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Alexandre Rademaker

A Proof Theory for Description Logics

TESE DE DOUTORADO

Thesis presented to the Postgraduate Program in Informatics of the Departamento de Informática, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Informática.

Advisor: Prof. Edward Hermann Haeusler

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Abstract

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Description Logics (DLs) is a family of formalisms used to represent knowledge of a domain. They are equipped with a formal logic-based semantics. Knowledge representation systems based on description logics provide various inference capabilities that deduce implicit knowledge from the explicitly represented knowledge. In this thesis we investigate the Proof Theory for DLs. We introduce Sequent Calculi and Natural Deduction for some DLs (ALC, ALCQ). Cut-elimination and Normalization are proved for the calculi. It is argued that those systems can improve the extraction of computational content from DLs proofs for explanations purpose.

Keywords

Proof theory. Sequent Calculus. Natural Deduction. Description Logics.

Resumo

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Lógicas de Descrição são uma família de formalismos usada para representação de conhecimento de um domínio. Elas são equipadas com uma semântica formal. Conhecimento representado em sistemas baseados em lógicas de descrição oferecem várias capacidades de inferência para dedução de conhecimentos implícitos a partir dos explicitamente representados. Nesta tese investigamos teoria da prova para DLs. Apresentamos Cálculos de Sequentes e Deduções Naturais para algumas DLs (ALC, ALCQ). Eliminação do corte e normalização são provadas para os sistemas apresentados. Argumentamos que tais sistemas podem melhorar a obtenção de conteúdo computacional de provas em DLs, facilitando a geração de explicações.

Palavras-chave

Teoria da Prova. Cálculo de Sequentes. Dedução Natural. Lógicas descritivas.

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