



**David Sotelo Pinheiro da Silva**

## **On the Permutation Flow Shop Scheduling Problem**

### **TESE DE DOUTORADO**

Thesis presented to the Postgraduate Program in Informatics of the Departamento de Informática, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Informática.

Advisor: Prof. Marcus Vinicius Soledade Poggi de Aragão

Rio de Janeiro  
May 2010



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To Bianca, Julia and Lilo.

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## Resumo

Silva, David Sotelo Pinheiro da; Poggi de Aragão, Marcus Vinicius Soledade. **Sobre o Problema de Escalonamento Permutation Flow Shop**. Rio de Janeiro, 2010. 80p. Tese de Doutorado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

Nos últimos cinquenta anos, o Problema de Escalonamento do tipo Permutation Flow Shop com minimização de makespan (PFS) tem se mostrado um problema central e intensamente estudado pela comunidade de otimização combinatória, conhecido por sua intratabilidade, relacionada tanto a aspectos teóricos quanto computacionais. Neste trabalho, três contribuições principais foram obtidas para o problema PFS.

A primeira contribuição consiste em um algoritmo aproximativo para o problema PFS com  $n$  tarefas e  $m$  máquinas. Este algoritmo apresenta um fator de aproximação de  $O(\sqrt{n+m})$  e pode ser executado em tempo linear. Este é o melhor fator de aproximação já obtido para o problema PFS no caso em que  $n = \Theta(m)$ . Além disso, uma nova conexão é estabelecida entre problemas em subsequências monótonas e o PFS, resultando em uma extensão do teorema de Erdős-Szekeres para subsequências monótonas com pesos.

O segundo resultado é um algoritmo mais rápido para o problema PFS com  $n$  tarefas e 2 máquinas (2-PFS). Nós fornecemos um algoritmo com complexidade  $O(n \log k)$  que determina soluções ótimas para o problema 2-PFS, onde  $k \leq n$  é o número mínimo de cliques necessárias para cobrir os vértices de um grafo de intervalo correspondente. Até onde nosso conhecimento permite afirmar, este é o primeiro avanço obtido no que se diz respeito à complexidade de tempo  $O(n \log n)$  do algoritmo clássico de Johnson.

A terceira contribuição deste trabalho é uma nova família de heurísticas determinísticas competitivas para o problema PFS. Quatro novas heurísticas são apresentadas como extensões da heurística clássica NEH. Tais heurísticas são baseadas em técnicas de poda na árvore de enumeração implícita do problema PFS. Resultados computacionais atestam que os novos métodos propostos figuram entre os mais efetivos para o problema PFS.

### Palavras-chave

Escalonamento permutation flow shop. Algoritmos aproximativos. Subsequências monótonas. Grafos de intervalo. Cobertura por cliques. Heurística NEH.

## Abstract

Silva, David Sotelo Pinheiro da; Poggi de Aragão, Marcus Vinicius Soledade. **On the Permutation Flow Shop Scheduling Problem**. Rio de Janeiro, 2010. 80p. DSc Thesis — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

In the last fifty years, the Permutation Flow Shop Scheduling Problem with makespan minimization (PFS) has been a central and well-studied problem in combinatorial optimization community, known by its intractability from theoretical and computational aspects. In this work, three major contributions were obtained for the PFS problem.

The first one is an approximation algorithm for the PFS problem with  $n$  jobs and  $m$  machines. This algorithm achieves an approximation guarantee of  $O(\sqrt{n+m})$  and runs in linear time. This is the best performance ratio already obtained for the PFS problem in the case of  $n = \Theta(m)$ . Furthermore, a novel connection between PFS and monotone subsequence problems is established, resulting on an extension of the Erdős-Szekeres theorem to weighted monotone subsequences.

The second result is a faster algorithm for the PFS with  $n$  jobs and two machines (2-PFS). We give an  $O(n \log k)$  algorithm that determines optimal solutions for the 2-PFS problem, where  $k \leq n$  is the minimum number of cliques necessary to cover the nodes of an underlying interval graph. From the best of our knowledge, this is the first improvement upon the  $O(n \log n)$  time complexity of the classical algorithm from Johnson.

The third contribution of this work is a new family of competitive deterministic heuristic for the PFS problem. Four new heuristics are introduced as extensions of the classical NEH heuristic. Such heuristics are based on pruning techniques on the implicit enumeration tree of the PFS problem. Computational results attest that the new proposed methods stand among the most effectives for the PFS problem.

## Keywords

Permutation flow shop scheduling. Approximation algorithms. Monotone subsequences. Interval graphs. Clique covering. NEH heuristic.

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