2
Shocks to trading volume, risk and return

2.1.
Introduction

The Efficient Markets Hypothesis (EMH) states that asset prices fully reflect all the information about future returns of an asset and, thus, investors can not make extraordinary profits based on available information.[1] Empirical results inconsistent with this hypothesis, that is, which report actual average returns statistically different than expected returns, are referred to as anomalies in finance literature.[2] As stated by Fama (1970), any test of the EMH is simultaneously testing the model used to estimate expected returns. Thus, any anomaly may either indicate a failure of the EMH, or an inadequacy of the underlying asset-pricing model. Despite this joint-test limitation, market efficiency literature (or anomaly literature) contributes to develop a better understanding of asset returns in the time-series and in the cross-section, as exposed by Fama (1991). This article focuses on a specific anomaly and questions the test used to reject the null hypothesis that the difference in expected returns is due to systematic risk.

The anomaly we focused on was reported by Gervais, Kaniel and Mingelgrin (2001), henceforth GKM. They show that trading volume (the number of shares traded), a public information available at low cost, can be used to predict expected returns for individual stocks. In their work, GKM form equal weighed portfolios based on volume. For instance, taking daily volumes, a stock is classified as high (low) volume if volume in the last day is among the top (bottom) five of the past 50 days. That is, it is set to the high (low) portfolio if it falls within the top (bottom) decile for the considered trading interval. Otherwise it is classified as normal volume. GKM have shown that the high (low) volume

[1] To avoid the discussion about the possibility of using private information to forecast returns, this hypothesis is frequently loosen, constraining the information set to public available information. Campbell, Lo and MacKinlay (1997) discuss alternative formulations of the Efficient Market Hypothesis (section 1.5) and the predictability of asset returns.

[2] Schwert (2003) provides a review of anomalies literature.

portfolio presents higher (lower) mean return than the normal volume portfolio, over periods that ranged from 1 to 50 days after portfolios are formed.

GKM argue that this "high-volume return premium" is not related to systematic risk. The underlying asset pricing model in GKM's work, to account for systematic risk, is the unconditional Capital Asset Pricing Model (CAPM).[3] Essentially they argue that they do not find a statistical significant difference between the betas of the high and low volume portfolios. However we pose two challenges to their conclusion.

First, the standard deviation of the difference between betas estimated by GKM is of the order of 0.05. Considering a risk premium of 10% per year, a difference of two standard deviations could explain 1% per year of the "high-volume return premium", and would still have a 50% probability of being considered statistically non significant (at 5% level), even though it is clearly economically significant. We improve the power of the test increasing the sample. This is done by using every daily returns to estimate betas, instead of using only one return measure for each 51 days. This multiplies by 50 the number of observations used to estimate beta.[4] We adjust the statistical procedure to account for possible autocorrelation of daily returns. Differently than GKM, we find that the difference between the betas of the high volume and low volume portfolios is positive, statistically significant, and of the order of 0.04. Thus, this test could not reject the contribution of systematic risk to the volume premium, even if the unconditional CAPM could satisfactory explain the differences in expected returns.

Second, Fama and French (1992) provide empirical evidence that the unconditional CAPM does not seem to adequately explain the cross-section variation in average returns, mainly when controlling for differences in firm sizes across portfolios.[5] Because GKM split the sample in three size groups, a sort of control for size is applied. Thus, their analysis of systematic risk is not adequate.

[3] For a review of the CAPM one can see, for instance, Cochrane (2005), Chapter 9.

[4] The number of observations is not multiplied by 51 because we ignore the daily return on the portfolio formation dates.

[5] Fama and French (1992) simultaneously classified each stock by its size (market capitalization) and its beta calculated in the five years (at least two for new firms) before portfolio formation. Stocks were classified into ten size deciles, and into ten beta deciles, forming 100 portfolios. For firms in the same size decile, even though there was a large spread in portfolios' betas, the betas could not explain the differences in average returns. Conversely, taking firms in the same beta decile, size could explain differences in average portfolio returns.

Jagannathan and Wang (1996) argue that a major problem with the usual tests of the CAPM, is that beta is assumed to be constant over time. They run tests of the conditional CAPM (CAPM with changing betas) on portfolios similar to Fama and Frenchs's (1992) and find that beta is capable of explaining 30% of the cross-section variation in average returns, against 1% by the unconditional CAPM.[6] This means that if instead of measuring a single beta for each portfolio, we measure multiple betas, we may find greater differences between the betas of the high volume and the low volume portfolios.

With the objective of capturing variations in beta that may be relevant for portfolios classified by volume, we define a state variable, which we label "market activity", and we measure two betas for each volume portfolio: one considering the trading intervals with positive "market activity" on the formation date, and other considering trading intervals with negative "market activity". Indeed, we observe that conditioned on positive "market activity" the difference between the betas of high volume and the low volume portfolios is greater than the difference between unconditional betas. (The difference gets about 0.10, compatible with a premium of about 1% per year.) Additionally, conditioned on negative "market activity", the difference becomes negative. This allows a more powerful test on the contribution of systematic risk to the "high-volume return premium". The high volume portfolio should present greater return conditioned on positive "market activity", and vice-versa. However, the return of the high volume portfolio remains greater, with a statistically significant difference even for negative "market activity". This gives a more robust support to GKM's conclusion that the "high-volume return premium" can not be explained by exposure to systematic risk.

The article proceeds as follows. In the next section, we describe the full procedure used to form portfolios based on GKM's volume classification, the data used in our analysis, and how returns are computed. In Section 2.3 we explain how we improved GKM's test for the difference in systematic risk and present our results. In Section 2.4 we motivate and define the variable "market activity", and

[6] Lewellen and Nagel (2006) provide evidence that "the conditional CAPM does not explain asset-pricing anomalies like B/M or momentum". However, they confirm that "beta fluctuates significantly over time", even after accounting for estimation uncertainty. Besides, they do not question that conditional CAPM explains the cross-section variation in average returns better than the unconditional CAPM.

show how differences between betas of volume portfolios dramatically change conditioned on this variable. In Section 2.5 we apply the test for exposure to systematic risk conditioning on this variable. Section 2.6 concludes.

2.2. Data and portfolio formation

For individual stocks, daily trading volumes[7], closing prices[8], shares outstanding, and adjusted returns[9] are taken from CRSP database, for all common stocks traded at NYSE between January 1963 and December 2007. Whenever we refer to "the" aggregate market index it is the CRSP values weighed index. Below, Table 1 presents summary statistics for these variables for four sub-periods that will be used later to test the high-volume return premium out of the sample. In Table 1 we see a relevant increase in turnover across the periods into which we split the sample. It indicates that trading volume of individual stocks has increased, despite the increase in the number of stocks available.

Following GKM we form consecutive, non-overlapping trading intervals of 50 transaction days. One day is skipped between every two consecutive trading intervals. The 50th day (the portfolio formation date) of each trading interval is used to classify each stock by volume (high, normal or low). This time sequence is depicted in Figure 1. Each stock is classified as high (low) volume if the volume falls in top (bottom) five volumes of the trading interval, that is, if it falls in the top (bottom) decile of the 50 daily volumes; otherwise it is classified as normal volume.[10] It is very important to notice that stocks are classified by volume according to its own time series, and not according to its rank in the cross-section. This implies that for each trading interval the number of stocks in the

[7] As usual, in CRSP database, daily trading volume for a stock is the number of shares of that stock traded on that day.

[8] Whenever there is no transaction of a stock on a trading day, the CRSP database uses the bid/ask average of that day, and indicates this by adding a negative sign to the price value. We chose to consider bid/ask averages as non-missing data, using the absolute value of prices.

[9] Daily returns are usually defined as the ratio between close prices of the current and previous trading day. Closing prices are ex-dividend. Thus, adjusted returns incorporate the additional return due to dividend payments.

[10] Whenever there is a tie, stocks with equal volume are arbitrarily ranked. Ties usually happen if there are many days with zero trading volume in a trading interval. We have also run all analysis eliminating all observations (one stock in one trading interval) with any daily volume equal to zero, either in the trading interval or in the test interval. All results were qualitatively mainained and suffered just small quantitative changes, mainly for the small firm sub-sample.

high volume and in the low volume portfolios may be different. Additionally, the percentage of stocks in each portfolio may vary over trading intervals.

Table 1 - Summary statistics for CRSP data (NYSE common stocks)

Panel A: CRSP value weighed index				
Sub-periods	08/15/1963 to 06/17/1975	06/19/1975 to 02/26/1986	02/28/1986 to 11/01/1996	11/05/1996 to 08/02/2007
Mean daily return	0,010%	0,041%	0,042%	0,035%
Standard deviation of daily returns	0,760%	0,776%	0,857%	1,106%

Panel B: Individual stocks				
Sub-periods	08/15/1963 to 06/17/1975	06/19/1975 to 02/26/1986	02/28/1986 to 11/01/1996	11/05/1996 to 08/02/2007
Mean number of stocks	1336	1528	1953	2672
Median of mean daily returns	0,036%	0,088%	0,055%	0,057%
Median of standard deviation of daily returns	2,218%	2,145%	2,021%	2,155%
Median of yearly turnovers	0,26	0,43	0,69	1,09
Maximum yearly turnover	4,18	32,0	155	769

Through the procedure described in the previous paragraph we end up with a data set containing observations identified by a stock and a trading interval. Each observation is classified by volume (high, low or normal). We remove from this data set observations for which:[11] a) there is volume data missing in the trading or test interval;[12] b) the same for price data; c) the same for return data; d)

[11] These exclusions are made to mimic as close as possible the analysis made by GKM. GKM also excluded observations for which the firm experienced a merger, a delisting, a partial liquidation, or a seasoned equity offering during or within one year prior to the formation date. We did not apply these exclusions because we did not have the data. But we believe these exclusions are arbitrary and do not affect results, or affect by a selection bias that we should avoid. The main reason is that effect of these corporate events on prices, returns and volumes occur on the announcement date. So, if, for instance, a seasonal offering is announced during the trading interval, but it occurs after the formation date, the stock would be affected, but would not be excluded from the data sample. Also, there may be news about corporate events, but that are not confirmed later. Anyway stock prices and volumes are affected by news, but they are not removed from the database.

[12] We consider here only missing data, because they make the analysis unreliable. When trading volume is zero, it is not missing, and is not removed from our sample. (We ran the analysis

the first non zero or missing volume in the CRSP database occurred at least one year before the first date of the trading interval; e) there was at least one day when the stock price fell below US$ 5,00 within the trading interval (excluding the formation date). To best mimic GKM's procedure, we take August 15, 1963, as the first date of the first trading interval, and we ignore the trading intervals with any date falling on the second semester of 1968, because the exchange was closed on Wednesdays, during this period, affecting the measures of trading volume.

To control for firm size effects, the sample was split into sub-samples classified by market capitalization at the end of the year preceding the formation date.[13] Stocks were set to the big firms sub-sample, if they fell in deciles 10 or 9; to the medium firms sub-sample, if they fell in deciles 6 through 8, and, finally, to the small firms sub-sample if they fell in deciles 2 through 5. Stocks in the lowest decile were ignored in the size sub-samples. The combination of volume and size classification forms nine portfolios (the three size sub-samples times the three volume classes).

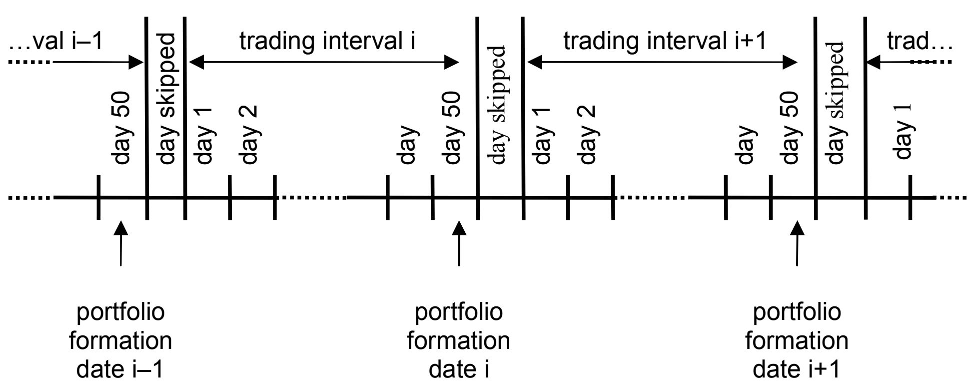

Figure 1 – Time sequence for daily trading intervals

The interval formed by the 50 trading days that immediately follow the formation date for trading interval *i* is called test interval *i*. We are interested on

presented in the sections ahead removing all observations with at least one daily volume equal to zero in the reference period. The results were virtually the same obtained when we do not remove these observations.)

[13] Market capitalization at year end is calculated as the product o closing price on the last trading day of the year by the number of shares outstanding on the same day. If the number of shares outstanding is missing on this day, we take the same product for the last day of the year on which shares outstanding is available.

the behavior of portfolio returns during the test interval[14]. As in GKM, portfolios are equal weighed. They are formed on the trading date and held until next trading date. Portfolios are not rebalanced over the test interval. The cumulative return ($R_{j,i,m}$) for an individual stock j, relative to test interval i, is:

$$R_{j,i,m} = \prod_{d=1}^{w} \left(1 + r_{j,i,d}\right) \tag{1}$$

where d indexes trading dates within the test interval and m refers to the number of trading days used to compute cumulative return for test interval i. The cumulative return ($R_{k,i,m}$) for portfolio k, relative to trading interval i, is:

$$R_{k,i,m} = \frac{\sum_{j \in k} R_{j,i,m}}{\sum_{j \in k} 1} \tag{2}$$

where k is equal to l, n or h for low, normal and high volume portfolios, respectively, within a size subsample. That is, we use equal weighed portfolios. The daily return ($r_{k,i,d}$) for portfolio k, on day d of trading interval i, is:

$$r_{k,i,d} = \frac{R_{j,i,d}}{R_{j,i,d-1}} - 1 \tag{3}$$

Some analyses are also made with 20 days trading intervals, again skipping one day between every two sequential trading intervals. This allows an increase in the number of observations. The considered test interval is the 20 days following the formation date (the holding period of each portfolio).[15] A stock is classified as high (low) volume if its volume, on the last day of the trading interval (formation

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[14] GKM considered a test interval with 100 days. But they show that the "high-volume return premium" is much less intense after the 50th day, even reverting for big firms. So we decided to focus our analysis on the 50 days interval.

[15] This analysis with 20 days trading intervals was not made by GKM. But according to them, the first 20 days after the formation date are the most relevant for the high-volume return premium. In their analysis, either with 50 days or with 10 weeks trading intervals, the cumulative premium in these 20 days is more than ten times the cumulative premium in following 30 days.

date), is among the top (bottom) two daily volumes of the trading interval, that is, if it falls on the top (bottom) decile.

2.3. Improving the power of the tests on differences between betas

Originally GKM estimated the CAPM's betas of the high volume and low-volume portfolios using the following seemingly unrelated regression (SURE) model:

$$r_{i,j,w} = \alpha_j + \beta^{*}_{j}\, r_{i,m,w} + \varepsilon_{i,j}, \tag{4}$$

where $r_{i,j,w}$ refers to the i^{th} test period, for portfolio j (where j may be h for high volume, l for low-volume, or m for market portfolio), and w is the number of days in the beginning of the test period (window) for which the cumulative equal weighed returns are calculated. Then they computed the difference ($\beta_h - \beta_l$), to account for differences in exposure to systematic risk between the high volume and low volume portfolios. They calculated betas for w = 1, 10 and 20. That is, in the regression of equation (4) they used only one return (the cumulative return over 1, 10 or 20 days) for each trading interval. The variance of the estimator of β_j increases with the variance of $\varepsilon_{i,j}$, and decreases with the number of periods. Because returns are very noisy, the 161 trading intervals in GKM's analysis may be insufficient.

To improve the analysis of systematic risk we use daily return of each portfolio over the 50 days after the portfolio formation date. On the 51st day the portfolios are updated. Thus we use daily returns, instead of cumulative returns. However, one may argue that using daily returns, our time series will present higher autocorrelation than if we had taken only one measure of return from each trading interval. To account for this possibility, we will compute directly the difference ($\beta_h - \beta_l$) in:

$$\begin{aligned} r_{i,d,h} - r_{i,d,l} &= (\alpha_h - \alpha_l) + (\beta_h - \beta_l)\; r_{i,d,m} + (\varepsilon_{d,h} - \varepsilon_{d,l}) = \\ &= \alpha^{*} + (\beta_h - \beta_l)\, r_{i,d,m} + \varepsilon^{*}_{i,d}, \end{aligned} \tag{5}$$

where $r_{i,d,p}$ is the daily return of portfolio p on day d (1 to 50) of trading interval i; α^*, β_h and β_l are parameters to be estimated, and $\varepsilon^*_{i,d}$ is an error term. The portfolio index p may be h, for high volume, l, for low volume, or m for marker portfolio, which is taken as the CRSP value weighed index. Estimating parameters from equation (5) we can use the Newey-West estimator for the covariance matrix to obtain standard errors robust to heteroskedasticity and autocorrelations.

Table 2 -
Differences between betas of high and low volume portfolios ($\beta_h - \beta_l$)

The sample period is split in trading intervals of 50 days, skipping one day between every two trading intervals. In each trading interval, stocks are classified according to size and trading volume. Stocks are allocated to the high (low) volume portfolio if its volume on the last day of the trading interval is among the top (bottom) 5 volumes of the stock among the 50 day of the trading interval. The differences ($\beta_h - \beta_l$) are estimated in the following expression:

$$r_{i,d,h} - r_{i,d,l} = \alpha^* + (\beta_h - \beta_l)\, r_{i,d,m} + \varepsilon_{i,d},$$

where $r_{i,d,p}$ is the daily return of portfolio p on day d (1 to 50) of trading interval i; α^*, and ($\beta_h - \beta_l$) are parameters to be estimated, and $\varepsilon^*_{i,d}$ is an error term. The portfolio index p may be h, for high volume, l, for low volume, or m for marker portfolio, which is taken as the CRSP value weighed index. The numbers in parentheses are z-statistics robust to auto-correlation and heteroskedasticity, computed using the Newey-West estimator with 10 lags.

Size Group	Small	Medium	Large
Period:			
08/15/1963 to 11/01/1996	0.0372	0.0363	-0.0004
(161 trading intervals)	(2.63)	(3.04)	(-0.03)
Period:			
08/15/1963 to 08/02/2007	0.0425	0.0418	0.0286
(215 trading intervals)	(3.67)	(4.04)	(2.1)

To improve the power of the test on the differences ($\beta_h - \beta_l$), we also expand our sample by using data out of GKM's sample. Specifically, their sample ends in 1996 and ours goes through 2007. Table 2 presents the results, for the periods ending in 1996 and 2007. In parentheses are z-statistics. Differences ($\beta_h - \beta_l$) are highly statistically significant for the sub-samples of small and medium firms. They are not statistically significant only for the sub-sample of large firms with the sample period ending in 1996, the same used by GKM. But it is worth to note that the high volume premium over the 50 days horizon of the test interval, for GKM's sample, is only statistically significant at 10% level. That is, we found differences ($\beta_h - \beta_l$) that were highly statistically significant whenever the high volume premium was also highly statistically significant. So, differently than GKM, we can not discard, so far, that the high-volume return premium is at least partially explained by systematic risk.

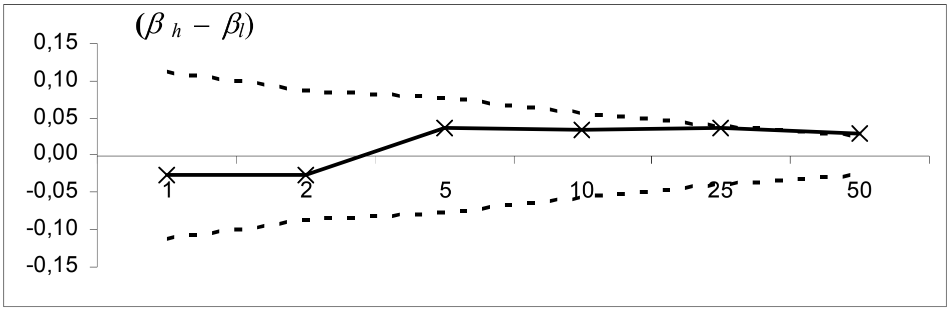

Figure 2.a. – Large firms

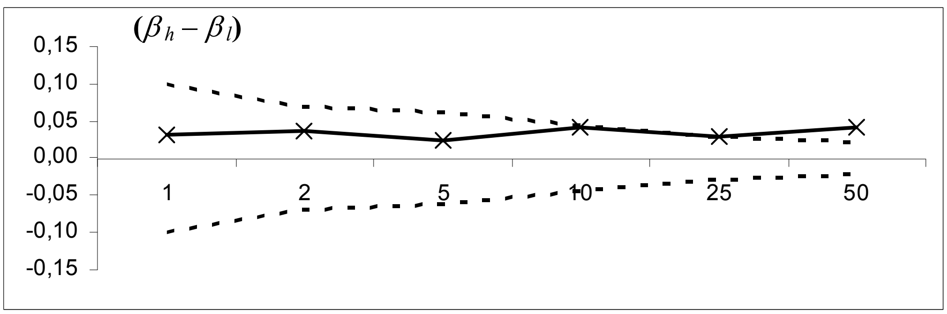

Figure 2.b. – Medium firms

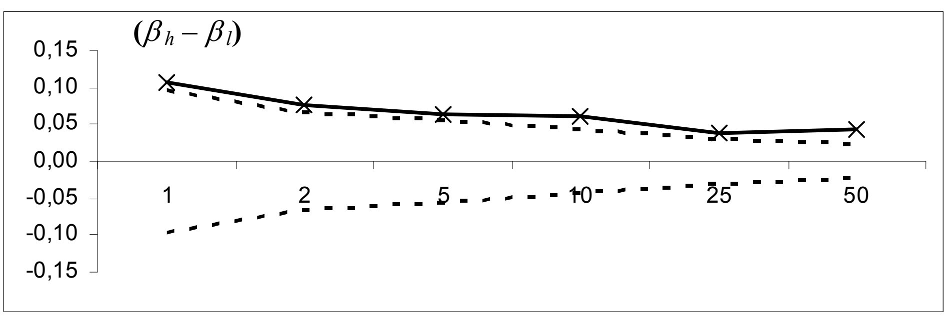

Figure 2.c. – Small firms

Figure 2 – Differences ($\beta_h - \beta_l$) versus number of daily returns. The graphs above plot in the solid line the evolution of the differences ($\beta_h - \beta_l$) between the high volume and low volume portfolios against the number of daily returns in each trading interval used to them, using equation (7). The sample period goes from 08/15/1963 to 08/02/2007. The dashed lines represent the limits of a two standard deviations confidence interval around zero. Whenever the solid line is above the dashed line, the difference ($\beta_h - \beta_l$) is statistically significant at 5% level.

2.4. Evaluating differences between betas conditioned on market activity

So far we have shown that the high volume portfolio has higher exposition to market risk factor than the low volume portfolio, that is, the difference between the betas of the high volume and low volume portfolios ($\beta_h - \beta_l$) in equation (5) is positive.[16] Thus, the "high-volume return premium" may be, at least partially, a compensation for holding higher non-diversifiable risk. But why high (low) volume in the formation date is associated with high (low) beta?

To answer this question, we will refer to the disagreement models literature. Hong and Stein (2007) provide a review of this literature. The starting point for us is that if news generates divergence of opinion among investors about the distribution of future cash flows of stocks, investors will also disagree on the fundamental value of the stocks. As in Harris and Raviv (1993), a recurrent reference in disagreement literature, speculative trading may arise from this divergence of opinion, being reflected in higher volume. Particularly, if there is news that affects aggregate market returns (as macroeconomic news), stocks with returns more correlated with the market index will be the ones subject to speculative trading. Since stocks' betas are essentially positive, stocks with greater betas will be more intensively traded.[17] One important point is that disagreement is about the impact of news, but investors must agree on the future covariance between stocks returns and aggregate market index returns, to choose for trading stocks with greater beta ex-post. (Note that we measure betas using returns after the formation date.)

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[16] Not necessarily the lower beta means lower exposition to systematic risk. Lower beta may stem from the lead-lag effect documented by Lo and MacKinlay (1990). The lead-lag effect is a pattern of cross-autocorrelation between stocks characterized by the positive correlation between the returns of small capitalization stocks with the lags of the returns of larger capitalization stocks. So, the returns of large firms would have higher covariance with returns of aggregate market index, but small firms are also exposed to market risk, with a delay. Since we have separated stocks into sub-samples by market capitalization size, this effect should be significantly reduced within each sub-sample.

[17] Since stocks' betas may theoretically be either positive or negative, selecting stocks with high absolute beta does not guarantee that high-volume portfolio's beta will be higher. Nevertheless, most of the firms for which a risk factor is relevant show similar response to changes in the risk factor. For instance, a reduction in exchange rates will be market wide positive, since it reduces the cost of capital for all firms. The trading volume will change more for high leverage firms, which will benefit the most. This behavior pattern seems to be pervasive, due to difficulty of forming a portfolio of stocks for which returns present a negative covariance with market returns. (Negative beta portfolios are usually formed by short selling stocks, or using other types of assets, such as gold.)

To explain why GKM's volume classification sorts stocks by its ex-post betas we have made two hypotheses: 1) volume of individual stocks is driven by disagreement among investors, and 2) this disagreement is driven by the release of relevant news. The first consequence of these hypotheses is that we should observe a negative correlation between the number of stocks in the low and high volume portfolios. This follows from stocks being classified by volume through comparison with their own recent volumes. Then, if there is news related to systematic risk factors, more stocks are subjected to disagreement, and consequently more stocks will be classified as high volume and less will be classified as low volume. Indeed this correlation is -0.444, -0.394, and -0.275, for the size groups of large, medium and small firms, respectively. (These correlations were calculated using the complete sample period, ending in 2007, and comprising 214 trading intervals.)[18]

So, in our framework, the difference between the number of stocks in the high and low volume portfolios is a proxy for the degree of disagreement stemming from the arrival of news related to aggregate market index returns.[19] We will define "market activity" in period *i* as:

$$A_i = (N_{h,i} - N_{l,i}) / N_i \tag{6}$$

where $N_{h,i}$ and $N_{l,i}$ are the number of stocks in the high volume and low-volume portfolios, respectively, and N_i is the total number of stocks in the sample, in formation period *i*. The denominator N_i is used to normalize our measure, so that A_i is comparable for different formation periods, despite the change in the number of stocks in the sample.[20]

[18] This negative correlation was mentioned, but not explained by GKM.

[19] Since we consider disagreement among rational investors, we can not define *a priori* what kind of news generates disagreement to test hypothesis 2 alone. Maybe, in future research, the opposite can be done, that is, using this proxy, one may try to document what kind of news generates disagreement.

[20] The variable A_i defined by expression (6) does not present trend when we regress it in a time variable (the sequential number of the trading interval) and the square of this time variable. (Results of this regression are not reported.) There is about half trading intervals with A_i >0 (and so about half with A_i ≤0) for all the sub-periods presented in Table 1. This balance between periods with A_i > 0 and A_i ≤ 0 holds considering trading intervals with either 50 days, or 20 days.

Table 3 –
Differences between betas of high and low volume portfolios ($\beta_h - \beta_l$)

The sample period is split in trading intervals of 50 days, skipping one day between every two trading intervals. In each trading interval, stocks are classified according to size and trading volume. Stocks are allocated to the high (low) volume portfolio if its volume on the last day of the trading interval is among the top (bottom) 5 volumes of the stock among the 50 day of the trading interval. The differences ($\beta_h - \beta_l$) are estimated in the following expression:

$$r_{i,d,h} - r_{i,d,l} = \alpha^* + (\beta_h - \beta_l)\, r_{i,d,m} + \varepsilon_{i,d},$$

where $r_{i,d,p}$ is the daily return of portfolio *p* on day *d* (1 to 50) of trading interval *i*; α^*, and ($\beta_h - \beta_l$) are parameters to be estimated, and $\square^*_{i,d}$ is an error term. The portfolio index *p* may be *h*, for high volume, *l*, for low volume, or *m* for marker portfolio, which is taken as the CRSP value weighed index. The numbers in parentheses are *z*-statistics robust to auto-correlation and heteroskedasticity, computed using the Newey-West estimator with 10 lags.

Panel A: period from 08/15/1963 to 11/01/1996 (161 trading intervals)			
Size Group	Small	Medium	Large
Unconditional	0.0372 (2.63)	0.0363 (3.04)	-0.0004 (-0.03)
Conditioning on $A_i > 0$	0.1105 (3.51)	0.1046 (6.48)	0.0628 (2.42)
Conditioning on $A_i \leq 0$	-0.0566 (-4.34)	-0.0509 (-4.14)	-0.0756 (-4.43)
Panel B: period from 08/15/1963 to 08/02/2007 (215 trading intervals)			
Size Group	Small	Medium	Large
Unconditional	0.0425 (3.67)	0.0418 (4.04)	0.0286 (2.1)
Conditioning on $A_i > 0$	0.1140 (4.88)	0.1164 (8.35)	0.1020 (4.14)
Conditioning on $A_i \leq 0$	-0.0419 (-4.17)	-0.0464 (-4.29)	-0.0545 (-4.14)

Another testable implication of the two hypotheses we have made is that the difference between betas of high and low volume portfolios ($\beta_h - \beta_l$) should be greater when A_i is higher. That is, the difference ($\beta_h - \beta_l$) should be greater when there is news related to systematic risk factors and, thus, the separation between stocks with high and low covariance with the market index takes place. In Table 3 we show the difference between high and low portfolios' betas obtained from equation (5), but we use only the returns for test periods with $A_i > 0$, or only the returns for $A_i \leq 0$. We present results for GKM's sample period (1963 – 1996), in Panel A, and for the extended period (1963 – 2007), in Panel B. Numbers in parentheses are z-statistics. The first line in each panel is a copy of the third line from Table 2, that is, the difference $\beta_h - \beta_l$ without conditioning on A_i.

We see in Table 3 that differences ($\beta_h - \beta_l$) at least double when conditioned on $A_i > 0$. Additionally, they become statistically significant at significance levels better than 0.1% even for the large firms sub-sample, and despite the reduction in the sample sizes, due to the split of the original sample according to A_i. The difference ($\beta_h - \beta_l$) conditioned on $A_i > 0$ is about 0.10. And it is about the double of the absolute value of the differences ($\beta_h - \beta_l$) conditioned on $A_i \leq 0$. Even estimating with one return measure per interval trading, as GKM did, the test of conditional differences ($\beta_h - \beta_l$) of this magnitude would probably reject the null hypothesis.

In the beginning of this section, we developed an argument to motivate the definition of variable A ("market activity"). Even if this argument is not valid, the relevance of conditioning differences ($\beta_h - \beta_l$) on this variable is clear. To see if GKM's anomaly can be explained by exposure to systematic risk, we must evaluate how average returns of high volume and low volume portfolios behave, conditioned on A_i. We do this in the next section.

2.5.
Measures of the "high-volume return premium"

An interesting aspect of Tables 3 is that the differences ($\beta_h - \beta_l$) are negative for $A_i \leq 0$. We argue that this happens for the same reason why a positive difference is observed when $A_i \leq 0$. Stocks with lower betas will not present high volume caused by shocks to systematic risk factors, on days within the reference period. Then there will be less days with abnormally high trading volumes in the reference period. This increases the probability of these stocks being included in the high volume portfolio, when there is no shock to a systematic risk factor on the formation period. The opposite is valid for high beta stocks. Thus, when $A_i \leq 0$, stocks with lower betas have higher probability of being classified as high volume.

But again, it does not matter whether this argument is valid or not. If the conditional CAPM holds, then the "high-volume return premium" should be negative, when conditioned on $A_i \leq 0$. To test this implication, we calculate the high-volume return premium according to GKM's procedure. First we calculate the cumulative return over *m* days of the test period, for each trading interval *i*, for both the high volume portfolio ($R_{h,i,\mathrm{m}}$) and the low volume portfolio ($R_{l,i,\mathrm{m}}$), as in expression (3). Following, we calculate the difference between the cumulative returns of high volume and low volume portfolios for each trading interval. And finally we calculate the high-volume return premium for *m* days in the test period $\left(\overline{HVP_m}\right)$ as the mean difference between the high volume and low volume portfolios:

$$\overline{HVP_m} = \frac{\sum_i \left(R_{h,i,m} - R_{l,i,m}\right)}{\sum_i 1} \qquad (7)$$

We then calculate high-volume return premium conditioned on A_i $\left(\overline{HVP_{m,I_A}}\right)$ as:

$$\overline{HVP_{m,A>0}} = \frac{\sum_i I_{A,i}\left(R_{h,i,m} - R_{l,i,m}\right)}{\sum_{j\in k} I_{A,i}} \qquad (8a)$$

$$\overline{HVP_{m,A\leq 0}} = \frac{\sum_i \left(1 - I_{A,i}\right)\left(R_{h,i,m} - R_{l,i,m}\right)}{\sum_{j\in k}\left(1 - I_{A,i}\right)} \qquad (8b),$$

where $I_{A,i}$ is a dummy that is equal to 1 if $A_i > 0$, and equal to zero otherwise. Table 4 presents the results. The numbers in parenthesis are the *t*-statistics for the mean difference. The first line contains unconditioned results.

We see in Table 4 that even for $A_i \leq 0$, there are statistically significant premiums for small and medium firms. Additionally, the premiums are of same magnitude as for $A_i > 0$. However, for large firms the high-volume premium is significant, at usual levels, only when $A_i > 0$ (and specifically for the 20 days horizon).

Table 4 - High-volume return premium - 50 days trading intervals

At the end of every 50th trading day between 08/15/1963 and 08/02/2007 (214 trading intervals)[21], equally weighed portfolios are formed according to the trading volume. The high volume return premium is computed as the mean difference between the returns of the high volume and low volume portfolios. It is computed over four different horizons following the formation date: 1, 10, 20, and 50 trading days. Numbers in parentheses are *t*-statistics robust to heteroskedasticity. Variable A_i is calculated as in expression (6).

Panel A: Large Firms				
Test period (in days):	1	10	20	50
Unconditional	0.00% (0.06)	0.17% (1.33)	0.30% (1.59)	0.10% (0.38)
Conditioning on $A_i > 0$	-0.05% (-0.70)	0.30% (1.52)	0.53% (1.93)	0.31% (0.77)
Conditioning on $A_i \leq 0$	0.06% (1.02)	0.05% (0.28)	0.06% (0.25)	-0.10% (-0.26)
Panel B: Medium Firms				
Test period (in days):	1	10	20	50
Unconditional	0.18% (4.80)	0.59% (5.23)	0.75% (4.63)	1.02% (4.46)
Conditioning on $A_i > 0$	0.20% (3.28)	0.45% (2.45)	0.69% (2.59)	1.06% (2.88)
Conditioning on $A_i \leq 0$	0.17% (3.60)	0.73% (5.69)	0.81% (4.39)	0.97% (3.63)
Panel C: Small Firms				
Test period (in days):	1	10	20	50
Unconditional	0.31% (7.00)	0.98% (8.74)	1.20% (7.95)	1.15% (4.87)
Conditioning on $A_i > 0$	0.26% (3.64)	0.96% (5.97)	1.14% (5.07)	0.91% (2.55)
Conditioning on $A_i \leq 0$	0.35% (7.20)	1.01% (6.37)	1.26% (6.24)	1.40% (4.50)

Because in this analysis a premium of 0.3% in a 50 days horizon (1.5% per year) is not statistically significant (see Panel A of Table 4), but is economically significant, we make a change to improve the power of the test. To increase the number of observations, we reduce the trading interval from 50 days to 20 days. This more than doubles the number of trading intervals (observations). The values obtained for the high-volume premium considering this change are presented in Table 5.[22] It now becomes clear that even for large firms we have a positive and significant premium for the high volume portfolio.

Table 5 - High-volume return premium - 20 days trading intervals

At the end of every 20th trading day between 08/15/1963 and 10/31/2007 (524 trading intervals)[23], equally weighed portfolios are formed according to the trading volume. The high volume return premium is computed as the mean difference between the returns of the high volume and low volume portfolios. It is computed over three different horizons following the formation date: 1, 10, and 20 trading days. Numbers in parentheses are *t*-statistics robust to heteroskedasticity. Variable A_i is calculated as in expression (6).

	Panel A: Large Firms		
Test period (in days):	1	10	20
Unconditional	0,12% (3,44)	0,41% (4,84)	0,60% (5,35)
Conditioning on $A_i > 0$	0,14% (3,09)	0,29% (2,95)	0,83% (4,90)
Conditioning on $A_i \leq 0$	0,11% (1,81)	0,50% (3,86)	0,31% (2,31)
	Panel B: Medium Firms		
Test period (in days):	1	10	20
Unconditional	0,18% (6,46)	0,51% (7,13)	0,54% (5,39)
Conditioning on $A_i > 0$	0,18% (4,01)	0,47% (5,49)	0,55% (3,57)
Conditioning on $A_i \leq 0$	0,18% (6,38)	0,54% (4,96)	0,52% (4,50)
	Panel C: Small Firms		
Test period (in days):	1	10	20
Unconditional	0,32% (11,33)	0,67% (9,80)	0,83% (9,14)
Conditioning on $A_i > 0$	0,30% (7,36)	0,72% (7,96)	0,79% (5,89)
Conditioning on $A_i \leq 0$	0,34% (9,06)	0,63% (6,32)	0,87% (7,53)

[21] Three trading intervals with dates falling on the second semester of 1968 were discarded.

[22] The differences between the betas of the high volume and low volume portfolio, using 20 days trading interval, are of the same magnitude of the ones obtained with 50 days trading intervals, either conditioning on $A_i > 0$, on $A_i \leq 0$, or not conditioning on A_i. (These results are nor presented in this report.)

[23] Six trading intervals with dates falling on the second semester of 1968 were discarded.

The results presented in tables 4 and 5 may either reject the conditional CAPM or support GKM's conjecture that the "high-volume return premium" is an anomaly to the efficient market hypothesis. However, it is not necessary that the conditional CAPM holds exactly to provide at least the expected sign for the premium. We thus have a more robust support to GKM's conclusion.

Table 6 - High-volume return premium for different sub-periods

At the end of every 20th trading day between 08/15/1963 and 10/31/2007 (524 trading intervals), equally weighed portfolios are formed according to the trading volume. The high volume return premium is computed as the mean difference between the returns of the high volume and low volume portfolios. It is computed over three different horizons following the formation date: 1, 10, and 20 trading days. Numbers in parentheses are *t*-statistics robust to heteroskedasticity.

Sub-periods	08/15/1963 to 05/21/1975	05/23/1975 to 03/11/1986	03/13/1986 to 12/24/1996	12/27/1996 to 10/31/2007
# trading intervals	134*	130	130	130
Panel A: Large firms				
$\overline{HVP_{20}}$	0,99% (4,56)	0,60% (2,66)	0,25% (1,43)	0,54% (2,03)
$\overline{HVP_{10}}$	0,71% (3,72)	0,55% (3,27)	0,16% (1,31)	0,20% (1,10)
$\overline{HVP_{1}}$	0,15% (2,33)	0,08% (1,51)	0,11% (2,30)	0,16% (1,47)
Panel B: Medium firms				
$\overline{HVP_{20}}$	0,98% (4,27)	0,56% (2,50)	0,37% (2,35)	0,21% (1,29)
$\overline{HVP_{10}}$	0,84% (4,78)	0,53% (4,39)	0,35% (2,67)	0,30% (2,31)
$\overline{HVP_{1}}$	0,27% (3,35)	0,21% (4,95)	0,13% (2,70)	0,11% (2,67)
Panel C: Small firms				
$\overline{HVP_{20}}$	1,15% (5,91)	0,90% (4,83)	0,84% (5,00)	0,40% (2,42)
$\overline{HVP_{10}}$	0,89% (6,10)	0,72% (4,55)	0,59% (5,04)	0,47% (4,00)
$\overline{HVP_{1}}$	0,50% (7,13)	0,36% (7,22)	0,28% (5,84)	0,12% (2,60)

* There are 140 trading intervals with 20 days, and one day skipped between each two in a sequence, but we excluded 6 trading interval with some date falling on the second semester of 1968, as GKM did, because exchange was closed on Wednesday during this period, affecting trading volume measures, used to classify stocks.

We may, however, ask if the anomaly persists out of the sample used by GKM. According to Schwert (2003), anomalies tend to disappear after they are first reported. To perform this analysis we break our sample into four sub-periods with approximately same number of trading intervals. (We leave the first sub-period with more trading intervals.) Because the number of trading intervals by sub-period is one fourth of the total number of trading intervals, there are left only 53 observations by sub-period, if we use 50 days trading intervals. Again, to increase the number of observations, we work with 20 days trading intervals. Results are presented in Table 6. Although we can see a declining trend in the volume premium, it continues statistically and economically significant after the sample period used by GKM.[24]

2.6. Conclusion

We have analyzed the "high-volume return premium", an anomaly to the efficient markets hypothesis reported by Gervais, Kaniel and Mingelgrin (2001), abbreviated GKM. They show that trading volume, which is public information obtained at low cost, can predict expected returns. This article questions the test used to reject the hypothesis that the difference in expected returns is due to systematic risk.

We increased the power of the test on the difference between the betas of the high volume and the low volume portfolios. The result was an economically and statistically significant difference, in direct contradiction with GKM. We also argued that when market activity is more intense, that is, when more stocks are traded with abnormally high volume, the difference between the betas of the two volume portfolios should be greater. We provided empirical evidence supporting this conjecture. Additionally, we have shown that when market activity is less intense, the beta difference is negative. This opened the opportunity to improve the test on the relation between the volume premium and systematic risk, because the negative beta difference is expected to be associated with a negative premium. This hypothesis was strongly rejected.

[24] GKM's paper was published in 2001, but early drafts were available since 1997.

Our result gave a more robust support to GKM's conclusion that the volume premium can not be explained by systematic risk.

Differently than other anomalies that disappear after they are reported, the "high volume return premium" survives after GKM's sample period. However when we break the period between January 1963 and December 2007 in four sub-periods, we see a declining trend in the value of the premium.