IX
Conclusion

IX.1 Contributions

Description Logics have well-known and mature proof procedures based on Tableaux for reasoning on Ontologies and Knowledge Bases. The task of understanding the outcomes of formal proof procedure or consistency tests is sometimes quite hard. Explanations on the reasons for some subsumptions either hold or not are demanding. The latter is in general supported by a human-readable translation of the witness construction obtained by the usual, first-order inspired, Tableaux DL procedure. For the former, however, an explanation should be obtained from the proof resulted by this very Tableaux procedure.

Considering the logical motivation of providing a purely propositional (not based on nominals) proof procedure for propositional DLs, we show two Sequent Calculus and two Natural Deductions defined by purely propositional terms. Considering the concrete use of DL reasoners, we believe that the use of a system that allow the use of non-analytic cuts (non-atomic cuts) is interesting whenever one takes into account the super-polynomial size of some cut-free proofs (such as the Pigeonhole Principle). Besides that, producing proofs of subsumptions inside a **TBOX**, without making use of the terminological gap imposed by the traditional Tableaux procedure, seems to an interesting step towards better explanation generations.

The main contributions of this thesis are twofold. Firstly, from the point of view of producing short proofs, we define proof systems that are able to produce proofs or derivations with cuts ($\mathrm{SC}_{\mathcal{ALC}}$, $\mathrm{SC}^{\square}{}_{\mathcal{ALC}}$ and its extension for $\mathcal{ALCQI}$) as well as non-normal proofs ($\mathrm{ND}_{\mathcal{ALC}}$ and its extension for $\mathcal{ALCQI}$). The elimination of the cut rule as well as the normalization theorem are mandatory proof-obligations performed in this thesis aiming to prove that the systems are minimally mechanizable. The other contribution made in this thesis relies on the fact that the Sequent Calculus as well as the Natural Deduction are not strongly based on first-order mechanisms and interpretations as the known Tableaux procedure are. The systems are purely propositional. In order to achieve this feature, a strong use of labeled formulas is made. Thus, both, the Sequent Calculus and the Natural Deduction are labeled deductive systems, following the tradition initiated by Dov Gabbay [29]. Both features are steps towards the possibility of generating quite human-readable explanations. Besides those previous mentioned contribuitions we think that presenting an alternative proof procedure for a well-know logic is a contribution in its own.

Regarding the Natural Deduction systems presented for $\mathcal{ALC}$ and $\mathcal{ALCQI}$, despite providing a variation of themes, the main motivation is the possibility of getting ride on a weak form of the Curry-Howard isomorphism in order to provide explanations with greater content. This last affirmative takes into account that the reading (explanatory) content of a proof is a direct consequence of its computational content.

We not only presented ND systems for $\mathcal{ALC}$ and $\mathcal{ALCQI}$ but also showed, by means of some examples, how they can be useful to explain formal facts on theories obtained from UML models. Instead of UML, ER could also be used according a similar framework. Regarding the examples used and the explanations obtained, it is worthwhile noting that the Natural Deduction proofs obtained are quite close to the natural language explanation provided. It is a future task to provide the respective natural language explanation for a comparison. We aimed to show that **ND** deduction systems are better than Tableaux and Sequent Calculus as structures to be used in explaining theorem when validating theories in the presence of false positives. That is, when a valid subsumption should not be the case. We also remark and show how normalization is important in order to provide well-structured proofs.

We brifey suggest how to use the structural feature of sequent calculus in favour of producing explanations in natural language from proofs. As it was remarked at the introduction, the use of the cut-rule can provide shorter proofs. The cut-rule does not increase the complexity of the explanation, since it simply may provide more structure to the original proof. With the help of the results reported in this thesis one has a solid basis to build mechanisms to provide shorter and good explanation for $\mathcal{ALC}$ subsumption in the context of a KB authoring enviroment. The inclusion of the cut-rule, however, at the implementation level, is a hard one. Presently, there are approaches to include analytical cuts in Tableaux, as far as we know there is no research on how to extend this to $\mathcal{ALC}$ Tableaux. This puts our results in advantage when taking explanations, and the size of the proofs as well, into account. There are also other techniques, besides the use of the cut-rule, to produce short proofs in the sequent calculus, see [31] and [26], that can be used in our context.

IX.2 Future Work

Future investigation must include the following topics:

- The extension of the calculi in order to deal with stronger Description Logics, mainly, $\mathcal{SHIQ}$ [1];
- The development of methods for proof explanation extraction from proofs;
- A proof of completeness for $\mathrm{ND}_{\mathcal{ALCQI}}$ and $\mathrm{SC}_{\mathcal{ALCQI}}$ should be obtained by extending the completeness proof for $\mathrm{SC}_{\mathcal{ALC}}$;
- The development of constructive (Intuitionistic) versions of $\mathrm{ND}_{\mathcal{ALC}}$ and $\mathrm{SC}_{\mathcal{ALC}}$. The starting point should be the study of some proposed constructive semantics for $\mathcal{ALC}$ [20, 8, 46].