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7
On the Solution of the Petroleum Supply Planning

7.1 Summary

In this chapter we describe how we solve the petroleum supply planning problem. Apart from using the results from Chapters 5 and 6, we present others basic developments that contribute to speed up its solution time. We show all the reformulations we have made to solve this problem, even though some of them have already been presented in previous chapters.

This chapter sums up all the contributions presented in this thesis, since everything that we developed in the course of this work had the solution of the petroleum supply planning problem as our primary goal. In fact, all previous chapters paved the way for the solution of this important problem and also helped us to bridge the gap from the theory to the real application.

Finally, in this chapter we stress the importance of first understanding the problem at hand, and second not only looking at practical problems in a holistic way, but examine them in detail in order to identify structures and subproblems that can be studied to find better formulations and/or algorithms to solve them.

7.2 Inventory Balance Reformulation at Production Sites

As one may appreciate, it is not obvious what can be done with the inventory balance inequalities at the production sites in order to strengthen the LP relaxation of the overall model presented in Chapter 3. However, as shown in Chapter 6, if we express the inventory at the production sites in an accumulated way, the following inequalities are obtained:

$$sp_{p,t} = ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau} - \sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} CAPT_{cl} \cdot bp_{p,cl,z,\tau} \qquad \forall p \in P,\ t \in T \tag{7.1}$$

Where ISP_p is the initial inventory at platform p

But the inventory variables at the production sites must be greater than or equal to zero. Thus,

$$ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau} - \sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} CAPT_{cl} \cdot bp_{p,cl,z,\tau} \geq 0 \quad \forall p \in P,\ t \in T \tag{7.2}$$

Or,

$$\sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} CAPT_{cl} \cdot bp_{p,cl,z,\tau} \leq ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau} \quad \forall p \in P,\ t \in T \tag{7.3}$$

Furthermore, the inventory variables at the production sites must also be less than or equal to their maximum storage capacities. Hence,

$$ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau} - \sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} CAPT_{cl} \cdot bp_{p,cl,z,\tau} \leq MSP_p \qquad \forall p \in P,\ t \in T \tag{7.4}$$

Or,

$$\sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} CAPT_{cl} \cdot bp_{p,cl,z,\tau} \geq ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau} - MSP_p \qquad \forall p \in P,\ t \in T \tag{7.5}$$

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Although the LP relaxation of our reformulation remains the same, there is an advantage in using it. As pointed out in Chapter 6, these new inequalities possess an interesting structure that defines cascading sets of knapsack inequalities which can be reinforced through associated Lifted Minimum Cover Inequalities (see [Gu98]). Indeed, this is automatically done in most commercial MILP solvers. To the best of our knowledge, the only work in the literature that has reported this idea before was the one from Liberatore and Miller [Lib85], where they presented a model for the production planning of a tile company. However, at that time they could not take advantage of the algorithms for presolving and generating cuts that we have at hand nowadays.

7.3 Valid Inequalities

The majority of binary variables used to model the petroleum supply planning problem are associated with the offloading of platforms and this is the weakest part of our model due to the economies of scale. Since the cost per volume decreases as the tanker capacities increase, the linear relaxation tends to use a fraction of a possible larger tanker available instead of trying a smaller tanker or an entire larger tanker. Considering the importance of this subproblem, we will present some inequalities that can tighten the problem formulation and help to speed up the solution process.

In order to tackle this problem and make our presentation clearer, we consider a platform p with the following characteristics:

- Set of classes of tankers that can offload the platform: cl_1, cl_2, cl_3
- Capacity of each class of tanker that can offload the platform, in increasing order: $CAPT_{cl1}, CAPT_{cl2}, CAPT_{cl3}$
- Set of terminals that the platform can ship crude oil to: z_1, z_2, z_3

Given the information above, it is easy to derive lower and upper bounds for the number of tankers that can offload platform p during a given time period. The upper bound can be obtained by the number of times platform p can completely load the smallest tanker until a given time t. Neglecting the offloadings, the total accumulated inventory at a given time t at platform p can then be calculated as:

$$sp_{p,t} = ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau}$$

And hence, an upper bound on the number of tankers that can offload platform p can be calculated as:

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$$UB_p = \left\lfloor \frac{ISP_p + \sum_{\tau=0}^{t-1} P_{p,\tau}}{CAPT_{cl_1}} \right\rfloor$$

Following the same reasoning, a lower bound on the number of offloadings can be obtained by the minimum number of times tankers need to visit platform p. This is calculated considering that always the largest tanker allowed to operate in platform p is used and that the platform is offloaded only when its inventory reaches its maximum storage capacity. A procedure to calculate the lower bound on the number of tankers used to offload platform p is shown by Algorithm 5.

Algorithm 5 Lower Bound on number of tankers

```
sp_{p,0} = ISP_p
θ = 0
LB_{p,θ} = 0
for t = 1 to T do
  sp_{p,t} = sp_{p,t-1} + P_{p,t-1}
  if sp_{p,t} ≥ MSP_p then
    LB_{p,θ+1} = LB_{p,θ} + 1
    θ = θ + 1
    sp_{p,t} = sp_{p,t} - CAPT_{cl3}
  end if
end for
```

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Figures 7.1 and 7.2 show more clearly how the lower and upper bounds are determined.

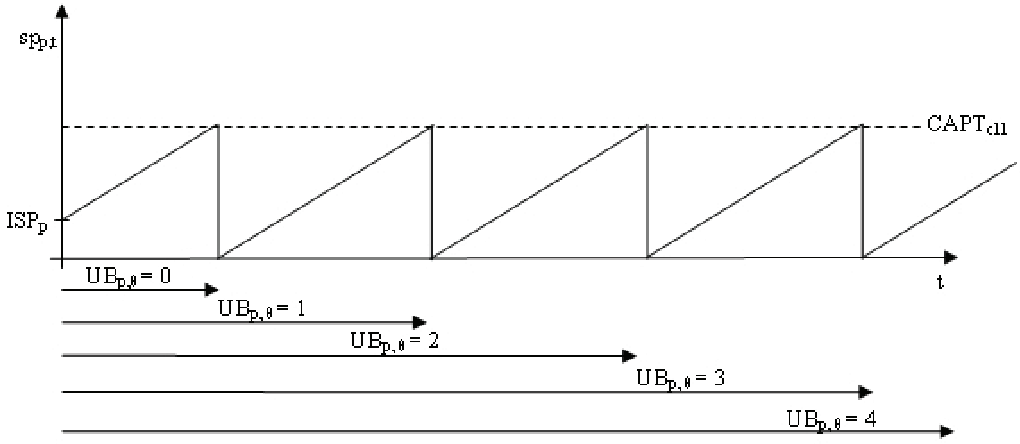

Figure 7.1: Representation of the algorithm to obtain the upper bound on the number of tankers that can visit a platform p

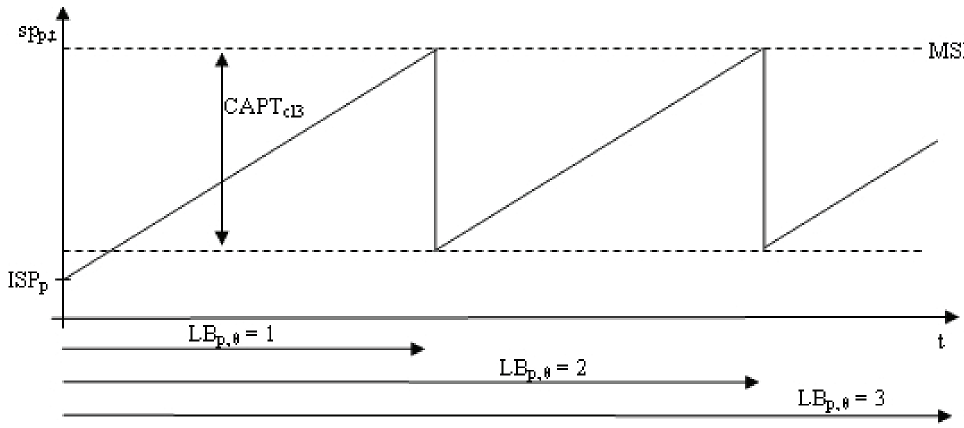

Figure 7.2: Representation of the algorithm to obtain the lower bound on the number of tankers that can visit a platform p

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In algorithmic terms, these valid inequalities can be implemented through the pseudo-code given by Algorithms 6 and 7.

Algorithm 6 Pseudo-code for generating valid inequalities for the maximum number of tankers that can offload a given platform p

for $p = 1$ to P **do**
  $\theta = 0$
  $UB_{p,\theta} = 0$
  $sp_{p,0} = ISP_p$
  if $sp_{p,0} \geq CAPT_{cl_1}$ **then**
    $UB_{p,\theta} = UB_{p,\theta} + 1$
    $sp_{p,0} = sp_{p,0} - CAPT_{cl_1}$
  end if
  for $t = 1$ to T **do**
    $sp_{p,t} = sp_{p,t-1} + P_{p,t-1}$
    if $sp_{p,t} \geq CAPT_{cl_1}$ **then**
      $Insert\left(\sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} bp_{p,cl,z,\tau} \leq UB_{p,\theta}\right)$
      $UB_{p,\theta+1} = UB_{p,\theta} + 1$
      $\theta = \theta + 1$
      $sp_{p,t} = sp_{p,t} - CAPT_{cl_1}$
    end if
  end for
end for

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Algorithm 7 Pseudo-code for generating valid inequalities for the minimum number of tankers that can offload a given platform p

for $p = 1$ to P **do**
  $\theta = 0$
  $LB_{p,\theta} = 0$
  $sp_{p,0} = ISP_p$
  for $t = 1$ to T **do**
    $sp_{p,t} = sp_{p,t-1} + P_{p,t-1}$
    if $sp_{p,t} \geq MSP_p$ **then**
      $LB_{p,\theta+1} = LB_{p,\theta} + 1$
      $\theta = \theta + 1$
      $Insert\left(\sum_{\tau=0}^{t-1} \sum_{cl \in CL} \sum_{z \in Z} bp_{p,cl,z,\tau} \geq LB_{p,\theta}\right)$
      $sp_{p,t} = sp_{p,t} - CAPT_{cl_3}$
    end if
  end for
end for

(a) Example

In the example below we show that the cuts presented in this subsection can be obtained by the Chvátal procedure for tightening formulations, and that they correspond to rank 1 Chvátal inequalities [Chv73].
Consider the following data,

Platform production: $5 \times 10^3 m^3/day$

Maximum platform's storage capacity: $40 \times 10^3 m^3$

Classes of tankers: $\begin{cases} cl_1 - capacity : 20 \times 10^3 m^3 \\ cl_2 - capacity : 30 \times 10^3 m^3 \end{cases}$

Set of terminals: z_1 and z_2

The reformulated inventory balance constraints, according to inequality (7.3) are given by

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$$
\begin{aligned}
&20bp_{p,cl_1,z_1,0} + 20bp_{p,cl_1,z_2,0} + 30bp_{p,cl_2,z_1,0} + 30bp_{p,cl_2,z_2,0} \leq 15 \quad (C_1)\\
&20bp_{p,cl_1,z_1,0} + 20bp_{p,cl_1,z_2,0} + 30bp_{p,cl_2,z_1,0} + 30bp_{p,cl_2,z_2,0} +\\
&+ 20bp_{p,cl_1,z_1,1} + 20bp_{p,cl_1,z_2,1} + 30bp_{p,cl_2,z_1,1} + 30bp_{p,cl_2,z_2,1} \leq 20 \quad (C_2)\\
&20bp_{p,cl_1,z_1,0} + 20bp_{p,cl_1,z_2,0} + 30bp_{p,cl_2,z_1,0} + 30bp_{p,cl_2,z_2,0} +\\
&+ 20bp_{p,cl_1,z_1,1} + 20bp_{p,cl_1,z_2,1} + 30bp_{p,cl_2,z_1,1} + 30bp_{p,cl_2,z_2,1} +\\
&+ 20bp_{p,cl_1,z_1,2} + 20bp_{p,cl_1,z_2,2} + 30bp_{p,cl_2,z_1,2} + 30bp_{p,cl_2,z_2,2} \leq 25 \quad (C_3)\\
&20bp_{p,cl_1,z_1,0} + 20bp_{p,cl_1,z_2,0} + 30bp_{p,cl_2,z_1,0} + 30bp_{p,cl_2,z_2,0} +\\
&+ 20bp_{p,cl_1,z_1,1} + 20bp_{p,cl_1,z_2,1} + 30bp_{p,cl_2,z_1,1} + 30bp_{p,cl_2,z_2,1} +\\
&+ 20bp_{p,cl_1,z_1,2} + 20bp_{p,cl_1,z_2,2} + 30bp_{p,cl_2,z_1,2} + 30bp_{p,cl_2,z_2,2} +\\
&+ 20bp_{p,cl_1,z_1,3} + 20bp_{p,cl_1,z_2,3} + 30bp_{p,cl_2,z_1,3} + 30bp_{p,cl_2,z_2,3} \leq 30 \quad (C_4)\\
&20bp_{p,cl_1,z_1,0} + 20bp_{p,cl_1,z_2,0} + 30bp_{p,cl_2,z_1,0} + 30bp_{p,cl_2,z_2,0} +\\
&+ 20bp_{p,cl_1,z_1,1} + 20bp_{p,cl_1,z_2,1} + 30bp_{p,cl_2,z_1,1} + 30bp_{p,cl_2,z_2,1} +\\
&+ 20bp_{p,cl_1,z_1,2} + 20bp_{p,cl_1,z_2,2} + 30bp_{p,cl_2,z_1,2} + 30bp_{p,cl_2,z_2,2} +\\
&+ 20bp_{p,cl_1,z_1,3} + 20bp_{p,cl_1,z_2,3} + 30bp_{p,cl_2,z_1,3} + 30bp_{p,cl_2,z_2,3} +\\
&+ 20bp_{p,cl_1,z_1,4} + 20bp_{p,cl_1,z_2,4} + 30bp_{p,cl_2,z_1,4} + 30bp_{p,cl_2,z_2,4} \leq 35 \quad (C_5)\\
&\vdots
\end{aligned}
$$

Using Algorithm 6, the cuts for the maximum number of offloadings are given by,

$$
\begin{aligned}
&bp_{p,cl_1,z_1,0} + bp_{p,cl_1,z_2,0} + bp_{p,cl_2,z_1,0} + bp_{p,cl_2,z_2,0} \leq 0 \quad (U_1)\\
&bp_{p,cl_1,z_1,0} + bp_{p,cl_1,z_2,0} + bp_{p,cl_2,z_1,0} + bp_{p,cl_2,z_2,0} +\\
&+ bp_{p,cl_1,z_1,1} + bp_{p,cl_1,z_2,1} + bp_{p,cl_2,z_1,1} + bp_{p,cl_2,z_2,1} +\\
&+ bp_{p,cl_1,z_1,2} + bp_{p,cl_1,z_2,2} + bp_{p,cl_2,z_1,2} + bp_{p,cl_2,z_2,2} +\\
&+ bp_{p,cl_1,z_1,3} + bp_{p,cl_1,z_2,3} + bp_{p,cl_2,z_1,3} + bp_{p,cl_2,z_2,3} +\\
&+ bp_{p,cl_1,z_1,4} + bp_{p,cl_1,z_2,4} + bp_{p,cl_2,z_1,4} + bp_{p,cl_2,z_2,4} \leq 1 \quad (U_2)\\
&\vdots
\end{aligned}
$$

We should notice that,

$$U_1 = \left\lfloor \frac{1}{20} C_1 \right\rfloor$$

This shows that inequality U_1 could also be obtained by applying the Chvátal procedure with $\mu_1 = 1/20$ for inequalities (C_1) and $\mu_i = 0$ for all

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inequalities (C_i) such that $i \neq 1$. The same can be shown with (U_2),

$$U_2 = \left\lfloor \frac{1}{20} C_5 \right\rfloor$$

This reasoning could also be followed for showing the same results in the case of cuts for the minimum numbers of offloadings.

One could argue that this procedure with these coefficients could be applied to all inequalities of the reformulation. However, it is easy to show that only for some inequalities the Chvátal procedure with these multipliers can effectively tighten the reformulation. Our procedure to generate valid cuts works as if the Chvátal procedure with the given multipliers is applied only to the best inequalities to tighten the reformulation.

7.4 Extended formulation

Rocha [Roc04] and Rocha et al. [Roc09] showed that the petroleum supply planning problem becomes easier if the classes of tankers to be used and the offloading times are known a priori for each platform. The basic idea of the extended formulation is to capitalize on this finding and impose a hierarchy in the decision variables associated with the platforms offloadings. In the model from Chapter 3, the variables $bp_{p,cl,z,t}$ define, besides the class of tanker and the offloading time, the terminal to where the petroleum p will be shipped to. We propose to split this variable into two, where the new variables $bship_{p,cl,t}$ will be in charge of defining the offloading of the production sites, and the original variables $bp_{p,cl,z,t}$ will simply be used to determine the terminal where to send this shipment. Moreover we branch first on the $bship_{p,cl,t}$ variables by giving them higher priority in the Branch-and-Bound algorithm. The Figure 7.3 clarifies the idea and shows the relation between the variables.

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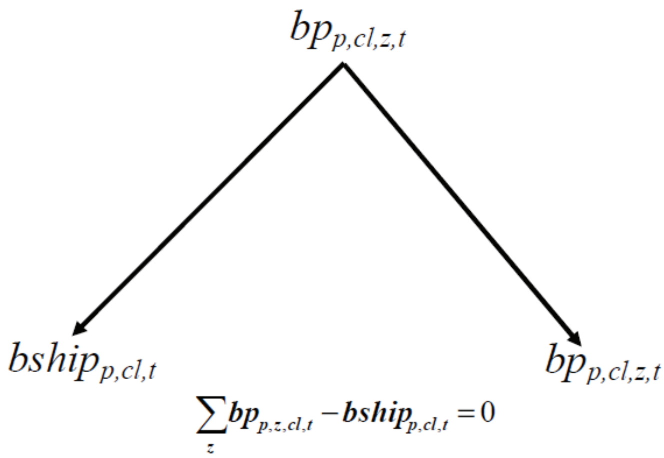

Figure 7.3: Basic idea of the extended formulation

7.5 Decomposition Algorithm

The decomposition algorithm presented in Chapter 5 was originally proposed to solve the petroleum supply planning problem. As pointed out earlier, this problem can be faced as several offloading platform subproblems and a refinery supply master problem (see Figure 7.4), where the challenge is to reconcile the demands of the refineries with the schedule of platforms offloadings. The application of this algorithm to this problem brings up an interesting economic interpretation of the petroleum supply process. Everything happens as if the refineries could ask for the best shipment schedules that fit best their market demands for final products and the platforms would verify whether this planning is feasible. In case the refineries proposals are infeasible, each platform has to explain the reasons of its infeasibility, sending back this information to the refineries through one or more constraints.

The model considered in the decomposition algorithm is the extended formulation. Furthermore, we use the improved idea of the decomposition algorithm where the local branching in the repairing MIP infeasibility procedure is only applied to the variables *bship*. The master problem is not solved to optimality at any time. Also, we start setting the integrality gap parameter to a larger value and we dynamically decrease it based on the progress of the lower bound as the algorithm proceeds. To check the feasibility of the subproblems we developed our own algorithm since the subproblems are fairly easy, and in this way, we can further expedite this process.

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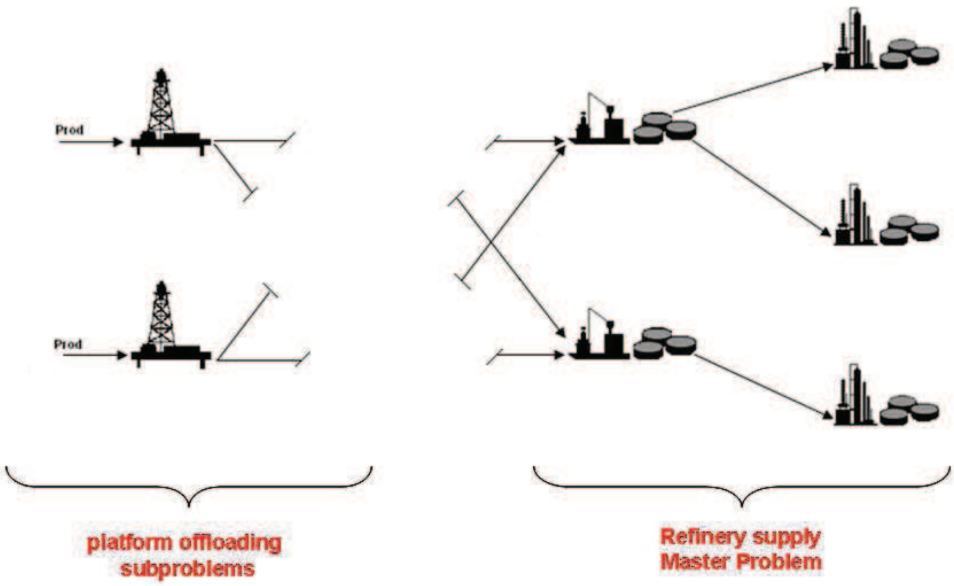

Figure 7.4: Decomposition Scheme

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7.6 Computational Results

Experiments were conducted on a Pentium Xeon 3.2GHz 2.0Gb of RAM and the code was implemented in C++ using the Concert Technology Library and compiled on Visual Studio 2008 under a Windows Platform. All models were solved using CPLEX 11.0. Our models and algorithm were tested on real instances of the problem involving roughly 20 platforms/crude oils, 7 different classes of tankers, 8 maritime terminals, 11 refineries, and 20 distillation units over a time horizon of 72 days. The typical instance gives rise to a model with approximately 40,000 binary variables, 45,000 continuous variables, and 32,000 constraints. We have tested and implemented the following models and algorithm:

- **InitModel**: Initial model as described in Chapter 3
- **InvRef**: Inventory balance reformulation inequalities substituted for the original inventory balance constraints in the initial model
- **CutInvRef**: **InvRef** model with the addition of valid inequalities from section 7.3
- **ExtModel**: **CutInvRef** model reformulated with the extended formulation idea presented in section 7.4
- **Decomp**: **ExtModel** solved using the decomposition algorithm and repairing MIP infeasibility from Chapter 5

- **HullRef**: **ExtModel** reformulated using the results from Chapter 6. This reformulation is only used for the cases where the number of classes of tankers offloading platforms is less than three.

The parameters used in all tests are the default from the CPLEX software, except for the relative gap and time limit that are set to 5% and 2 hours, respectively. These values were chosen following a tradeoff between solution quality and computational time. We believe that 2 hours is a reasonable time for the user to wait to get a solution with a gap at least 5% from the optimum, which is a fairly good solution. Additionally, for the **ExtModel** model, we have adopted a branching rule that considers the *bship* variables with higher priority, setting first to one the variables related to an earlier time. To compare the models we use ten real instances for three different scenarios, namely, scenario 1 where more than two classes of tankers can offload the platforms, scenario 2 where exactly two classes of tankers can offload the platforms, and finally, scenario 3 where only one class of tanker can offload the platforms. Apart from this difference, all the other data for instances in different scenarios are exactly the same, for instance, all data for instances 01, 11 and 21 are equal except for the number of classes of tankers that can offload the platforms.

Table 7.1: Comparison of linear relaxations for more than two classes of tankers

instance	Best known solution	**InitModel/InvRef**		**CutInvRef/extModel**	
		LP Sol.	gap	LP Sol.	gap
01	34543.9	31132.8	9.9	31881.0	7.7
02	62385.0	55663.0	10.8	57123.1	8.4
03	63646.9	57907.9	9.0	59011.5	7.3
04	53734.3	49288.0	8.3	50493.2	6.0
05	48568.5	45614.2	6.1	46384.3	4.5
06	35546.3	32903.4	7.4	33516.9	5.7
07	83125.9	76430.7	8.1	77665.8	6.6
08	63426.7	60150.4	5.2	61355.3	3.3
09	78955.2	72730.8	7.9	73984.4	6.3
10	20723.6	18698.6	9.8	19072.2	8.0

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Table 7.2: Comparison of linear relaxations for two classes of tankers

instance	Best known solution	**InitModel/InvRef** LP Sol.	gap	**CutInvRef/extModel** LP Sol.	gap	**HullRel** LP Sol.	gap
11	36485.0	33689.8	7.7	34502.3	5.4	35257.0	3.4
12	65157.4	60602.1	7.0	61422.8	5.7	63081.6	3.2
13	66582.7	61523.9	7.6	62649.0	5.9	64187.4	3.6
14	58297	54414.4	6.7	55535.5	4.7	56367.1	3.3
15	53900.3	46656.6	13.4	48057.5	10.8	48501.3	10.0
16	39449.0	36452.6	7.6	37779.5	4.2	38344.1	2.8
17	91180.0	85110.0	6.7	86326.1	5.3	88039.0	3.4
18	75885.6	72053.1	5.1	73667.6	2.9	74395.5	2.0
19	92853.8	86246.9	7.1	88079.8	5.1	89698.0	3.4
20	25435.1	23966.4	5.8	24668.5	3.0	25082.7	1.4

Table 7.3: Comparison of linear relaxations for one class of tanker

instance	Best Known solution	**InitModel/InvRef** LP Sol.	gap	**CutInvRef/extModel** LP Sol.	gap	**HullRel** LP Sol.	gap
21	40294.3	38032.2	5.6	39908.9	1.0	39909.5	1.0
22	71681.9	68209.7	4.8	71359.1	0.5	71359.9	0.4
23	72991.1	69148.9	5.3	72673.1	0.4	72747.3	0.3
24	64117.8	60856.2	5.1	63526.8	0.9	63600.7	0.8
25	59009.9	56538.4	4.2	58577.8	0.7	58607.1	0.7
26	46021.5	43462.1	5.6	45600.1	0.9	45637.5	0.8
27	101568.0	95531.6	5.9	99892.9	1.6	100008.0	1.5
28	89145.3	85296.0	4.3	88597.3	0.6	88599.1	0.6
29	103484.0	97020.7	6.2	101830.0	1.6	101963.0	1.5
30	28365.0	26802.4	5.5	28083.2	1.0	28083.2	1.0

Tables 7.1, 7.2 and 7.3 present a comparison of the linear relaxation for the different proposed models. As we can see, the **InitModel** and the **InvRef** have the same linear relaxation as explained before. The same happens with the **CutInvRef** and the **extModel** since the latter is the former with a disaggregation of variables. The valid inequalities in the **CutInvRef**, though simple, are effective to close the gap of the **InitModel**. On average the initial gap for the **CutInvRef** formulation is 45% smaller than the **InitModel**. The **HullRel** formulation is the tighest model for instances where the number of classes of tankers available to offoad the platforms is less than three. In particular, for instance with exactly two classes of tankers to offload the platforms, the initial gap for the **HullRel** formulation is 54% and 34% smaller than the **InitModel** and the **CutInvRef/extModel**, respectively. Another interesting point to observe is the fact that for instances with one class of tanker to offload the platforms, the **CutInvRef/extModel** formulations are approximately equivalent to the **HullRel** as supported by their initial gaps. The tigheness of the **CutInvRef/extModel** and the **HullRel** are in great part responsible for the speedup that we achieve solving the petroleum supply planning problem as reported in the sequel.

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Table 7.4: Comparison of all different models for more than two classes of tankers

instance	InitModel		InvRef		CutInvRef		extModel	
	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)
01	34543.9	7200*	34621.5	6530	34826.0	6661	34873.7	6541
02	62385.0	7200*	62453.1	6579	62057.6	6549	62981.4	6610
03	63834.1	6899	63646.9	4156	64815.0	6547	64308.0	1945
04	55836.8	7200*	53734.3	6521	53973.6	6699	54460.0	996
05	48568.5	6997	49194.8	4397	49297.8	2747	48766.5	1385
06	35546.3	7200*	35778.9	1782	35765.0	832	35839.8	6502
07	83125.9	6654	84406.0	6520	84596.6	6778	84291.8	6532
08	63629.5	7006	63426.7	4751	64467.8	1296	64494.0	528
09	78955.2	6927	79537.7	6698	79434.4	2127	80395.2	5902
10	20930.2	7200*	20726.2	6718	20723.6	6964	20755.4	1602

* Reached the time limit without closing the 5 % gap

Table 7.5: Comparison of all different models for two classes of tankers

instance	InitModel		InvRef		CutInvRef		extModel		HullRel	
	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)
11	36749.2	6570	36966.7	4010	37077.1	1304	37008.8	168	36485.0	54
12	67206.6	7200*	66580.9	1217	667083.5	6600	65919.9	271	66582.7	66
13	67481.9	6497	67862.5	449	68079.3	1037	67955.8	252	66582.7	65
14	58674.7	6627	58297	5005	59087.3	6504	59104.4	888	58611.4	207
15	54463.1	7200*	53966.2	368	53900.3	371	54182.4	281	54232.2	107
16	39673.5	7200*	39449.0	725	40000.2	3287	40318.3	466	40175.7	101
17	91813.1	6487	91587.4	603	92245.7	726	92693.9	713	91180.0	33
18	75885.6	318	76775.5	59	76929.8	69	76376.3	112	75974.3	23
19	93057.2	6645	94035.1	6489	94399.3	400	93932.9	337	92853.8	31
20	25435.1	7046	25810.2	359	25875.0	308	26035.8	226	26368.1	22

* Reached the time limit without closing the 5 % gap

Table 7.6: Comparison of all different models for one class of tanker

instance	InitModel		InvRef		CutInvRef		ExtModel		HullRel	
	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)	Solution	Time(s)
21	40858.6	6611	40294.3	9	41908.6	9	41329.7	8	41329.7	8
22	72941.5	5907	72514.0	10	71681.9	9	73067.4	7	73067.4	7
23	74206.6	892	72991.1	7	74182.9	8	73878.3	7	73878.3	7
24	64393.4	1252	64117.8	8	66274	5	64494.1	8	64494.1	8
25	59194.5	104	59511.5	4	59713.3	6	59009.9	11	59009.9	11
26	46028.1	774	46889.8	4	47854.7	5	46021.5	12	46021.5	12
27	101919.0	793	101568.0	5	105058.0	6	103145.0	6	103145.0	6
28	89444.1	102	91998.2	4	91615.3	4	89145.3	8	89145.3	8
29	103484.0	6556	106180.0	5	104555.0	7	103364.0	10	103364	9
30	28365.0	6798	29040.1	9	29285.4	6	28390.3	10	28390.3	10

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Tables 7.4, 7.5 and 7.6 compare the different models in regard to the computational time to solve the instances for different scenarios. From these results we can draw the following conclusions:

- **More than two classes of tankers** - The **ExtModel** is the fastest option for almost all instances, while the **initModel** formulation is the slowest alternative to solve the problem. In fact, this last formulation could only close the 5% gap for 5 out of 10 instances. It is impressive to see that such a practical simple idea implemented in the **ExtModel** formulation can make such a big difference in the solution time to solve this problem.
- **Exactly two classes of tankers** - In this situation, the model **HullRel** is on average twice as fast as the second fastest solution alternative. The figure is not so clear for the second solution option for this case. The only model that is out of question to be considered as a solution alternative is the **InitModel** as its solution time is always the worse.
- **One class of tanker** - Apart from the **InitModel**, all the other models are a good alternative for the solution of these instances. This test shows that for these instances all reformulations based on the **InvRef** model do not pay off the price and we should stick to this last formulation if we are to solve this problem for this scenario instances. It should be noticed that the results for **ExtModel** and **HullRel** are exactly the same since, as mentioned in Chapter 6, the coefficient reduction algorithm from CPLEX turns the formulation **ExtModel** precisely into the **HullRel**. We see that even for these much easier instances, the **InitModel** has some difficult in closing the 5% gap in some instances.
- The number of classes of tankers that can offload the platforms plays an important role in the complexity of this problem. Indeed, there is a drastic reduction in the computational time to solve the problem for the cases where the number of classes of tankers is less than three.

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Table 7.7: Comparison of the Best Formulation and the Novel Decomposition for more than two classes of tankers

instance	**Best Formulation**			**Decomp**	
	Model	Solution	Time(s)	Solution	Time(s)
01	InvRef	34621.5	6530	34917.0	543
02	CutInvRef	62057.6	6549	63653.7	6797
03	extModel	64308.0	1945	63989.4	548
04	extModel	54460.0	996	54206.7	1820
05	extModel	48766.5	1385	48611.3	844
06	CutInvRef	35765.0	832	35768.0	592
07	InvRef	84406.0	6520	85227.4	558
08	extModel	64494.0	528	64207.4	562
09	CutInvRef	79434.4	2127	80852.5	1300
10	extModel	20755.4	1602	21336.6	450

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Table 7.8: Comparison of the Best Formulation and the Novel Decomposition for two classes of tankers

instance	**Best Formulation**			**Decomp**	
	Model	Solution	Time(s)	Solution	Time(s)
11	HullRel	36485.0	54	37249.7	543
12	HullRel	66582.7	66	65157.4	391
13	HullRel	66582.7	65	67983.1	1163
14	HullRel	58611.4	207	58775.5	339
15	HullRel	54232.2	107	54710.0	148
16	HullRel	40175.7	101	39941.6	312
17	HullRel	91180.0	33	93110.8	540
18	HullRel	75974.3	23	76527.0	210
19	HullRel	92853.8	31	93174.3	1108
20	HullRel	26368.1	22	26262.2	73

Table 7.9: Comparison of the Best Formulation and the Novel Decomposition for one class of tanker

instance	**Best Formulation**			**Decomp**	
	Model	Best Solution	Time(s)	Solution	Time(s)
21	ExtModel/HullRel	41329.7	8	40353.3	22
22	ExtModel/HullRel	73067.4	7	74243.2	58
23	InvRef/ExtModel/HullRel	72991.1	7	74347.4	16
24	CutInvRef	66274	5	64527.6	54
25	InvRef	59511.5	4	59982.9	12
26	InvRef	46889.8	4	47669.6	10
27	InvRef	101568.0	5	105114.0	8
28	InvRef/CutInvRef	91615.3	4	89347.5	10
29	InvRef	106180.0	5	106999	7
30	CutInvRef	29285.4	6	28906.5	11

Tables 7.7, 7.8 and 7.9 compare the best formulation to the petroleum supply planning problem and the novel decomposition algorithm with respect to the computational time to solve the instances for different scenarios. As one can see, the **Decomp** algorithm is better for the hardest instances, i.e., where platforms can be offloaded by more than two classes of tankers. For all other instances that are not so hard to solve, the implementation of the **Decomp** does not pay off in terms of computational time, since the proposed reformulations proved to be very effective to solve these instances.

7.7 Conclusions

In this chapter we presented how we have solved the petroleum supply planning problem. We showed the effectiveness of simple ideas based on how this problem is tackled by the people in charge of this activity at PETROBRAS. In particular, the extended formulation combined with the branching priority on the offloading variables has proved to be a powerful scheme to speed up the computation solution time to solve this problem. We tested all theoretical developments we have made in the previous chapters, which allowed us to establish the best solution option for different number of classes of tankers to offload the platforms. We showed that for scenarios where the number of classes of tankers is greater than two, the new decompostion algorithm is the best way to go if one is to solve the petroleum supply planning problem. For cases where the number of classes of tankers is equal to two, the best solution option is to reformulate the problem using the convex hull reformulation idea, while for instances with exactly one class of tanker the **InvRef** formulation is the fastest alternative. It is interesting to observe that in practice the petroleum supply planning specialists consider only one class of tanker to offload each production site. Under this circumstance, the **InvRef** formulation is able to solve our problem in less than one minute, while the initial formulation takes almost two hours in most of the cases.

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