5
Conclusion and Future Work

5.1 Contribution of this work

This thesis is related to the Permutation Flow Shop Scheduling Problem with makespan minimization (`PFS`). In this work, three major contributions were obtained for this problem.

The first one is an approximation algorithm for the `PFS` problem with n jobs and m machines. This algorithm achieves an approximation guarantee of $O(\sqrt{n+m})$ and runs in linear time. This is the best performance ratio already obtained for the `PFS` problem in the case of $n = \Theta(m)$. Furthermore, a novel connection between PFS and monotone subsequence problems is established, resulting on an extension of the Erdös-Szekeres theorem to weighted monotone subsequences.

The second result is a faster algorithm for the `2-PFS` problem. We give an $O(n \log k)$ algorithm that determines optimal solutions for the `2-PFS` problem, where $k \leq n$ is the minimum number of cliques necessary to cover the nodes of an underlying interval graph. From the best of our knowledge, this is the first improvement upon the $O(n \log n)$ time complexity of the classical algorithm from Johnson.

The third contribution of this work is a new family of competitive deterministic heuristic for the `PFS` problem. Four new heuristics are introduced as extensions of the classical NEH heuristic. Such heuristics are based on pruning techniques on the implicit enumeration tree of the `PFS` problem. Computational results attest that the new proposed methods stand among the most effectives for the `PFS` problem.

5.2 Open problems

At this section, some open problems related to this thesis and the PFS problem will be pointed out.

First of all, determining the approximation factor of the well-performing NEH heuristic is an interesting open problem. Despite the large interest on finding theoretical arguments that justify the good practical performance of NEH, little advance has been made on proving the approximation guarantee of this popular heuristic.

Another point concerns obtaining a PTAS for PFS when the number of machines is fixed. This problem was only solved for $m = 3$ machines by Hall [30]. A recent paper from Jansen et al. [33] introduced an equivalent approximation scheme for job shops. There is a strong belief in scheduling community that the techniques used in this paper can be adapted to flow shops.

Obtaining inapproximability results for the PFS problem is another interesting research line. In particular, how the PCP theorem could by applied to the PFS in order to prove its inapproximability at some point?

Regarding to the 2-PFS problem, could the optimal makespan of a 2-PFS instance be calculated in linear time, without determining the optimal permutation schedule? Is there a polynomial time characterization of the 2-PFS optimal solutions?

The current state-of-art on exact methods for solving the PFS problem does not comprise the use of linear programming to obtain good lower bounds. A reason for this phenomena comes from the fact that the integrality gaps of the existing LP formulations for PFS are considerably high. Furthermore, its execution times are quite expensive to be used in methods like branch-and-cut and branch-and-price. Are there new LP formulations for PFS that achieve better integrality gaps and/or run faster?

Finally, there is a new combinatorial optimization problem that emerges from the analysis of PFS by a worst-case perspective. This problem can be defined as follows: given a PFS instance I, what would be the worst possible permutation schedule for I? In particular, could this bad schedule be determined in polynomial time? If the problem is *NP-hard*, does it admit an approximation algorithm with a constant factor or a PTAS?