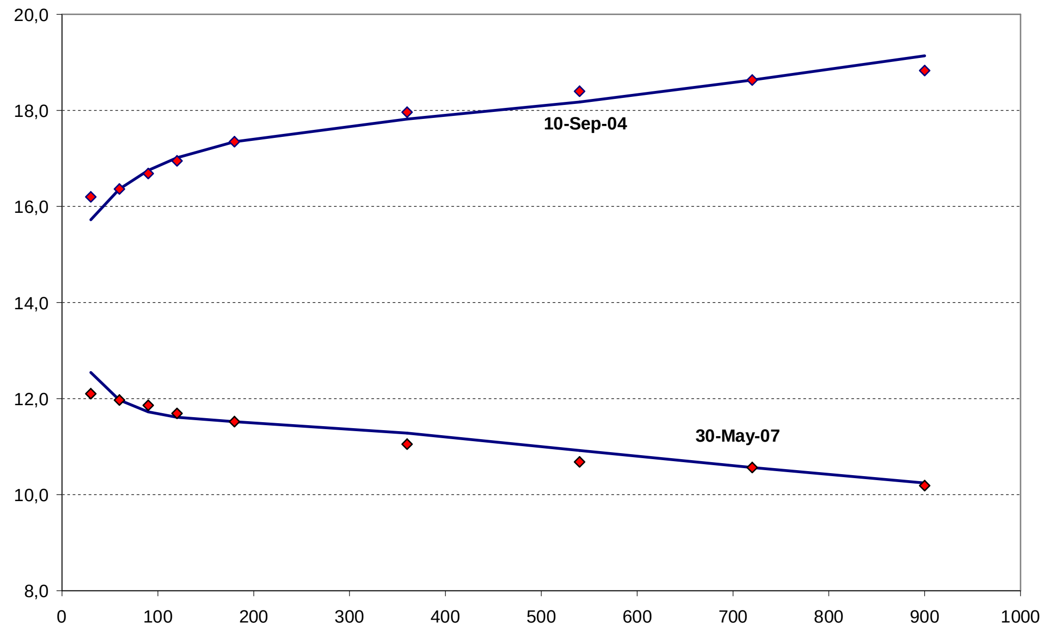


Figure 6: Impulse Response
-100bps E(e) + 100bps CRP



PUC-Rio - Certificação Digital Nº 0410928/CA

Figure 7: In-Sample Curve Estimation



PUC-Rio - Certificação Digital Nº 0410928/CA

Figure 8: Latent Model - Forecast Out-of-Sample (60 days)
360 days yield

Figure 9: Macro Model - Forecast Out-of-Sample (60 days)
360 days yield

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

Trade Disclosure and Strategic Behavior in a Three Stage Inter-Dealer Model*

Sylvio Klein Trompowsky Heck†
PUC-Rio

January 21, 2009

Abstract

The paper examines an inter-dealer model where the disclosure of information is modeled as noisy informative signals. We consider an additional inter-dealer channel of negotiation and let an informed market-maker optimize offsetting his position trough both rounds of trade. The results challenge market regulators policy recommendations on more disclosure of information and the risk sharing measured by the volume traded in the first round decreases with the precision of information in both inter-dealer stages. When we endogenize the precision of the third round informative signal allowing for a threshold volume in the second round that triggers a more precise signal in the third round, the risk sharing will be even lower as both the public investor and the winning dealer play strategically to avoid revealing further information.

JEL Codes:

Keywords: Inter-Dealer Trading, Trading Disclosure, Strategic Trading, Linear Equilibria, Risk-Sharing.

*We thank Antonio Pedro Linhares, Leonardo Breder and Gabriel Srour for valuable institutional insights. The usual disclaimer applies

†Ph.D.Candidate, Department of Economics PUC-Rio. E-mail: sheck@econ.puc-rio. br

PUC-Rio - Certificação Digital Nº 0410928/CA

1 Introduction

Most of the dealership models in the market microstructure literature assume a single dealer market rather than competitive multiple dealers. The literature concentrates on equity markets, and particular the NYSE, a specialist (single dealer) market. However, other important equity markets such as NASDAQ and the London Stock Exchange (LSE), as well as government securities markets in most developed countries, are structured as multiple dealer markets[1]. In this paper we explore a multiple dealership evironment where trades take place in three consecutive stages. Dealers will compete for the initial trade and the winning dealer will use the inter-dealer market to offset his inventory in two consecutive opportunities. We let noisy informative signals be revealed in each stage and evaluate the risk-sharing and welfare implications conditioned on the precision of the signals. The asymmetry of information will be key reason here behind trades. Precision parameters can also be made endogenous and informed dealers will play strategically to avoid revealing information.

As opposed to a single dealer setup, multiple dealers will compete for their share on the customer's order flow, for example a Hedge Funds, which can be informative concerning the asset's final payoff. Trades between customers and dealers will take place in a public trading environment. The existence of an inter-dealer market offers for each dealer the option of managing their inventory via transactions with each other. Indeed, in the inter-dealer market dealers have the choice of either trading bilaterally with each other or indirectly via an inter-dealer broker system. Ho and Stoll (1983), Leach and Madhavan (1993), Vogler (1997) and Naik, Neuberger and Viswanathan (1999), among others, work with multi-dealer models. None of them, however, has proposed a multi-stage inter-dealer setup. Ho and Stoll (1983) was the first to model inter-dealer trading, but allowed customer trades and inter-dealer trades to be carried out in an identical manner. As most of the articles that followed, they assume a high degree of transparency in the trading environment and that transaction and dealer inventories are public information[2]. Viswanathan and Wang (2004) is the only paper we found that works with several inter-dealer stages of trade. They extend the traditional single inter-dealer round of trade to a sequential auction with multiple rounds of unit-auctions. Results show that liquidity falls and the seller is better off with more rounds of auctions, rationalizing the use of sequential auctions and explaining the phenomenon of "hot potato" trading.

This paper intends to revisit the topic and discuss an alternative scheme for the information disclosure process in a more realistic multi-stage inter-dealer market structure. We propose a three stage model where the market making sector possess two inter-dealer rounds to manage his position after a initial public trade. Our model is based on the two stage model proposed by Naik, Neuberger

[1] Gravelle (2002) examines the structural diferences between multiple dealer equity markets and government securities.

[2] Discussing the empirical results of foreign exchange market models Lyons (1996a) states tha: "a microstrucutral understanding of this market requires a much richer multiple-dealer theory than now exists"

and Viswanathan (1999). The extension we propose is made to accommodates, in a certain way, real direct trading and a broker's screens channels of negotiation. Instead of the sequential auction model proposed by Viswanathan and Wang (2004), we prefer to include only one additional stage to work as the broker's screens channel just mentioned and focus on the information revealing process and the strategic behavior on those trades. The asymmetry of information will be the key reason here behind trades. The motivation differs from the usual inventory models in the literature where dealers charge fees in the form of bid-ask spreads to provide liquidity. Ho and Stoll (1983) and .

Another important assumption of the model is that the information revealing process is noisy and individuals are not able to learn perfectly from the moves of other individuals. Although not standard in the literature, we believe this framework is a very realistic description of the trading environment. We follow Angeletos and Werning (2006) and admit that it captures in a parsimonious way the idea that players revise their beliefs based on others' actions and that it may takes more than one period to invert the full information content[3]. Indeed, the two inter-dealer stages will be strategically used by informed players to avoid revealing information and extract rents from uninformed parties.

We depart from the base inter-dealer literature and let a public investor receives an informative signal that induces the trade with one out of the many dealers operating in a given market making sector. Dealers will compete for the initial trade as this reveals public investor's private information concerning the asset's payoff that can be exploited in subsequents rounds of trade. The winning dealer will offer liquidity for private investors using proprietary inventory and use the inter-dealer market to offset his positions. Dealers who do not win the initial trade are assumed to play competitively in both inter-dealer rounds and are only assumed to trade with the winning dealer. We do also allow information in the model to be released as noisy informative signal as in Dasgupta (2006) where the precision parameter can be controlled and used to run alternative scenarios. When compared with the usual extreme full disclosure or non-informative assumptions as in Naik, Neuberger and Viswanathan (1999), this modification makes it possible to simulate how trades evolve between different channels of transaction and the way risk-sharing in the economy changes as the asymmetry of information varies. As far as we know, no other multi dealership model has sucesfully managed to capture asymmetry of information and strategic behavior in a two-stage inter-dealer market.

The risk sharing here will depend on the asymmetry of information as dealers who did not win the initial trade will be less willing to hold the risky asset the noisier the signal they receive in both rounds of trades. The volume traded will be higher the greater the differential of information among players. Simulations are conducted under an exogenous and an endogenous setup. When we allow a certain threshold volume traded in the first inter-dealer stage to trigger a higher precision parameter in the final round, both public investor and winning dealer will play strategically to restrict trades in the second round. The idea here is

[3] Minelli and Polemacharkis (2003) work similar theme.

PUC-Rio - Certificação Digital Nº 0410928/CA

that dealers infer from the amount of transaction but it takes one period to process information and actually use this in the final round. In general, this endogenous setup limits the winning dealer capacity to extract rents from the remaining dealers and thus reduces his willingness for trade in the initial round. On the other hand, the public investor anticipates the transactions that take place in inter-dealer stages and the risk shearing measured by the volume traded in the first period further decreases.

The results we obtain contribute to the discussion on policy recommendations on transparency and trade disclosure on both equity and government securities dealer markets.

This paper is organized as follows. In section 2, we develop an exogenous precision model and solve for the equilibrium in each one of the three stages considered. Section 3 simulate a numerical example for the base model and discuss the effects of different precision parameters and implied asymmetry of information in equilibrium. In section 4 we propose an endogenous precision model and show how the problem changes when players can play strategically to avoid the revelation of information. Section 5 concludes.

2 Base Model

The base model is built on the Two-Stage Model proposed by Naik, Neuberger and Viswanathan (1999). We let a Public Investor (PI) receives a noisy informative signal concerning the payoff of a given asset and quote a trade with one of the $K+1$ identical dealers (or market makers) that operate in an inter-dealer market. Both PI and the dealers are risk averse and for simplification reasons have zero starting inventory, such that short-salles will be allowed without restriction. Dealers will compete for the initial trade to gather information that lead to an informational advantage that can be exploited in future rounds of trade. We let MM_0 denotes the winning dealer.

The inter-dealer market is open for trade in two consecutive opportunities. Right after the initial trade MM_0 can trade directly with other market participants, and some information possessed by MM_0 is revealed as a noisy signal. A third and final round is open for trade in broker's screens. This will be MM_0's last opportunity to offset his position in the inter-dealer market before the asset's terminal payoff. The same K remaining dealer will be willing to trade once again as the information conditioning their beliefs will be made upon an additional informative noisy signal generated in the screens. The trade-off between both channels of trade comes from the informational content and the optimization problem and pricing rule applied by the remaining dealers while trading with MM_0. This point will be made clear as we now go further in the model.

The information process that determines the asymmetry of information is built on a key assumption concerning the precision of the informative signals generated in each stage. We make this precision purely exogenous in the base formulation of the model and evaluate the effects of greater transparency in equilibrium. In section 4 we show an extension to the baseline where the pre-

PUC-Rio - Certificação Digital Nº 0410928/CA

cision of the informative signal in the final round will depend on the volume traded in the previous round.

A.Setup

Stage 1: Public Investor receives informative signal and goes for a trade

PI receives an informative signal $v = w + \frac{1}{\varkappa}\epsilon$ regarding the payoff w of the asset, where $\epsilon \sim N(0,1)$ and $\varkappa$ is the precision parameter. The public investor is assumed to have a CARA utility function with absolute risk aversion coefficient of θ and have zero inventory before to the initial trade. A positive signal motivates the initial trade as the public investor wishes to buy a fraction y of shares that balances the expected return and the risk associated with the updated conditional distribution on the payoff.

PI is able to quote for a trade with $K+1$ identical dealers that operate in the inter-dealer market but is allowed, for simplicity, to trade with a single dealer only. We let this initial trade to be fully informative for the winning dealer regarding the signal received by PI.

Stage 2: MM_0 trades q_2^D and the remaining dealers set competitive p_2.

The first opportunity for MM_0 to offset his position after the initial trade will be through direct trading with the remaining K dealers in the market. We call this the second round of trade. Some information will be revealed here for the remaining dealers within the strategy adopted by the winning dealer and the quantity traded. We assume that the market can infer relevant information v received by MM_0 in the first stage as a noisy signal $s_i(v,\tau)$, where the precision τ is made a variable of the model[4]. The remaining dealers will update their conditional distribution of the payoff of the share based on this signal and set each one a linear price schedule p_2 for the fraction of trade received[5]. Since all the remaining dealers are identical we let each receive an equal share of the trade in the second round. They will behave competitively, optimize under a CARA utility function with absolute risk aversion coefficient of γ and have zero inventory and receive their reservation utility while trading in the second round.

Stage 3: MM_0 trades q_3^S and the remaining dealers set competitive p_3.

The winning dealer does not need to offset his entire position in the second round. We consider a third and final round that will take place in broker's screens. The existence of an additional stage of trading allows him to balance his adjustments between both inter-dealer stages. Once again, we assume that the remaining dealers will behave competitively in the final round and do not consider the trade made in the previous round as inventory. The remaining are also only allowed to trade with MM_0. The basic structure here is the same and the major difference will be an additional informative signal $s_i(v,\delta)$ that we let update the second stage conditional distribution and which will cause an inevitably lower asymmetry of information in the third round. Even though

[4] Instead of considering extreme assumptions for the learning process like perfect learning or no learning at all, the signal structure we choose is able to capture the dynamics of the model for a wide range of informational asymmetry. (The advantages of this setup will be made clear later on in the paper)

[5] We show later that a linear equilibrium exists and assure a linear dependence of the informative component z that will be updating the remaining dealers beliefs.

PUC-Rio - Certificação Digital Nº 0410928/CA

would prefer deals in a less informative second round, MM_0 faces a trade-off between the two channels due to the increasing linear pricing schedule considered by the remaining dealers.

Table 1 shows the mais variables of the model.

Table1: Variables of the Model

Variable	*Description*
w	True value of asset payoff
y	Buying order placed by IP in the first round of trade
γ	MM_0's coefficient of risk aversion
θ	PI's coefficient of risk aversion
p_1	Price in stage 1 (public trade)
p_2	Price in stage 2 (direct trading)
p_3	Price in stage 3 (broker's screens)
q_2^D	Quantity traded by MM_0 in the second stage
q_3^S	Quantity traded by MM_0 in the third stage
$K+1$	#dealers in inter-dealer market
σ_I^2	PI and MM_0 updated estimate of variance of the payoff
σ_w^2	Asset's variance of the payoff
χ	PI's informative signal precision parameter
τ	Direct Trading precision parameter
δ	Broker's Screens precision parameter

B.Solving the Game

We solve the model by backward induction. Starting from the last round of trade, MM_0 takes the prices and quantities traded in the previous rounds as given and solve for the final volume he wishes to buy back. In the second round MM_0 takes once again as given the equilibrium in the first round round, but incorporates the equilibrium in the final round and consider q_3^S as a reaction functions of the choices in the second round. In both inter-dealer stages the remaining dealers react setting competitive prices.

In the first stage MM_0 will now anticipate the game that follows in the inter-dealer market and set a competitive price p_1 for the share y brought by the public investor in terms of the equilibrium prices and quantities in the second and third stages. The public investor will solve an equivalent problem and set the amount y that he wishes to buy. His optimization problem will assume the price function defined by MM_0 that ultimately depends on the volume y itself, and so the strategic move defined by PI in this setup will be essential for the entire dynamics of the model.

2.1 First Stage Winning Dealer Optimization Problem

The winning dealer (MM_0) optimization problem is choosing the volumes he wishes to buy back trough both direct (q_2^D) and Inter-Dealer broker screens (q_3^S) channels after selling y quotes in the first stage of trading for the public

PUC-Rio - Certificação Digital Nº 0410928/CA

investor (PI). The initial trade generates a positive cash flow for MM_0 in the first round while the volumes bought back in the inter-dealer market represent costs and the terminal payoff will depend on the effective shares he ends up carrying after the last opportunity to trade in the third stage. Conditioned on the full informative signal revealed by the public trade to MM_0 the problem can be stated as:

$$E(U_0) = -E\left[\exp\left(-\left(\left(q_3^S + q_2^D - y\right)w + yp_1 - q_3^S p_3 - q_2^D p_2\right) \mid v\right)\right]$$

Alternatively, the problem can be equivalently written under a mean-variance representation:

$$\begin{aligned}
&-\frac{1}{\gamma}\log\left(E\left[U_0(q_3^S, q_2^D) \mid v\right]\right) \\
&= E\left[\left(q_3^S + q_2^D - y\right)w + yp_1 - q_3^S p_3 - q_2^D p_2 \mid v\right] \\
&-\frac{\gamma}{2}Var\left[\left(q_3^S + q_2^D - y\right)w + yp_1 - q_3^S p_3 - q_2^D p_2 \mid v\right] \\
&= \left(q_3^S + q_2^D - y\right)v + yp_1 - q_3^S p_3 - q_2^D p_2 \\
&-\frac{\gamma}{2}\left(q_3^S + q_2^D - y\right)^2 \sigma_I^2
\end{aligned}$$

where $E(w/v) = v$ and $Var(w/v) \equiv \sigma_I^2$ represent the conditional distribution of the true values of the shares given the perfect information assumption provided by the public trade. We follow addressing the problem by backward induction.

2.1.1 Inter-Dealer Broker Screens (Third Stage)

MM_0 problem on the third stage will be maximize his utility with respect to q_3^S. Taking as given the variables defined in the previous rounds of trade and once again conditioning in the information revealed by the public trade:

$$\begin{aligned}
&-\frac{1}{\gamma}\log\left(E\left[U_0(q_3^S) \mid v\right]\right) \qquad (1)\\
&= \left(q_3^S + q_2^D - y\right)v + yp_1 - q_3^S\left(\alpha_3 + \beta_3\frac{q_3^S}{K}\right) - q_2^D p_2 \\
&-\frac{\gamma}{2}\left(q_3^S + q_2^D - y\right)^2 \sigma_I^2
\end{aligned}$$

We assume that the K remaining dealers apply a linear price rule $p_3 = \alpha_3 + \beta_3 q_{3,i}^S$, where $q_{3,i}^S = \frac{q_3^S}{K}$. It will be shown later that a simple price schedule like that will generate an equilibrium. The first order condition here with respect to q_3^S is:

$$\begin{aligned}
q_3^S &= \frac{K}{2\beta_3 + K\gamma\sigma_I^2}\left(v - \alpha_3 - \gamma\sigma_I^2\left(q_2^D - y\right)\right) \qquad (2)\\
q_{3,i}^S &= \frac{1}{2\beta_3 + K\gamma\sigma_I^2}\left(v - \alpha_3 - \gamma\sigma_I^2\left(q_2^D - y\right)\right)
\end{aligned}$$

PUC-Rio - Certificação Digital Nº 0410928/CA

or equivalently:

$$q_3^S = K\Psi\left(v - \alpha_3 - \gamma\sigma_I^2\left(q_2^D - y\right)\right) \tag{3}$$
$$\Psi = \frac{1}{2\beta_3 + K\gamma\sigma_I^2}$$

2.1.2 Direct Trading (Second Stage)

In the second stage MM_0 optimization problem will take into account the equilibrium in the final round and take as given the initial trade such that:

$$\begin{aligned} & -\frac{1}{\gamma}\log\left(E\left[U_0(q_2^D, q_3^S(q_2^D)) \mid v\right]\right) \qquad (4) \\ = & \left(q_3^S + q_2^D - y\right)v + yp_1 - q_2^D\left(\alpha_2 + \beta_2\frac{q_2^D}{K}\right) \\ & -q_3^S\left(\alpha_3 + \beta_3\frac{q_3^S}{K}\right) - \frac{\gamma}{2}\left(q_3^S + q_2^D - y\right)^2\sigma_I^2 \end{aligned}$$

Once again, considering a linear price schedule for the trades conducted outside screens $p_2 = \alpha_2 + \beta_2 q_{i,2}^D$. The first order condition or $q_2^D = Kq_{i,2}^D$,substituting for q_3^S obtained in the following stage:

$$\begin{aligned} & v\left[1+\Omega\right] - \alpha_2 - \frac{2\beta_2 q_2^D}{K} - \Omega\alpha_3 - \frac{2\beta_3\Omega q_3^T}{K} \qquad (5) \\ & -\gamma\sigma_I^2\left(q_3^S + q_2^D - y\right)(1+\Omega) = 0 \end{aligned}$$

where;

$$\frac{dq_3^S}{dq_2^D} = -K\Psi\gamma\sigma_I^2 = \Omega \tag{6}$$

The equation that defines q_2^D as a function of the exogenous variables of the model can be written as:

$$q_2^D = \Phi\left(\alpha_2 + \Omega\alpha_3 - v\left(1+\Omega\right) - 2\gamma\sigma_I^2\Psi\beta_3 q_3^S + \gamma\sigma_I^2\left(1+\Omega\right)\left(q_3^S - y\right)\right) \tag{7}$$
$$\Phi = \frac{-K}{2\beta_2 + K\gamma\sigma_I^2\left(1+\Omega\right)}$$

2.1.3 Public Investor Initial Trade (First Stage)

In the first round of trade we assume that all the negotiating power resides with the public investor. The $K+1$ dealers in the market will compete for the initial trade such that all the rents provided by the asymmetry of information will be

PUC-Rio - Certificação Digital Nº 0410928/CA

captured by PI. The expected utility of MM_0 equals:

$$
\begin{aligned}
& -\frac{1}{\gamma}\log\left(E\left[U_0(p_1(q_2^D, q_3^S, y) \mid v\right]\right) \qquad (8) \\
= & \left(q_3^S + q_2^D - y\right) v + y p_1 - q_3^S\left(\alpha_3 + \beta_3 \frac{q_3^S}{K}\right) - q_2^D\left(\alpha_2 + \beta_2 \frac{q_2^D}{K}\right) \\
& -\frac{\gamma}{2}\left(q_3^S + q_2^D - y\right)^2 \sigma_I^2 = 0
\end{aligned}
$$

which yields the first stage price that reflects the ability of MM_0 to manage his position and exploit the informational advantage in the inter-dealer market:

$$
p_1 = \frac{1}{y}\left[q_3^S p_3 + q_2^D p_2 + \frac{\gamma}{2}\left(q_3^S + q_2^D - y\right)^2 \sigma_I^2 - \left(q_3^S + q_2^D - y\right) v\right] \qquad (9)
$$

2.2 The K Remaining Dealers Problem

We address the optimization problem of the K others market makers assuming a competitive behavior in both rounds of inter-dealer trading[6]. They will condition their information on different informative signals about the asset's payoff, inverted with some noise from the orders received in each inter-dealer stage. In the base model we treat the precision of these signals purely exogenous and let an alternative endogenous model for Section 3. Conditioned on the information received and the updates made, the inter-dealer market will solve for a similar maximization problem and determine the price schedule in both rounds.

2.2.1 Updating Beliefs

It follows from (3) and (7) that the volumes traded in the second and third round depend linearly on the informational content $v = w + \frac{1}{\varkappa}\epsilon$ received by MM_0 in the first round. This information will be transmitted trough the transactions conducted in both following stages with some noise. We let the technology that invert this informational content from the quantities be represented by an noisy signal s_i. In the second stage the precision will be measured by an exogenous parameter τ and can represent as:

$$
s_i\left(v, \tau\right) = v + \frac{1}{\tau}\eta_i \qquad (10)
$$

where η_i is distributed Standard Normal and independent of v. The noisy component will add to the signal variance. The inter-dealer market will update his conditional distribution of the payoff w following a Gaussian signal:

$$
\begin{aligned}
E\left(w \mid s_i\left(v, \tau\right)\right) &= \mu_w + \varphi_2\left(s_i\left(v, \tau\right) - \mu_v\right) \qquad (11) \\
Var\left(w \mid s_i\left(v, \tau\right)\right) &= \left(1 - \rho_2^2\right)\sigma_w^2
\end{aligned}
$$

[6] We follow Subrahmanyam (1991) and assume that each of the remaining dealers receive an equal share of the trade in both rounds of trade and do not extract any rent from the information received in the second round.

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

where μ_w, μ_v and σ_w^2 are the initial priors before the trade and:

$$\varphi_2 = \frac{cov(w, s_i(v,\tau))}{var(s_i(v,\tau))} = \frac{\sigma_w^2}{\sigma_w^2 + \frac{1}{\varkappa^2} + \frac{1}{\tau^2}}$$
$$\rho_2 = \frac{cov(w, s_i(v,\tau))}{\sqrt{\sigma_w^2 \sigma_{s_i}^2}} = \frac{\sigma_w^2}{\sqrt{\sigma_w^2\left[\sigma_w^2 + \frac{1}{\varkappa^2} + \frac{1}{\tau^2}\right]}}$$

In the broker screens we let δ represent the precision of similar informative signal:

$$s_i(v,\delta) = v + \frac{1}{\delta}\omega_i \tag{12}$$

where ω is also distributed Standard Normal and independent of v. The update in this stage will be made over the conditional distribution obtained after the second round. Once again, following a Gaussian signal $s_i(v,\delta)$ the distribution will be:

$$E(w \mid s_i(v,\delta), s_i(v,\tau)) = E(w \mid s_i(v,\tau)) + \varphi_3 (s_i(v,\delta) - E(v \mid s_i(v,\tau)) \tag{13}$$
$$Var(w \mid s_i(v,\delta), s_i(v,\tau)) = \left(1-\rho_3^2\right)\sigma^2_{w/s_i(v,\tau)}$$

such that:

$$\varphi_3 = \frac{cov(w, s_i(v,\delta))}{var(s_i(v,\delta))} = \frac{\sigma_w^2}{\sigma_w^2 + \frac{1}{\varkappa^2} + \frac{1}{\delta^2}}$$
$$\rho_3 = \frac{cov(w, s_i(v,\delta))}{\sqrt{\sigma_w^2 \sigma_{s_i}^2}} = \frac{\sigma_w^2}{\sqrt{\sigma_w^2\left[\sigma_w^2 + \frac{1}{\varkappa^2} + \frac{1}{\delta^2}\right]}}$$

2.2.2 Inter-Dealer Trading Prices

The inter-dealer market will set up prices in each stage based on the volume demanded by MM_0 and the updated conditional distributions. In the third stage the remaining dealers apply a linear pricing rule as function of the fraction $q_{3,i}^S$ brought to the broker screens and negotiated with each one that can be represented as:

$$p_3 = \alpha_3 + \beta_3 q_{3,i}^S \tag{14}$$

Considering K identical risk averse market makers with negative exponential utility and risk aversion coefficient of γ we solve the problem for a representative dealer with zero starting inventory. We follow Subrahmanyam (1991) and let the inter-dealer market compete for the transaction with MM_0. The competition contributes for a zero expected utility in the third stage, where both informative signals will be updating the remaining dealer's beliefs. In mean-variance representation and ignoring the outcome of the previous round:

$$-\frac{1}{\gamma}\log E\left[U_i\left(p_3\left(q_3^S\right)\right) \mid s_i(v,\delta), s_i(v,\tau)\right] = 0 \tag{15}$$
$$q_3^S (p_3 - E(w \mid s_i(v,\delta), s_i(v,\tau))) - \frac{1}{2}\gamma\left(q_3^S\right)^2 Var(w \mid s_i(v,\delta), s_i(v,\tau)) = 0$$

The price in the third stage can be implicitly defined as:

$$p_3 = \frac{1}{2}\gamma q_3^T Var\left(w \mid s_i\left(v,\delta\right), s_i\left(v,\tau\right)\right) + E\left(w \mid s_i\left(v,\delta\right), s_i\left(v,\tau\right)\right) \tag{16}$$

Comparing (16) and (14) we may write α_3 and β_3 in terms of the conditional distribution:

$$\begin{aligned} \alpha_3 &= E\left(w \mid s_i\left(v,\delta\right), s_i\left(v,\tau\right)\right) \\ \beta_3 &= \frac{K}{2}\gamma Var\left(w \mid s_i\left(v,\delta\right), s_i\left(v,\tau\right)\right) \end{aligned} \tag{17}$$

In the direct trading round MM_i will apply an equivalent pricing rule:

$$p_2 = \alpha_2 + \beta_2 q_{i,2}^D \tag{18}$$

The expected utility here will be conditioned in a single informative signal where the updates are given by (11).Once again, the competition contributes for a zero expected utility in mean-variance representation:

$$q_2^D\left(p_2 - E\left(w \mid s_i\left(v,\tau\right)\right)\right) - \frac{1}{2}\gamma\left(q_2^D\right)^2 Var\left(w \mid s_i\left(v,\tau\right)\right) = 0 \tag{19}$$

such that the price set by the each of the remaining dealers for their fraction of the trade is:

$$p_2 = \frac{1}{2}\gamma q_2^D Var\left(w \mid s_i\left(v,\tau\right)\right) + E\left(w \mid s_i\left(v,\tau\right)\right) \tag{20}$$

and the components α_2 and β_2 as a function of the conditional distribution in the second round:

$$\begin{aligned} \alpha_2 &= E\left(w \mid s_i\left(v,\tau\right)\right) \\ \beta_2 &= \frac{K}{2}\gamma Var\left(w \mid s_i\left(v,\tau\right)\right) \end{aligned} \tag{21}$$

2.3 The Public Investor Problem

We endogenize the public trade size y modeling the public investor's trading strategy. We endow the investor with an informative noisy signal $v = w + \frac{1}{\varkappa}\epsilon$ regarding the payoff of the shares, where $\epsilon \sim N(0,1)$ and $\varkappa$ plays the precision. No initial inventory is considered. PI will condition his beliefs on the informative signal v, update $E\left(w/v\right) = v$ and $Var\left(w/v\right) \equiv \sigma_I^2$ and maximize a negative exponential utility with risk aversion coefficient of θ.

$$\begin{aligned} &\underset{y}{Max} - \frac{1}{\theta}\log E\left[U\left(y\right) \mid v\right] \\ &= y(v - p_1) - \frac{\theta}{2}y^2\sigma_I^2 \end{aligned} \tag{22}$$

PUC-Rio - Certificação Digital Nº 0410928/CA

This informative signal will update MM_0's beliefs such that the price p_1 offered for trade in the first round will incorporate v and must be such that in competition ensures an expected utility for the winning dealer just as high as the expected utility for the market maker who does no get the trade. Any excess would be enough for one of the K remaining dealers to offer a slightly better price in the first round and win the trade. Considering identical market makers this competition for the trade will lead for a zero expected utility for the winning dealer in the entire three stage game that follows. Taking the price p_1 offered by MM_0 in the first round, conditioned on the volume y demanded by PI and defined in (9), the first order condition will be:

$$\begin{aligned} v - p_1 - \frac{dq_3^S}{dy}\left(\alpha_3 + \frac{2\beta_3}{K}\right) - \frac{dq_2^D}{dy}\left(\alpha_2 + \frac{2\beta_2}{K}\right) \\ -\gamma\sigma_I^2\left(q_3^S + q_2^D - y\right)\left(\frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1\right) \\ + \left(\frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1\right)v - \theta\sigma_I^2 = 0 \end{aligned} \tag{23}$$

where we consider a fully revealing assumption in the public trade such that:

$$\frac{dq_3^S}{dy} = K\Psi\gamma\sigma_I^2\left(1 - \frac{dq_2^D}{dy}\right)$$

$$\frac{dq_2^D}{dy} = \Phi\frac{\gamma\sigma_I^2\left(1+\Omega\right)\left(K\Psi\gamma\sigma_I^2 - 1\right) - 2K\Psi^2\gamma^2\beta_3\sigma_I^4}{1 - \Phi K\Psi\gamma^2\sigma_I^4\left(2\Psi\beta_3 - 1 - \Omega\right)}$$

3 Comparing Equilibria for Different Signal's Precision Scenarios

In order to highlight MM_0's optimal choices among different informational asymmetry we run numerical simulations considering sufficiently wide ranges for the precision parameters to describe the dynamics of the base model. The simulations help us to understand how all these variables interact and the equilibrium evolves under different assumptions for the two exogenous precision parameters in inter-dealer market. We calibrate the model for a set values commonly find in equivalent calibrations in the literature: γ=0.20, θ=0.20, K=20,v=10, σ_w^2=1.00, μ_w=μ_v=5, χ=10, τ=1 and δ=2. Results do not change significantly when we chose diferent set of calibrations.

3.1 Inter-Dealer Broker's Screens Precision Parameter (δ)

Figure 1 examines the relationship between transaction prices in all stages of trading. The contribution of the precision parameter δ will be straightforward in the third stage as the informative signal is directly related to the price formation in the final round through the updates made by the remaining dealers. On the

PUC-Rio - Certificação Digital Nº 0410928/CA

other side, the effects on the decisions made in the first and second round come from the strategic behavior of all the players considered. Solving the model by backward induction we allow players in the preceding rounds to incorporate the conditions met in the final round to their plans of trading.

Third's round price curve will be increasing in the precision parameter, becoming flat for sufficiently large δ. The updates made over more precise informative signals reduce the risk premia perceived by the remaining dealers and contribute for an upward pressure in prices. The volume left for trade in the final round decreases with the precision, becoming slightly negative (MM_0 selling) as the asymmetry of information vanishes and the risk taking profile considered for the winning dealer (γ=0.20) dominates. Figure 2 makes the point. The conditional mean in the final round increase in a way that it will not be optimal for MM_0 to split his trades in both channels of trade. The asymmetry in the second round will be such that it will be better to carry almost all the trade in the second round and avoid higher prices set by the remaining dealers in the third round.

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

Figure1: Transaction Prices versus Broker's Screens Precision Parameter (δ)

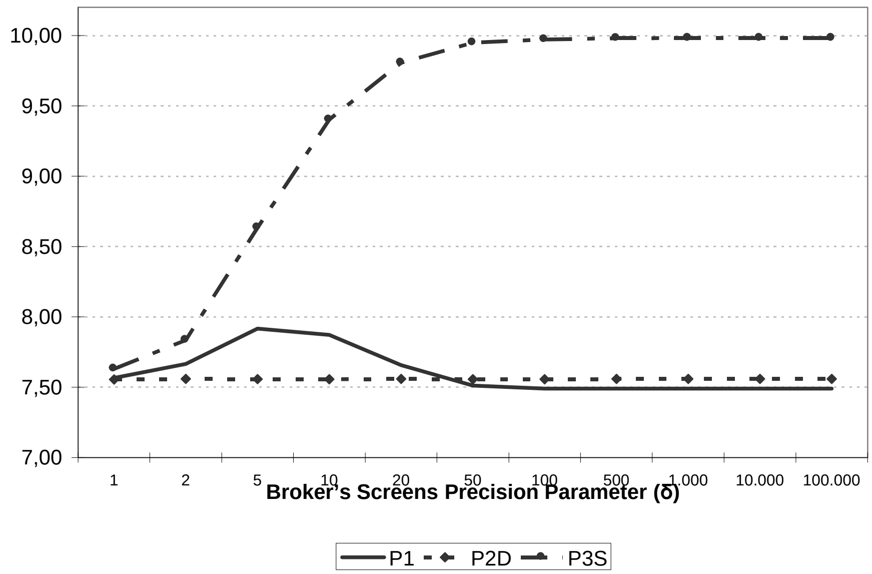


Figure2: Transaction Volumes versus Broker's Screens Precision Parameter (δ)

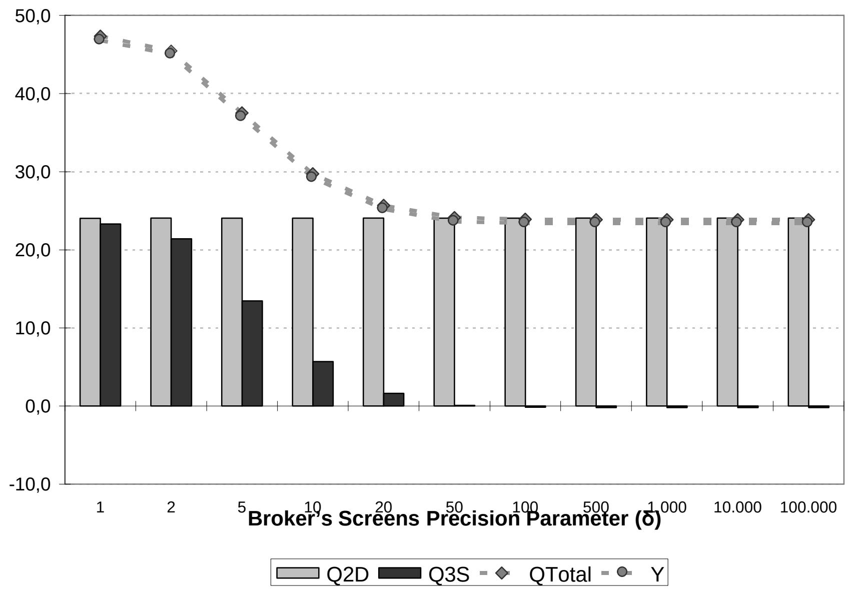
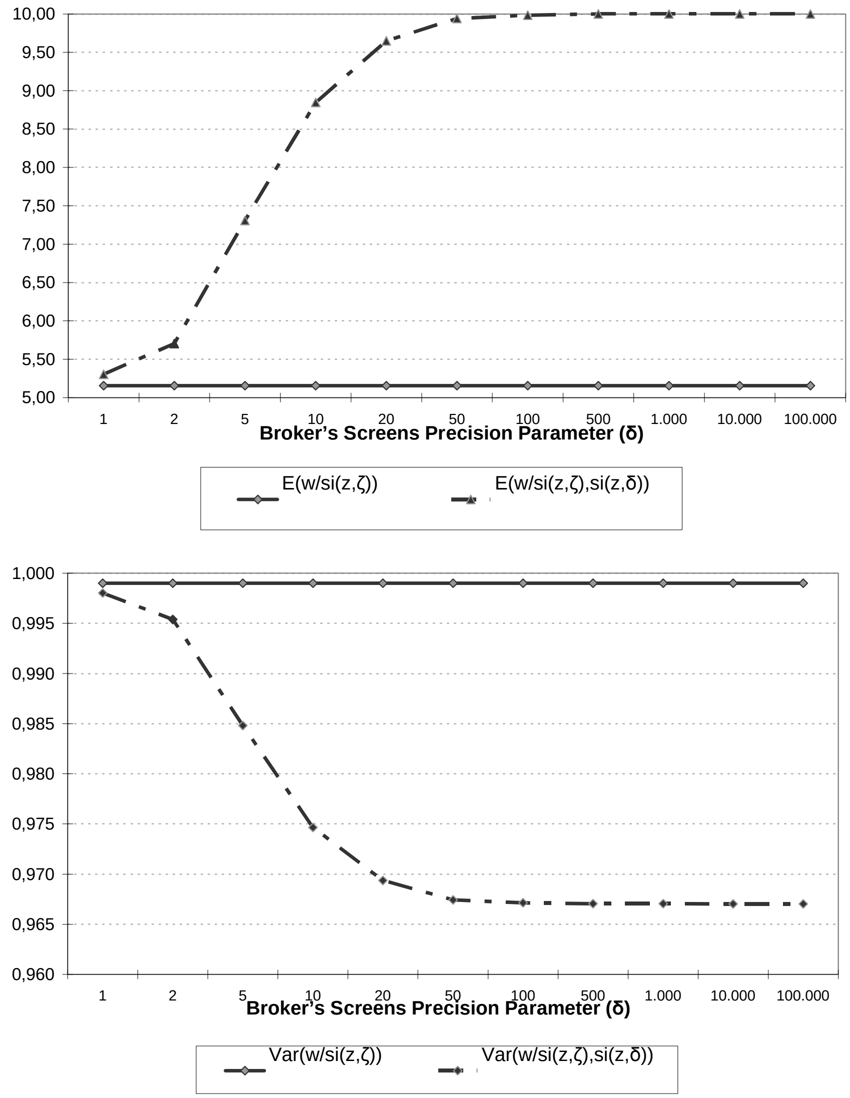


Figure3: Payoff's Conditional Distribution versus Broker's Screens Precision Parameter (δ)



PUC-Rio - Certificação Digital Nº 0410928/CA

3.2 Direct Trading Precision Parameter (τ)

We now simulate the effects of different precision parameters for the signal generated in the second round and that updates the beliefs of the K remaining dealers in both rounds of inter-dealer trade. The main difference between the precision in the second and third round is that the higher the precision here, the similar the asymmetry in both rounds as the signal in the second round is carried to final round. On the other hand, the greater the precision in the third round, and the lower the precision in the second, the greater the difference in the informational content in the two rounds of trade.

Figure 4 shows the evolution of the prices in all three stages. The asymmetry of information in both inter-dealer's rounds decreases with the precision parameter and the prices converge to the expected payoff value $E(w/x) = 10$. Quantities traded in both stages also decrease together as the informational advantage collapses. For sufficiently high precision parameters the asymmetry is almost completely eroded and the trades simply cease to exist. No transaction may take place in the first place.

PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

Figure4: Transaction Prices versus Direct Trading Precision Parameter (τ)

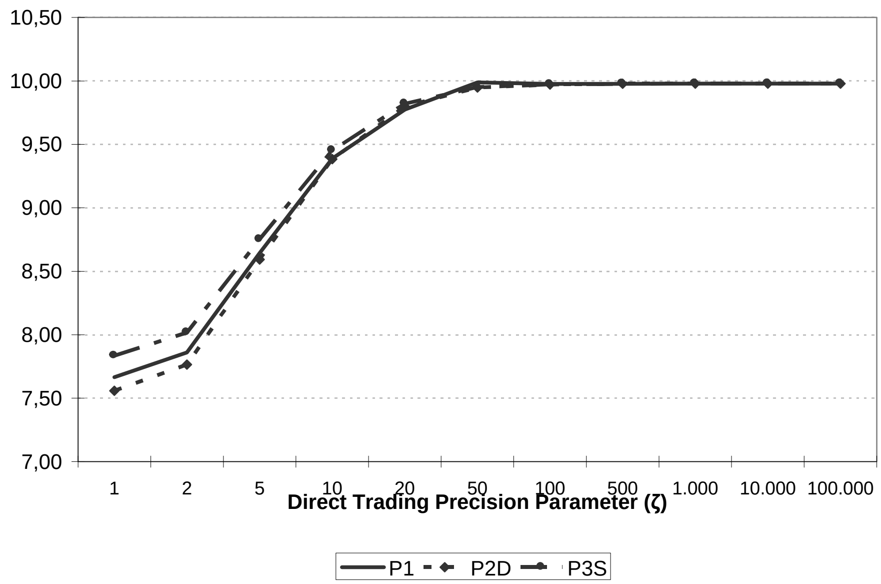


Figure5: Transaction Volumes versus Direct Trading Precision Parameter (τ)

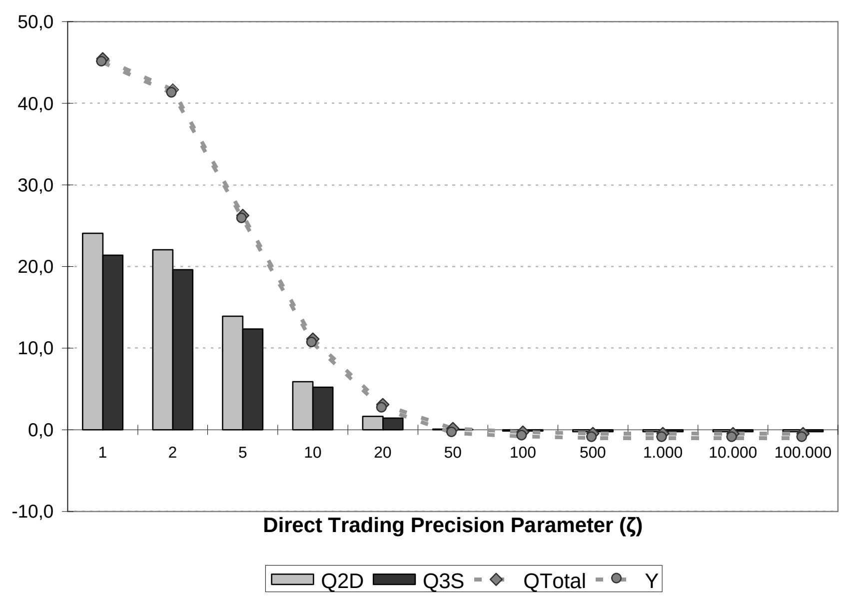
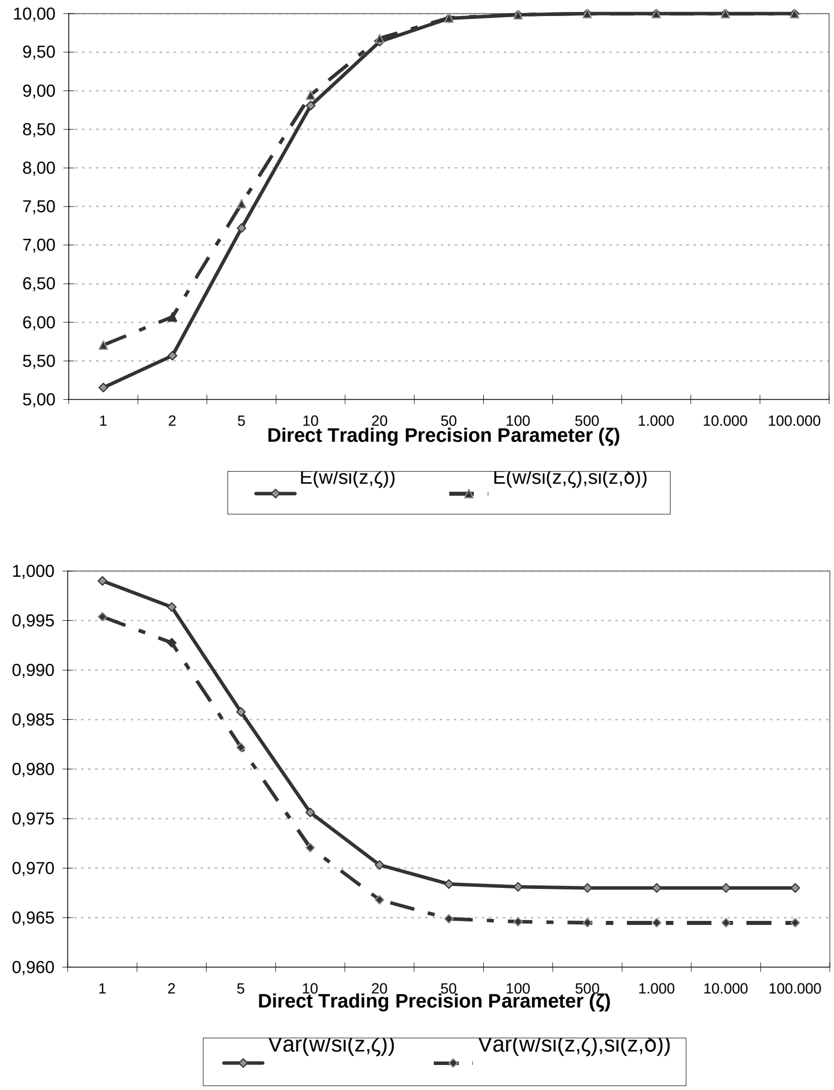


Figure6: Payoff's Conditional Distribution versus Direct Trading Precision Parameter (τ)



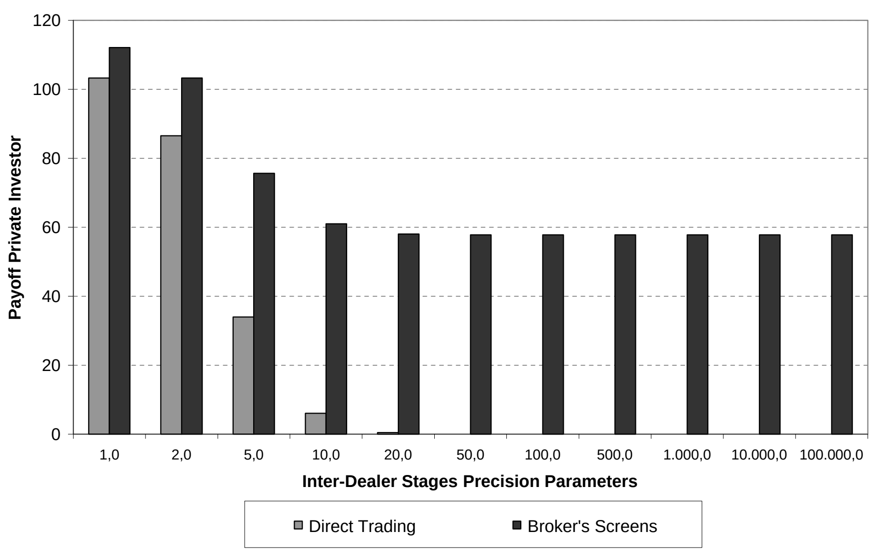
PUC-Rio - Certificação Digital Nº 0410928/CA

PUC-Rio - Certificação Digital Nº 0410928/CA

3.3 Welfare Comparison

To investigate the welfare implications of different precision parameters scenarios we compare the payoffs derived by the private investor. Assuming that the $K+1$ dealers will compete for the trade in the first round of negotiation and derive zero expected utility, all the bargaining power will be with the public investor. The investor ability to extract rents from the winning dealer will be higher the greater the asymmetry of information between the winning dealer and the remaining players in both inter-dealer rounds of trade. The greater the uncertainty about the payoff of the asset, the lower the price uninformed dealers would be willing to sell their inventories. In the first round, the winning dealer will be such that his informational advantage the will be exploit trading with the remaining dealers will be reverted to the private investor.

Figure7: Private Investor Welfare Comparison



4 An Endogenous Precision Model

The simulations ran so far shows that the rents for negotiation in the first round of trade is higher the greater the asymmetry of information generated by noisier signals in both inter-dealer trading stages. This asymmetry leads MM_0 to extract surplus from the K remaining dealers witch, due to competition, are ultimately transferred to PI in the first round of trade. Thus, the higher asymmetry the more favorable conditions for PI to buy in the first round and consequently the higher the volume traded.

So far we have assumed that the precision concerning the three informative signals in the model were purely exogenous. We now consider a variation of the model where the precision in the third stage depends on the volume traded in the previous stage. There will be a threshold volume in the second stage $\overline{q_2^D}$ above witch the precision in broker screens increase to $N\delta$ ($N > 1$). Both PI and MM_0 optimization problems will be constrained by the volume $\overline{q_2^D}$ that triggers a higher precision in the final stage. The motivation behind the endogenous framework is straightforward. We assume that the volume itself is revealing, once it may induce the remaining dealers to re-trade and generate a self-fulfilling sequence of transactions that could be informative.

As before, we solve the model by backward induction. The resolution will be almost identical to the base model, except that now both PI and MM_0 will choose between two optimization problems. In the second stage, after selling y units MM_0 will have to decide between restricting his trade in the second round and avoid a higher precision in the third round or exceeding the threshold $\overline{q_2^D}$ and generating a more precise signal in the final round. In the base model the choice of volumes traded in each inter-dealer stage would not impact the information revealed.

The PI problem in the first would be even more complex as we assume that his initial demand for trade will necessarily be met by $MM_0.PI$ will need to consider MM_0 optimization problem as this will determine the price $\widehat{p_1}$ for wich he will be willing to trade. In the first round PI will consider the strategic trade by MM_0 in the inter-dealer market that follows his initial move and maximize choosing a similar problem between a constrained and an unconstrained maximization.

4.0.1 Inter-Dealer Broker Screens (Third Stage)

We once again address the problem by backward induction. Beginning in the final round, we take as given the equilibrium variables in the preceding rounds and thus MM_0's optimization problem when we allow the precision to be dependent of the volume traded in the second round will be:

$$\begin{aligned} \underset{q_3^S}{Max} - \frac{1}{\gamma} \log \left(E\left[U_0(q_3^S)/v \right] \right) \\ st : \delta \Longleftrightarrow q_2^D = \widehat{q_2^D} \leq \overline{q_2^D} \\ N\delta \Longleftrightarrow q_2^D = \widetilde{q_2^D} > \overline{q_2^D} \end{aligned}$$

The optimal choices for q_3^S conditioned on the volume in the previous round will be:

$$\begin{aligned} \widetilde{q_2^D} &> \overline{q_2^D} \Longrightarrow \widetilde{q_3^S} = K\widetilde{\Psi}\left(v - \widetilde{\alpha_3} - \gamma\sigma_I^2 \widetilde{q_2^D}\right) \\ \widehat{q_2^D} &\leq \overline{q_2^D} \Longrightarrow \widehat{q_3^S} = K\Psi\left(v - \alpha_3 - \gamma\sigma_I^2 \widehat{q_2^D}\right) \end{aligned}$$

where $\widetilde{\alpha_3}$ and $\widetilde{\Psi}$ consider $N\delta$ instead of δ. In general, the endogenous precision scheme will increase the volume traded in the broker screens if the restriction

PUC-Rio - Certificação Digital Nº 0410928/CA

apply and it becomes optimal for MM_0 not to exceed $\overline{q_2^D}$. This is the case we will be interested. When it is optimal to exceed $\overline{q_2^D}$ and generate greater transparency in the third round, the informational advantage lost will lead inevitably to a reduction in q_3^S.

Proposition 1 *When it is optimal for MM_0 to restrict $\widehat{q_2^D} \leq \overline{q_2^D}$ and avoid $N\delta$, the amount negotiated in the final round will be at least as large as it would be in the base model, where there were no such conditions on the precision of the informative signal.*

Proof. *If for a given public trade y the trigger $\overline{q_2^D}$ does not apply and $\widehat{q_2^D} < \overline{q_2^D}$, then the quantity left for the third round will be exactly the same as in the base model. If, however, the initial trade y is such that the it now becomes optimal to restrict the transactions in the second stage and preserve the asymmetry to be exploited in the final round, then the fraction of the position did not offset in the second round will add in the next round.* ■

4.0.2 Direct Trading (Second Stage)

The real optimization problem faced by MM_0 will be in the second round. The outcome here will determine the precision in the following round and ultimately the potential asymmetry of information to be exploited. As before, MM_0 will compare the utilities resulting two maximizations. First, we consider the equilibrium when the volume traded here is bounded by $\overline{q_2^D}$, the precision in the third stage is kept δ and the third stage output is given by $\widehat{q_3^S}$:

$$\begin{gathered}
\underset{q_2^D}{Max} - \frac{1}{\gamma} \log\left(E\left[U_0(q_2^D, q_3^S(q_2^D))/v\right]\right) \\
st : \delta \\
q_3^S = \widehat{q_3^S} \\
\widehat{q_2^D} \leq \overline{q_2^D}
\end{gathered}$$

The optimal response here will be:

$$\widehat{q_2^D} = \left\{ \begin{array}{cc} \begin{array}{c} \Phi(\alpha_2 + \Omega\alpha_3 - v\left(1+\Omega\right) - 2\gamma\sigma_I^2\Psi\beta_3 q_3^S \\ +\gamma\sigma_I^2\left(1+\Omega\right)\left(q_3^S - y\right)) \end{array} & ;\ if \quad \widehat{q_2^D} \leq \overline{q_2^D} \\ \overline{q_2^D} & ;\ if \quad \widehat{q_2^D} = \overline{q_2^D} \end{array} \right\}$$

The alternative optimization problem that MM_0 consider is letting $q_2^D > \overline{q_2^D}$,

PUC-Rio - Certificação Digital Nº 0410928/CA

increase the precision to $N\delta$ and take the third stage output $\widetilde{q_2^D}$. This leads to:

$$\underset{q_2^D}{Max} - \frac{1}{\gamma}\log\left(E\left[U_0(q_2^D, q_3^S(q_2^D))/v\right]\right)$$
$$st : N\delta$$
$$q_3^S = \widetilde{q_3^S}$$
$$\widetilde{q_2^D} > \overline{q_2^D}$$

which yields;

$$\widetilde{q_2^D} = \widetilde{\Phi}\left(\alpha_2 + \widetilde{\Omega}\widetilde{\alpha_3} - v\left(1+\widetilde{\Omega}\right) - 2\gamma\sigma_I^2\widetilde{\Psi}\widetilde{\beta_3}\widetilde{q_3^S} + \gamma\sigma_I^2\left(1+\widetilde{\Omega}\right)\left(\widetilde{q_3^S} - y\right)\right)$$

4.0.3 Public Investor Initial Trade (First Stage)

All the interactions and information disclosure strategies that take place in the inter-dealer market come down to the first round of trade. We assume that PI can fully anticipate the strategic behavior behind the inter-dealer market and use this to optimize in the first round. Once again, the optimal volume y is determined by comparing the utility derived from different optimization problems. PI will chose between an unrestricted maximization, where it becomes optimal for MM_0 to make $\widetilde{q_2^D}$, and a restricted problem where the amount y does not trigger the threshold $\overline{q_2^D}$ in the second stage of trading.

The restricted problem can be represented as:

$$\underset{y}{Max} - \frac{1}{\theta}\log E\left[U\left(y\right)/v\right]$$
$$st : \delta$$
$$p_1 = \widehat{p_1}$$
$$q_2^D = \widehat{q_2^D}$$

where the price p_1 reflects the strategic behavior by MM_0 that follows the public trade. When y is such that avoids a higher disclosure of information in the third stage, the price asked by MM_0 will be:

$$\widehat{p_1} = \frac{1}{y}\left[\widehat{q_3^S}p_3 + \widehat{q_2^D}p_2 + \frac{\gamma}{2}\left(\widehat{q_3^S} + \widehat{q_2^D} - y\right)^2\sigma_I^2 - \left(\widehat{q_3^S} + \widehat{q_2^D} - y\right)v\right]$$

and the volume $\widehat{y}$ defined by the optimization follows implicitly from an equivalent first order condition derived in (23).

$$\begin{aligned} v - \widehat{p_1} - \frac{dq_3^S}{dy}\left(\alpha_3 + \frac{2\beta_3}{K}\right) - \frac{dq_2^D}{dy}\left(\alpha_2 + \frac{2\beta_2}{K}\right) \\ -\gamma\sigma_I^2\left(\widehat{q_3^S} + \widehat{q_2^D} - \widehat{y}\right)\left(\frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1\right) \\ + \left(\frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1\right)v - \frac{\theta}{2}\sigma_I^2 = 0 \end{aligned} \tag{24}$$

PUC-Rio - Certificação Digital Nº 0410928/CA

The unrestricted problem $\widetilde{y}$ will be identical to the base model, except that the precision will now increase to $N\delta$. The threshold equilibrium $\overline{q_2^D}$ will always be triggered whenever y is high enough to make it optimal to share the trade between both rounds. As mentioned before, we focus on situations where will be optimal to condition the volume traded to restrict the information revealed in the last stage of trade, such that the outcome of the unrestricted problem will not be optimal for the parameters we set. We can always set $\overline{q_2^D}$ and N to make it optimal for PI to restrict the volume traded in the first round.

In order to highlight the effects of an endogenous precision model to the risk sharing in the economy we consider a numerical example by specifying the following parameter values: γ=0.20, θ=0.20, K=20, v=10, σ_w^2=1.00, μ_w=5, μ_z=5, χ=10, τ=1, δ=2 and N=2. Figure 8 presents the relationship between payoff and volume traded in the first round for PI unrestricted (base model), $\overline{q_2^D}$=20 and $\overline{q_2^D}$=15. The risk sharing in the economy measured by the initial public trade is lower when we set a binding threshold $\overline{q_2^D}$ in the second round. More than that, the initial trade decreases the lower the threshold $\overline{q_2^D}$. For the range of y simulated it will always be optimal for MM_0 to avoid a greater transparency in the final round when the threshold is $\overline{q_2^D}$=20. Once we set $\overline{q_2^D}$=15 the restricted optimization will only be optimal up to $y < 60$. If the initial trade is made greater than that, it will become optimal for MM_0 to chose $\widetilde{q_2^D}$ even if it generates a more precise signal in the next round.

Figure 9 shows the range of y that makes it optimal for MM_0 to restrict the volume in the second round to $\widehat{q_2^D} = \overline{q_2^D}$ and keep the signal less informative. MM_0 will partially compensate in the final round the volume did not trade in the previous round. When y is high enough to make it worth to exceed $\overline{q_2^D}$ and reduce the asymmetry, the volume in the second round will then abruptly increase to extract the most from the second round and to compensate for the worst environment in the third round.

PUC-Rio - Certificação Digital Nº 0410928/CA

Figure8: Payoff PI versus Volume Traded in the First Stage

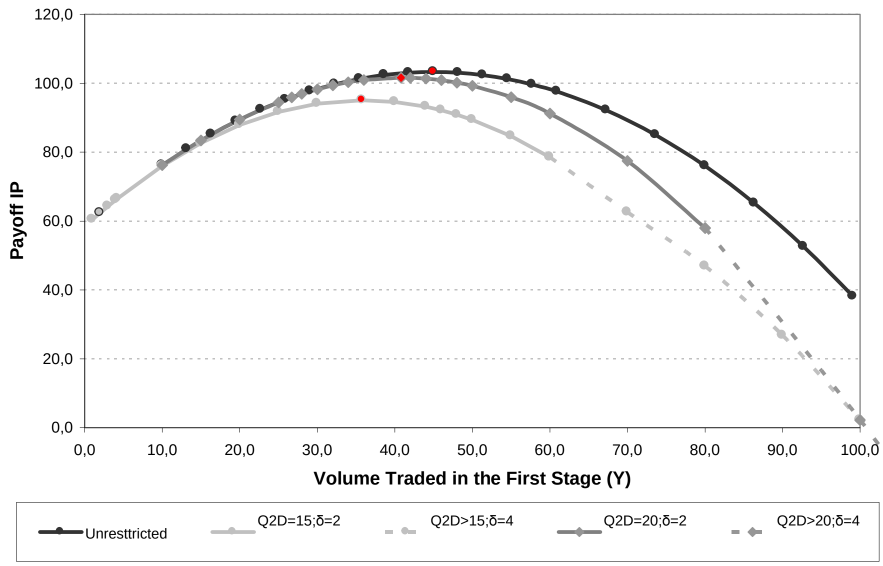
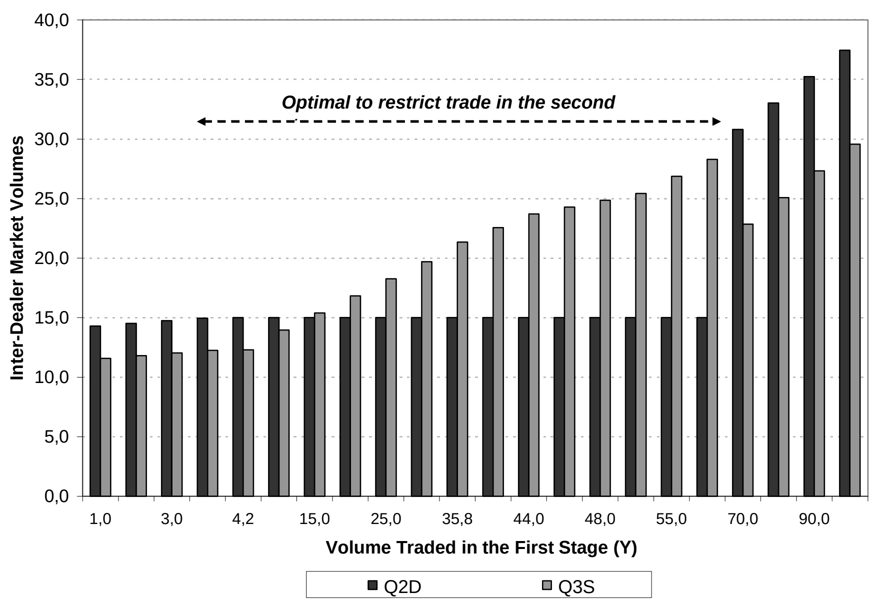


Figure9: Inter-Dealer Market Optimal Volumes
(Direct and Broker's Screens Trading when $\overline{q_2^D}$=15)



PUC-Rio - Certificação Digital Nº 0410928/CA

5 Conclusion

In this paper we propose a multi-stage inter-dealer model where trades in the inter-dealer market are conducted in two sequential stages. Dealers will compete for the public trade in the first round, which will reveal some private information. The winning dealer will be able to offset his unbalanced inventory in the inter-dealer market, that will be open for negotiation in two consecutives rounds of trade. We identify the first inter-dealer stage as bilateral direct trades between dealers.

Winning dealers will have a final opportunity to trade in a subsequent round of trade, an indirectly inter-dealer broker system that we believe captures in a certain way the inter-dealer broker's screens. Anonymous inter-dealer broker systems are specially popular among government securities, accounting for more than 90% of the inter-dealer US Treasuries market[7]. Both rounds will be informative of the asset's final payoff for the remaining dealers in the marketplace, although this information revealing process will be modeled here as noisy signals, not standard in the microstructure literature, as we believe this captures in a parsimonious way the idea that players revise their beliefs based on others' actions and that it may takes more than one period to invert the full information content. The precision of the noisy signal will be made both exogenous and endogenous. We run simulations showing that when we let the volume traded in the first inter-dealer stage defines the precision of the informative signal in the final stage of trade, dealers will play strategically to avoid revealing information. This will restrict the ability of the market makers to trade in the initial public round of trade and decrease the risk-sharing in first place.

References

[1] Angeletos, G; Werning, I. 2006 "Crises and Prices: Information Aggregation, Multiplicity and Volatility". American Economic Review, 96, 1720-1736.

[2] Bloomfield, R; O'Hara, M. 1999 "Market Transparency: Who wins and Who Loses?". The Review of Financial Studies, 12, 5-35.

[3] Bloomfield, R; O'Hara, M. 2000 "Can Transparent Markets Survive?". Journal of Financial Economics, 55, 425-459.

[4] Dasgupta, A. 2006 "Coordination and Delay in Global Games ". Forthcoming in the Journal of Economic Theory.

[5] Dattels, P. 1995 "The Microstructure of Government Securities Markets". IMF Working Paper, 95/117.

[7] Fleming and Remolona (1990) and Dattels (1995).

PUC-Rio - Certificação Digital Nº 0410928/CA

[6] Fleming, M; Remolona, E. 1999 "Price Formation and Liquidity in the US Treasury Market: The Response to Public Information". Journal of Finance, 54, 1901-1915.

[7] Gravelle, T. 2002 "The Microstructure of Multiple-Dealer Equity and Government Securities: How They Differ". Bank of Canada. Working Paper 2002-9.

[8] Ho, T.S.Y; Stoll, H.R. 1983 "The Dynamics of Dealer Market Under Competition". The Journal of Finance, 38, 1053-1074.

[9] Kyle, A. 1985 "Continuous Auctions and Insider Trading". Econometrica, 53, 1315-1335.

[10] Leach, J; Madhavan, A. 1993 "Price Experimentation and Security Market Structure". Review of Financial Studies, 6, 373-404.

[11] Lyons, R. 1996 "Optimal Transparency in a Dealer Market with an Application to Foreign Exchange". Journal of Financial Intermediation, 5, 225-254.

[12] Lyons, R. 1997 "A Simultaneous Trade Model of the Foreign Exchange Hot Potato". Journal of International Economics, 42, 275-298.

[13] Minelli, E; Polemarchakis, H. 2003 "Information at Equilibrium". Economic Theory, 21, 573-584.

[14] Nayk, N; Neuberger, A; Viswanathan, S. 1999 "Trade Disclosure Regulation in Markets with Negotiated Trades". Review of Financial Studies, 12, 4, 873-900.

[15] Subrahmanyam, A. 1991 "Risk Aversion, Market Liquidity and Price Efficiency". Review of Financial Studies, 4, 417-441.

[16] Viswanathan, S; Wang, J. 2004 "Inter-Dealer Trading in Financial Markets". Journal of Business, 77, 4.

[17] Vogler, K. 1997. "Risk Allocation and Inter-Dealer Trading". European Economic Review, 41, 1615-1634.

PUC-Rio - Certificação Digital Nº 0410928/CA