2
Central Bank Communication and Price Setting

2.1.
Introduction

There is a vast literature that documents a recent phenomenon - central banks around the world have changed their communication policies toward increasing transparency.[12] What is the rationale behind this change in attitude? How does information interfere on price setting? What is the role of communication on inflation and welfare? In order to address these questions, we exam how central bank communication alters firms' expectations about the current state of the economy and, consequently, pricing decisions, inflation, and welfare.

We introduce public signals in an imperfect information model where firms face strategic complementarity on their pricing decisions. Strategic complementarity forces firms to infer one another's actions as their payoffs depend not only on their own prices, but also on the prices set by other firms. In this context, firms take their pricing decisions using information to build expectations on the prices set by other firms and on the current state of aggregate nominal demand - the fundamental of our economy.

Our baseline framework is the sticky-dispersed information (SDI) model of chapter 1. This model mixes the sticky information model of Mankiw and Reis (2002) with dispersed information models like Morris and Shin (2002) and Angeletos and Pavan (2007).[13] As in Mankiw and Reis (2002), information is sticky, once only a fraction of firms update their information set at each period. This assumption creates persistence on the aggregate price index, since new information diffuses slowly. We also consider that information is heterogeneous between agents. As in Morris and Shin (2002) and Angeletos and Pavan (2007),

[12]See for instance Blinder et al. (2008) and Dincer and Eichengreen (2009).

[13]See Mankiw and Reis (2010) for the most recent survey of the literatures of dispersed information and sticky information models.

we assume that firms receive private signal of the fundamental when they update. To this framework, stated in chapter 1, we incorporate public information assuming that central bank releases new information about the current state of the economy every period. This information is available to all firms, including those that were not selected to update their information set.

Public information has a much stronger influence on inflation dynamics than private information. In chapter 1, it was shown that private signals modify inflation dynamics only through the strategic interaction among differently informed agents and their beliefs about the state. The rationale comes from the fact that idiosyncratic socks die out when we consider the aggregate price index. In contrast, our model shows that, as all firms use public information to support their pricing decisions, shocks from public signals have a direct impact on inflation. Although our structure incorporates stickiness, this result is in line with the dispersed information literature.[14] Nevertheless, stickiness allows us to study how public information drives inflation. We show that, if information is sticky, the impact of public signals on inflation last forever. However, as public signal becomes more precise, past shocks becomes relatively less important than current shocks, making inflation less persistent.

Following Angeletos and Pavan (2007), we use the *ex-ante* total profit as a welfare measure to evaluate the inefficiency created when firms set their prices without considering that they affect pricing decision of the other agents. We show that taxation helps to improve social welfare, just as in Angeletos and Pavan (2009). However, it has a difficult implementation. On the other hand, social welfare improves with the precision of the public signal, suggesting that central bank should improve transparency and the quality of the information it releases to the public as a means of making communication more precise.

We introduce the model in the next section and characterize the equilibrium in Section 2.3. We discuss the implications of the model for inflation in Section 2.4 and for welfare in Section 2.5. Section 2.6 draws the concluding remarks. All derivations that are not in the text can be found in the Appendix.

[14]See, for instance, Morris and Shin (2002) and Angeletos and Pavan (2007).

2.2. The Model

We incorporate a public signal to the sticky-dispersed information model studied at chapter 1. As before, we depart from Mankiw and Reis' (2002) standard sticky-information model by allowing information to be not only *sticky* but also heterogeneous and *dispersed*.

Pricing Decisions

There is a continuum of firms, indexed by $z \in [0,1]$. Every period $t \in \{1, 2, ...\}$, each firm z chooses its price $p_t(z)$. We can derive from a model of monopolistic competition à la Blanchard and Kiyotaki (1987) that the (log-linear) price decision that solves firms' profit maximization problem, p_t^*, is the same for all firms and given by

$$p_t^* = rP_t + (1-r)\theta_t, \tag{2.1}$$

where $P_t \equiv \int_{[0,1]} p_t(z)dz$ is the aggregate price level, θ_t is the nominal aggregate demand, the current state of the economy, and r is the degree of strategic complementarity.

Information

At period t, only a fraction λ of firms is selected to update their information sets about the current state of the economy. For simplicity, the probability of being selected to adjust information sets is the same across firms and independent of history. A firm that updates its information set receives information regarding the past states of the economy as well as a *private* signal about the current state. Additionally to this structure, already described at chapter 1, we assume that there is a *public* signal that is available every period at no cost.

If firm z is selected to update its information set in period t, it observes all *previous* periods realizations of the state,

$$\Theta_{t-1} \equiv \{\theta_{t-k}\}_{k=1}^{\infty},$$

and a noisy *private* signal about the current state,

$$x_t(z) \equiv \theta_t + \xi_t(z).$$

The shock $\xi_t(z)$ is idiosyncratic to each firm z and is distributed according to $N(0,\beta^{-1})$. We also consider that every period the central bank obtains new information about the fundamental growth,

$$r_t = \theta_t - \theta_{t-1} + u_t.$$

The central bank can get this information either from surveys or models. Whatever the origin, the shock u_t reflects that central banks' information about the change in the state is imprecise. Furthermore, communication is imperfect. That is, the public signal that is available to all firms, including those who have not been selected to update their information sets, is

$$y_t = r_t + v_t.$$

To simplify our notation, we write this signal as

$$y_t = \theta_t - \theta_{t-1} + \eta_t,$$

where $\eta_t \equiv u_t + v_t$ is a composite shock. As a result, the information set of a firm z that was selected to update its information j periods ago is

$$\Im_{t-j}(z) = \{x_{t-j}(z), \Theta_{t-j-1}, Y_t\},$$

where $Y_t = \{y_{t-k}\}_{k=0}^{\infty}$.

Firms also know that the state θ_t follows the process

$$\theta_t = \theta_{t-1} + \varepsilon_t.$$

Finally, we assume that all errors are independent of one another,

$$\varepsilon_t \perp \xi_{t+i}(z) \perp \eta_{t+k}, \forall(t,i,k,z),$$

and are distributed according to $\varepsilon_t \sim N(0,\alpha^{-1})$, $\xi_t(z) \sim N(0,\beta^{-1})$, and $\eta_t \sim N(0,\gamma^{-1})$.

2.3. Equilibrium

Using (2.1), the best response for a firm z that was selected to update its information j periods ago -- and, therefore, has $\Im_{t-j}(z)$ as its information set -- is its forecast of p_t^*, given the available information $\Im_{t-j}(z)$:

$$p_{j,t}(z) = E[p_t^* \mid \Im_{t-j}(z)]. \tag{2.2}$$

When we aggregate this expression, we obtain the price index P_t as

$$\begin{aligned} P_t &= \int_{[0,1]} p_t(z) d\mu \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} p_{j,t}(z) d\mu, \end{aligned} \tag{2.3}$$

where Λ_{t-j} is the set of firms that last updated its information set at period $t-j$ and μ is the Lebesgue measure. Computing the equilibrium requires finding the set of individual prices, $p_{j,t}(z)$, that satisfies (2.2), considering that P_t in (2.1) is given by (2.3). If firms knew θ_t, the equilibrium would be $p_{j,t}(z) = \theta_t = P_t = p_t^*$, $\forall z$. As firms do not know θ_t, this complete information equilibrium does not hold. The incomplete information equilibrium requires that firms use their information set to make forecasts about the current state of the economy and the price level. But, as P_t encompasses the equilibrium prices set by other firms, firm z must also predict the behavior of the other firms in the economy by making forecasts of these firms' forecasts about the state, forecasts about the forecasts of these firms' forecasts about the state, and so on and so forth.

This recursiveness shows the importance of computing high order beliefs to obtain the equilibrium. Mathematically, we can express the equilibrium price level as function of the beliefs by plugging (2.1) into (2.3) whenever P_t appears on the expression. This recursive procedure results in

$$P_t = (1-r) \sum_{k=1}^{\infty} r^{k-1} \bar{E}^k [\theta_t], \tag{2.4}$$

where the k -th order belief is given by

$$\begin{aligned} &\bar{E}^0[\theta_t] = \theta_t, \text{ and} \\ &\bar{E}^k[\theta_t] = \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[\bar{E}^{k-1}[\theta_t] \mid \Im_{t-j}(z)\right] d\mu. \end{aligned}$$

This is the same result we found at chapter 1. However, now $\Im_{t-j}(z)$ includes a public signal. This fact changes the way firms compute their expectations and, consequently, the equilibrium. As all firms share part of the information, prices will be partially synchronized. This behavior will be clear when we find expressions for the equilibrium prices.

2.3.1. Expectations

In order to compute the equilibrium, it is necessary understand how a firm z that updated its information set at $t-j$ computes its beliefs about a fundamental θ_{t-m}. Since firm z observes all previous states at the moment it adjusts its information set, it knows for sure the value of θ_{t-m} when $m > j$ (i.e., $\theta_{t-m} \in \Theta_{t-j-1}$, $\forall m > j$). Therefore, $E[\theta_{t-m} \mid \Im_{t-j}(z)] = \theta_{t-m}$. For $m \le j$, θ_{t-m} is not in the information set of firm z. However, it knows that

$$\theta_{t-m} = \theta_{t-j-1} + \sum_{i=m}^{j} \varepsilon_{t-i}.$$

Since $\theta_{t-j-1} \in \Im_{t-j}(z)$, it computes $E[\theta_{t-m} \mid \Im_{t-j}(z)]$ as

$$E[\theta_{t-m} \mid \Im_{t-j}(z)] = \theta_{t-j-1} + \sum_{i=m}^{j} E[\varepsilon_{t-i} \mid \Im_{t-j}(z)].$$

As the process is Markovian, past values of θ does not help to predict ε_{t-i}. Furthermore, since $y_{t-k} = \varepsilon_{t-k} + \eta_{t-k}$, ε_{t-i} is independent of y_{t-k}, $\forall k \ne i$. Similarly, if we define $v_{t-j} \equiv x_{t-j}(z) - \theta_{t-j-1} = \varepsilon_{t-j} + \xi_{t-j}(z)$, we get that ε_{t-i} is independent of v_{t-j}, if $i \ne j$. Therefore,[15]

$$\begin{aligned} E[\theta_{t-m} \mid \Im_{t-j}(z)] &= \theta_{t-j-1} + E[\varepsilon_{t-j} \mid y_{t-j}, v_{t-j}] + \sum_{i=m}^{j-1} E[\varepsilon_{t-i} \mid y_{t-i}] \\ &= (1-\delta) x_{t-j}(z) + \delta\theta_{t-j-1} + \delta\kappa y_{t-j} + \kappa \sum_{i=m}^{j-1} y_{t-i}. \end{aligned} \tag{2.5}$$

where

$$\delta = \frac{\alpha + \gamma}{\alpha + \beta + \gamma} \text{ and } \kappa = \frac{\gamma}{\alpha + \gamma}.$$

It is important to analyze this result. When $m = j$, $E[\theta_{t-m} \mid \Im_{t-j}(z)] = (1-\delta) x_{t-j}(z) + \delta\theta_{t-j-1} + \delta\kappa y_{t-j}$. We can write this expression as a convex combination of three different signals of fundamental θ_{t-j}: $x_{t-j}(z) = \theta_{t-j} + \xi_{t-j}(z)$, $w_{t-j} \equiv y_{t-j} + \theta_{t-j-1} = \theta_{t-j} + \eta_{t-j}$, and $z_{t-j} \equiv \theta_{t-j-1} = \theta_{t-j} - \varepsilon_{t-j}$. These signals are the only relevant piece of information firm z has on its information set to predict the state θ_{t-j}, since θ_{t-j} is

[15] See Appendix A for details.

independent of y_{t-j+k}, for all $k > 0$. Therefore,

$$\theta_{t-j} \mid x_{t-j}(z), w_{t-j}, z_{t-j} \sim N\Big((\alpha+\beta+\gamma)^{-1}[\beta x_{t-j}(z) + \gamma w_{t-j} + \alpha z_{t-j}], (\alpha+\beta+\gamma)^{-1}\Big).$$

This well-known result shows us that relative weights are functions of the precision associated to each of these signals. For $m > j$, firm z modifies this result just by adding the information released by the central bank. The weight κ captures the importantce of ε_{t-k} on the signal $y_{t-k} = \varepsilon_{t-k} + \eta_{t-k}$.

2.3.2. Computing the Equilibrium

We establish that there is a unique linear equilibrium in the game by computing the aggregate price level in period t as an weighed average of all (average) higher order beliefs about the state θ_t, as stated in (2.4).

Beliefs

In the Appendix B, we use (2.5) and the recursion (2.4) to derive the following useful result:

Lemma *The average k -th order forecast of the state is given by*

$$\bar{E}^k[\theta_t] = \sum_{m=0}^{\infty} (1-\lambda)^m \{\lambda[a_{m,k}\theta_{t-m} + b_{m,k}\theta_{t-m-1}] + \kappa c_{m,k} y_{t-m}\}, \qquad (2.6)$$

with the weights $(a_{m,k}, b_{m,k}, c_{m,k})$ are recursive defined for $k \geq 1$

$$\begin{bmatrix} a_{m,k+1} \\ b_{m,k+1} \\ c_{m,k+1} \end{bmatrix} = A_m \begin{bmatrix} a_{m,k} \\ b_{m,k} \\ c_{m,k} \end{bmatrix} + [1-(1-\lambda)^m]^k \begin{bmatrix} (1-\delta) \\ \delta \\ \rho \end{bmatrix},$$

where the initial weights are $(a_{m,1}, b_{m,1}, c_{m,1}) \equiv (1-\delta, \delta, \rho)$, $\rho \equiv 1 - \lambda(1-\delta)$ *and the matrix A_m is given by*

$$A_m = \begin{bmatrix} (1-\delta)\big[1-(1-\lambda)^{m+1}\big] + \delta[1-(1-\lambda)^m] & 0 & 0 \\ \delta\big[\big[1-(1-\lambda)^{m+1}\big] - [1-(1-\lambda)^m]\big] & \big[1-(1-\lambda)^{m+1}\big] & 0 \\ \lambda\rho(1-\lambda)^m & 0 & 1 \end{bmatrix}.$$

Price Level

From (2.6), we obtain the expression for the aggregate price level P_t, stated in (2.4), as a linear function of the current and past state and all public signals. We have then shown:[16]

Proposition *The equilibrium aggregate price level in period* t, P_t, *is linear in the states* $\{\theta_{t-j}\}_{j=0}^{\infty}$ *and in* $\{Y_{t-j}\}_{j=0}^{\infty}$, *i.e.*

$$P_t = \sum_{m=0}^{\infty} c_m \theta_{t-m} + \sum_{m=0}^{\infty} d_m y_{t-m}, \tag{2.7}$$

where the coefficients are given by

$$c_k \equiv \begin{cases} \frac{(1-r)(1-\rho)}{1-r(1-\rho)} & \text{if } k = 0 \\ (\frac{1-r}{r})\left[\frac{1}{1-r+r\rho(1-\lambda)^k} - \frac{1}{1-r+r\rho(1-\lambda)^{k-1}}\right] & \text{if } k \geq 1, \end{cases} \quad \text{and}$$

$$d_k \equiv \kappa\left[\frac{\rho(1-\lambda)^j}{1-r+r\rho(1-\lambda)^j}\right].$$

If we accurately guessed that the equilibrium price level was given by (2.7), we would obtain the same result using a much simpler method: matching coefficients. This method is presented in Appendix D.

Individual Prices

Using (2.7) and (2.2) we obtain the equilibrium price of a firm z that last updated information at $t-j$ as

$$\begin{aligned} p_{j,t}(z) = {} & [1 - r(1 - C_j)][(1-\delta)x_{t-j}(z) + \delta\theta_{t-j-1} + \delta\kappa y_{t-j}] \\ & + \kappa \sum_{k=0}^{j-1}[1 - r(1 - C_k)]y_{t-k} + r\sum_{m=j+1}^{\infty} c_m \theta_{t-m} + r\sum_{m=0}^{\infty} d_m y_{t-m} \end{aligned} \tag{2.8}$$

where $C_m \equiv \sum_{j=0}^{m} c_j$.[17] It is obvious form this expression that, if public signal did not exist, all terms related to y would die out. The last term, that is common to all firms, would disappear. Therefore, public signals help firms to coordinate their prices. The idea that a public signal helps agent to coordinate their actions is the workhorse of the dispersed information literature and has been extensively

[16] See Appendix C for details.

[17] See appendix E for details.

discussed.[18]

2.4. Inflation

We use (2.7) to obtain the expression for the dynamics of inflation as a function of independent shocks, such that

$$\begin{aligned} \pi_t &= P_t - P_{t-1} = \sum_{j=0}^{\infty} c_j(\theta_{t-j} - \theta_{t-j-1}) + \sum_{j=0}^{\infty} d_j(y_{t-j} - y_{t-j-1}) \\ &= \sum_{j=0}^{\infty} c_j \varepsilon_{t-j} + \sum_{j=0}^{\infty} d_j(\varepsilon_{t-j} + \eta_{t-j} - \varepsilon_{t-j-1} - \eta_{t-j-1}) \\ &= \sum_{j=0}^{\infty} (c_j + l_j)\varepsilon_{t-j} + \sum_{j=0}^{\infty} l_j \eta_{t-j} \end{aligned} \tag{2.9}$$

where

$$l_j = \begin{cases} d_j & = \kappa(1 - c_0) \quad , \text{ if } j = 0 \\ d_j - d_{j-1} & = -\kappa c_j \qquad , \text{ if } j \geq 1, \end{cases} . \tag{2.10}$$

This expression explicitly shows that public information affects inflation. This result would never appear in Mankiw and Reis (2002), as they do not show how to compute expectations from the model. Comparatively to this result, private signals have a relatively mild influence on prices. The rationale behind this observation comes from the fact that idiosyncratic shocks die out when we aggregate them. The only effect that remains comes from the modification on the strategic interaction that occurs when firms compute the equilibrium. In contrast, shocks that come from public signals last forever, as shown in (2.9). Combining (2.9) and (2.10), we can write inflation as

$$\pi_t = (\kappa + (1 - \kappa)c_0)\varepsilon_t + \kappa(1 - c_0)\eta_t + (1 - \kappa)\sum_{j=1}^{\infty} c_j \varepsilon_{t-j} - \kappa \sum_{j=1}^{\infty} c_j \eta_{t-j} \tag{2.11}$$

Therefore, when the precision of the public signal grows, $\gamma \to \infty$, we have $\kappa \to 1$ and $\eta_{t-j} \xrightarrow{p} 0$, $\forall j$. In this limit case, we obtain $\pi_t = \theta_t - \theta_{t-1}$. This is the inflation rate that would prevail if firms had complete information about the fundamental ($p_t(z) = p_t^* = P_t = \theta_t, \quad \forall z$). This observations it is easy to understand: as $y_{t-i} = \theta_{t-i} - \theta_{t-i-1}$ when $\gamma \to \infty$, a firms z that last updated its information set at period $t - j$ assess θ_t through

[18] See, for instance, Morris and Shin (2002) and Angeletos and Pavan (2007).

$$\theta_t = \theta_{t-j-1} + \sum_{i=0}^{j} y_{t-i}.$$

Therefore, no matter how long has been since firm z last updated its information set, it will always know the state θ_t.

When $\gamma \to 0$, we have $\kappa = 0$ and, consequently, $d_j = 0$, $\forall j$. In this situation, inflation does not depend on the public signal, y_t, being exclusively under the influence of θ. This result is the same we found in chapter 1, when we studied a model with sticky-dispersed information and no public signal. That is, firms ignore the public signal when, for being so imprecise, it does not give any information about the state. Since

$$\lim_{j\to\infty} \sum_{i=0}^{\infty} c_i = \lim_{j\to\infty} C_j = \lim_{j\to\infty} \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r+r\rho(1-\lambda)^j} - 1\right] = 1,$$

we can write

$$[\kappa + (1-\kappa)c_0] = [1-(1-\kappa)(1-c_0)] = \left[1-(1-\kappa)\sum_{j=1}^{\infty} c_j\right].$$

Equation (2.11) shows that for the intermediate case, $0 < \gamma < \infty$ (or $0 < \kappa < 1$), the influence of the past shocks ($\varepsilon_{t-i}, \eta_{t-i}$, for $i > 0$), is transferred to (ε_t, η_t). This result shows that, as public signal becomes more precise, it diminishes the persistence of inflation, even without changing the degree of informational persistence, λ.

2.4.1. Inflation Variance

We can use equation (2.11) and the definition of κ to obtain the inflation variance as

$$\begin{aligned} Var(\pi_t) &= \left[\kappa^2 + 2\kappa(1-\kappa)c_0 + (1-\kappa)^2 \sum_{j=0}^{\infty} c_j^2\right]\alpha^{-1} + \kappa^2\left[1 - 2c_0 + \sum_{j=0}^{\infty} c_j^2\right]\gamma^{-1} \\ &= \frac{1}{(\alpha+\gamma)^2}\left[\frac{\gamma^2}{\alpha} + 2\gamma c_0 + \alpha\sum_{j=0}^{\infty} c_j^2\right] + \frac{1}{(\alpha+\gamma)^2}\left[\gamma - 2\gamma c_0 + \gamma\sum_{j=0}^{\infty} c_j^2\right] \\ &= \frac{1}{(\alpha+\gamma)}\left[\frac{\gamma}{\alpha} + \sum_{j=0}^{\infty} c_j^2\right] \end{aligned}$$

Table 2.1 show the baseline calibration we use to study both inflation impulse responses to the shocks and the evolution of $Var(\pi_t)$ with the precision of public information, γ. The model's structural parameters are r, λ, α, β, and γ. Following Mankiw and Reis (2002), we use $\lambda = 0.25$ and $r = 0.9$ as our baseline values. The value $\lambda = 0.25$ implies that firms adjust their private information once a year, which is compatible with the most recent microeconomic evidence on price-setting.[19] For the remaining parameters, we set $\alpha = \beta = 1$ as our benchmark value to keep the baseline calibration as neutral as possible regarding the relative importance of each type of information.

Table 2.1: Baseline calibration

Parameter	Description	Range	Benchmark Value
r	Degree of strategic complementarity	$[0,1]$	0.90
λ	Degree of informational stickiness	$[0,1]$	0.25
α	Precision of the demand shock ε_t	$\mathbb{R}_+$	1.00
β	Precision of the private information shock ξ_{t-j}	$\mathbb{R}_+$	1.00
γ	Precision of the public information shock η_t	$\mathbb{R}_+$	1.00

Figure (F0c) shows how inflation evolves after a shock for three different values of γ. Panel (a) shows that after a demand shock, ε_t, inflation increases. When γ is small, inflation rises smoothly until reaching a peak and decreases afterwards. As γ increases, inflation rises almost instantaneously, becoming concentrated at $t = 0$. This observation is consistent with the analysis just made: as the precision of public information increases, stickiness becomes less important as inflation converges to the complete information equilibrium. Panel (b) shows how inflation evolves after a communication shock. When this shock occurs, firms raise their prices assuming that a demand shock has occurred. This fact is more relevant at $t = 0$, since no firm has information about the state. Afterwards, however, more and more firms find out that what they observed was actually a communication flaw and, consequently, that they should not have raised their prices. As public signal becomes more precise, i.e. γ grows, the influence of this shock is amplified. It is important to highlight that in both cases inflation becomes

[19] See, for example, Klenow and Malin (2009).

more volatile when the precision of public information increases

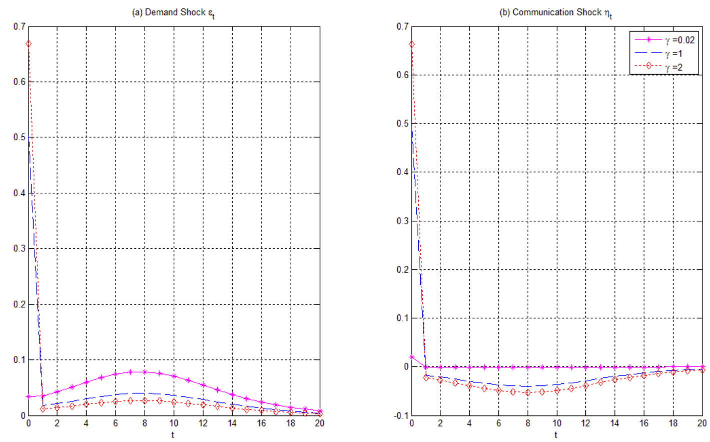

Figure 2.1: Inflation impulse responses to shocks ε_t and η_t.

Figure (F1c) shows how the inflation variance changes with γ. It is clear that inflation variance grows with γ.[20]

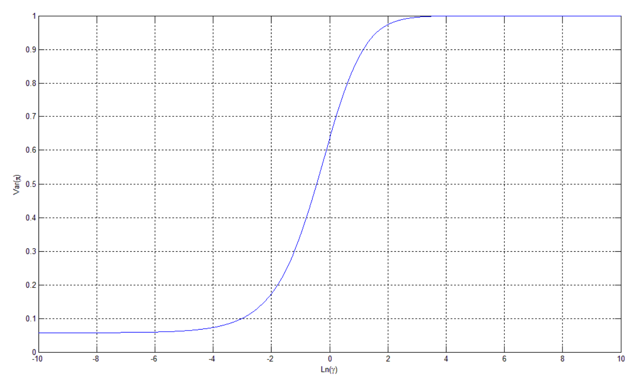

Figure 2.2: Evolution of inflation variance with γ

This fact is not surprising once individuals prices become more synchronized as the precision of the public signal increases, meaning that small

[20]Although this result was obtained only numerically, it holds for a huge combination of parameters. Only when we consider extreme values for the parametres (ex: $\lambda = r = 0.95$), we obtain a small region where the inflation variance decreases with γ. But, even for those cases, inflation variance increases for almost all values of γ.

shocks on the public information pushes inflation greatly. Furthermore, private information, which helps to desynchronize prices as idiosyncratic shocks are not correlated, becomes relatively less important as the precision of the public signal increases.

2.5. Policies

Firms set prices without internalizing how their own choices affect the information of others. We use a modified version of the efficiency criterion proposed in Angeletos and Pavan (2007) to study how public policies can help to offset this externality, improving social welfare.

2.5.1. Efficiency Criterion

We use an efficiency benchmark that addresses whether higher welfare could be obtained if agents were to use their available information in a different way than they do in equilibrium. Following Angeletos and Pavan (2007), we adopt as our efficiency benchmark the strategy that maximizes *ex ante* utility subject to the sole constraint that information cannot be transferred from one agent to another. We modify their efficiency criterion to nest the assumption that information is sticky. The Lagrangian for our problem is

$$E\Pi = -\lambda \int_{(\Theta_t, I_t)} \left[\sum_{j=0}^{\infty} (1-\lambda)^j \int_{x_{t-j}} u(x_{t-j}, \Theta_t, Y_t) dF(x_{t-j} \mid \Theta_t, Y_t) \right] dF(\Theta_t, Y_t) + \int_{(\Theta_t, I_t)} \eta(\Theta_t, Y_t) h(\Theta_t, Y_t) dF(\Theta_t, Y_t)$$

where $u(x_{t-j}, \Theta_t, Y_t)$ is the "utility" function of the firm, and $\eta(\Theta_t, Y_t)$ is the Lagrange multiplier associated to the constraint

$$h(\Theta_t, Y_t) \equiv P_t(\Theta_t, Y_t) - \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \int_{x_{t-j}} p_t(x_{t-j}, \Theta_{t-j-1}, Y_t) dF(x_{t-j} \mid \Theta_t, Y_t).$$

This criterion may be understood as measure of social welfare, if "welfare is now evaluated from the perspective of firms. We know that p_t^* is obtained as the first order condition of $u(x_{t-j}, \Theta_t, Y_t)$. But since many different functions can generate the same first order condition, this social welfare measure can vary. For instance, in Morris and Shin (2002), (2.1) appears as the first order condition of a beauty-context utility function.[21] Using this function, Morris and Shin (2002) showed that the provision of public information may diminish social welfare. Nevertheless, we follow Woodford (2002) and assume that

$$u(x_{t-j}, \Theta_t, Y_t) \equiv (p_t(z) - p_t^*)^2.$$

This function guarantees profit maximization in a way that is consistent with the approach presented in Blanchard and Kiyotaki (1987).

2.5.2. Fiscal Policy

In order to eliminate the externality caused by the fact that firms do not internalize how their own choices affect the information of others, we solve the following program

$$\min_{\{p_t(z)\}_{z\in[0,1]}} E\Pi_t.$$

The set of individual price schedules that solves this program is given by

$$p_t(z) = E[(1 - r^*)\theta_t + r^* P_t(\Theta_t, Y_t) \mid \Im_{t-j}(z)], \tag{2.12}$$

where $(1-r^*) \equiv (1-r)^2$.

This solution is the same that was found in Angeletos and Pavan (2007) for a model without sticky information. Therefore, we need to find some tax scheme that implements this optimal strategy as equilibrium. When the tax scheme is announced, the profit function of firm z becomes

$$\Pi_t(z) = E\left[-(p_t(z) - ((1 - r)\theta_t + rP_t))^2 - \tau(p_t(z)) \mid \Im_{t-j}(z)\right].$$

As the optimal behavior of the firm must equal (2.12), we get

$$\frac{d\tau(p_t(z))}{dp_t(z)} = 2r(1 - r)(\theta_t - P_t).$$

That is,

[21] In the working paper version of Angeletos and Pavan (2007), the function stated in Morris and Shin (2002) is written as $u(x_{t-j}, \Theta_t, Y_t) \equiv (1-r)(p_t(z) - \theta_t)^2 + r(p_t(z) - P_t)^2 - r\sigma_k^2$, where $\sigma_k^2 = \int (p_t(z) - P_t)^2 dz$.

$$\tau(p_t(z)) = 2r(1-r)(\theta_t - P_t)p_t(z).$$

The implementation of this tax scheme is rather difficult. If $(\theta_t,\ P_t)$ was revealed at the end of each period t, as in Angeletos and Pavan (2009), there would be no informational stickiness, meaning that we would have a complete different model and, consequently, a complete different equilibrium. To avoid this problem, we could assume that taxes are collected only when firm z updates its information set. Although this assumption solves the problem of revealing information (firm z receives Θ_{t-1} when it updates), it is not convincing that firm z has probability λ of paying taxes every period. Furthermore, if we assume that this tax is in fact a cost that firms should pay in order to update their information sets, firms would decide the best moment to update their information sets. This approach requires having λ endogenous, which, once again, generates a complete different model and equilibrium.

The optimal solution would also be implemented if the social planner charged

$$\tau(p_t(z)) = 2r(1-r)E[\theta_t - P_t \ \mid\ \Im_{t-j}(z)]p_t(z).$$

But, in this case, the social planner must know the information each firm has in order to implement this tax scheme. Therefore, in model with sticky-dispersed information taxation is not a good way to improve social welfare.

2.5.3. Communication

Communication is usually regarded as a means of improving social welfare, although Morris and Shin (2002) has shown that for some special cases this may not occur. We need to verify if this intuition holds for this model. We write our efficiency criterion as a function of γ, the precision of the public signal, in order to verify how it affects welfare. Therefore,

$$E\Pi(\gamma) = -\left(\frac{\lambda}{\alpha+\beta+\gamma} + \frac{1-\lambda}{\alpha+\gamma}\right)\sum_{j=0}^{\infty}(1-\lambda)^j\Omega_j^2$$

where

$$\Omega_j(\rho(\gamma)) = \left[\frac{1-r}{1-r\left[1-\rho(1-\lambda)^j\right]} \right]$$

It is easy to verify that the first derivative of this expression with respect to γ is always positive. This proves that communication is an effective means of improving social welfare. As we have already mentioned when we analyzed inflation, when $\gamma \to \infty$, we have the complete information equilibrium: $p_t(z) = p_t^* = P_t = \theta_t$, $\forall z$. This makes $u(x_{t-j}, \Theta_t, Y_t) = 0$, proving that the maximum social welfare is reached for the case of complete information equilibrium.

The whole analysis we presented here considers exclusively the precision of the information *available to firms*, η_t, which of a combination of two different shocks: u_t and v_t. We read $r_t = \theta_t - \theta_{t-1} + u_t$ as the information central bank has, no matter if it comes from market surveys or from models. Therefore, the quality of this information is represented by $\mu \equiv 1/\operatorname{var}(u_t)$. Recalling that $y_t = r_t + v_t$, transparency on the communication is represented by $\omega = 1/\operatorname{var}(v_t)$. Assuming that u_t and v_t are independent shocks, we have that $\gamma^{-1} = \mu^{-1} + \omega^{-1}$. Therefore, making $\gamma \to \infty$, is equivalent to have $\mu^{-1} \to 0$ and $\omega^{-1} \to 0$. This simple analysis shows that central bank should improve both the quality of the information it produces and transparency to improve social welfare.

It is important to highlight that communication does not eliminate the externality caused by the fact that firms do not internalize how their own choices affect the information of others. However, it improves social welfare.

2.6. Conclusions

We studied how price decisions change when we incorporate a public signal, available every period to all agents, to the sticky-dispersed information model studied in chapter 1. Firms use the information they have to predict the state of the economy and the behavior of other firms. Nevertheless, information is heterogeneous due to stickiness (as proposed in Mankiw and Reis (2002), only a fraction λ of firms updates their information set every period) and dispersion (as in Morris and Shin (2002), firms receives private signal of the current state when they update).

Public information has a much stronger influence on inflation dynamics than private information. In chapter 1, it was shown that private signals modify inflation dynamics only through the strategic interaction among differently informed agents and their beliefs about the state. The rationale comes from the fact that idiosyncratic socks die out when we consider the aggregate price index. In contrast, our model shows that, as all firms use public information to support their pricing decisions, shocks from public signals have a direct impact on inflation. Although our structure incorporates stickiness, this result is in line with the dispersed information literature. Nevertheless, stickiness allows us to study how public information drives inflation. We show that, if information is sticky, the impact of public signals on inflation last forever. However, as public signal becomes more precise, past shocks becomes relatively less important than current shocks, making inflation less persistent, even without changing the degree of informational persistence.

We evaluated welfare from the firms' point of view. Following Angeletos and Pavan (2007), we use the *ex-ante* total profit as our welfare measure in order to evaluate the inefficiency that comes from the fact that firms do not internalize the effect that their prices have on other firms pricing decisions. As in Angeletos and Pavan (2009), we show that taxation can offset this externality. However, it has a difficult implementation, since in our model the social planner must know the information each firm uses to set their prices. Although communication does not eliminate the externality by itself, it affects welfare in a much simpler manner: welfare increases with the precision of public signal. When precision becomes infinity, we obtain the prefect information equilibrium, which generates the same level of welfare that would prevail if there was no externality.