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Conclusions

In this thesis we have modeled and solved the petroleum supply planning problem at PETROBRAS. This is a problem of considerable importance for its petroleum supply chain integration since it provides a link between the strategic and operational decision levels. Therefore, we have built our model with the requirement of integration in mind. For this purpose, we tackled this problem as a whole and did not solve it by a hierarchical approach to avoid introducing suboptimality. Moreover, we have captured all the details of this activity in such a way that its solution is in consonance with the PETROBRAS's strategic plan and provides sufficient detailed information to be used at the operational level. Unfortunately, this comes at a cost. The model turns out to be a large scale problem for which is hard to find a good feasible solution in a reasonable computation time using commercial solvers. We proposed a series of reformulations and a novel decomposition algorithm that allowed us to solve this problem for a wide range of possible scenarios. This mathematical model is being tested at PETROBRAS and has proved to be an effective tool for what-if scenarios as well as for closing the gap between the strategic and operational decision levels. Figure 8.1 shows an upshot of its use in the Petroleum Supply Planning System, called ALOPE. This new tool is expected to be in production in the coming semester.

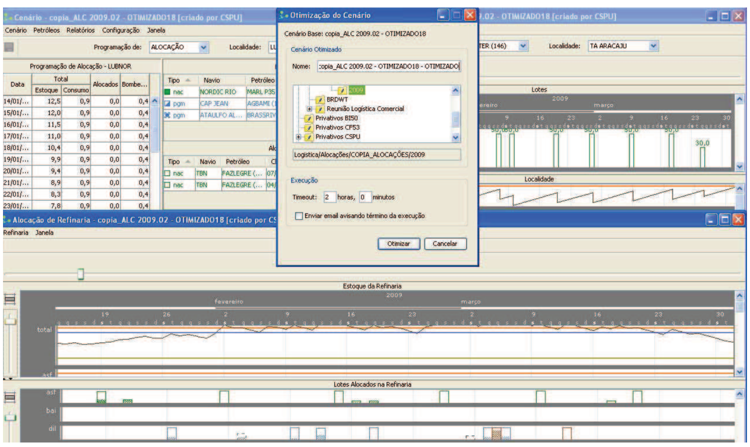

Figure 8.1: Petroleum Supply Planning System - ALOPE

8.1 Thesis Contributions

The main contributions of this work are twofold. The first, and our main goal, is the definition and solution of a real industrial problem. The second is the general theoretical developments that come as byproduct of this primary endeavor.

Concerning the solution of the petroleum supply planning, we clearly defined and stated the problem in Chapter 2. We also extended the real problem introducing flexible campaigns and using more than one class of tankers to offload the platforms. These extensions are important because on the one hand we can find better solutions with them, and on the other hand, they help to integrate the tactical and operational levels by providing solutions that can be used as input to the tanker routing problem. Additionally, we proposed a mixed integer programming model to solve the resulting problem in Chapter 3. As this initial model could not give a satisfactory solution for all real instances in the time limit set to solve them, we proposed a series of reformulations that allowed us to efficiently reduce their solution times. We started with some reformulations based on practical observations and went on to use the development on the Cascading Knapsack Inequalities from Chapter 6 to further improve the computational time for cases where the number of classes of tankers to offload the platforms is less than or equal to two. We also solved our problem using the novel decomposition algorithm that was very effective to provide good solutions to the hardest instances. With these results, we proposed a hierarchy of alternative solutions to solve the petroleum supply planning problem depending on the number of classes of tankers considered to offload the platforms. This part of the work was instrumental to show how we can close the gap between theory and practice. In the thesis from Saxena at Tepper School of Business at Carnegie Mellon University [Sax09], he defined Integer Programming as a technology. However, we observe a disconnection between the theory and practice in this area, and as a consequence, this technology is still far from having full application in the industry. We believe we have contributed to close this gap by proposing problem reformulations that can be used for other problems and that can be easily implemented by users in industry.

On the theoretical side, we demonstrated the usefulness of the inventory reformulation in inventory-production-distribution problems in Chapter 6. This is a structure that is often disregarded in the literature even by some prominent researchers as in the paper by Lejeune and Margot [Lej08]. We also introduced the term Cascading Knapsack Inequalities and provided effective reformulations by describing the convex hull of single knapsack inequalities with fewer integer variables. Another important contribution was the development and implementation of a novel decomposition algorithm that can be applied to a wide range of practical problems. Resorting to disjunctive programming theory to build the cut separation procedure, this method overcomes the difficulty present in traditional decomposition algorithms when both the master and the subproblem are integer programs. Finally, we should mention that these theoretical contributions were very exciting because they came after deeply understanding the real problem, proving that theory and practice can and must go hand in hand.

8.2 Future Work

There are some topics that could not be investigated in this thesis, but that we believe their study would further improve the performance of our decomposition algorithm and extend the range of application of the convex hull relaxation idea. These topics are as follows:

- **Improve the performance of the cut generation procedure**: In the decomposition algorithm we need to consider all variables in the original problem to generate the cuts. Depending on the number of variables, the cut separation problem can be too large and consequently the time to solve it can be prohibitive to be applied in practice. Perregaard [Per03] had shown that we can solve this separation only for a subset of variables and lift the remaining variables using a simple closed formula. In our case, we can solve the separation only for the variables assuming value of one in a given solution and subsequently lift the obtained cut to include the other variables.
- **Strengthen the cuts generated**: Balas [Bal98] has proposed a method for strengthening cutting planes derived from disjunctions in the case when, besides the disjunction which applies to the basic variables, there are also integrality constraints on some of the nonbasic variables. This is particularly useful for our decomposition algorithm for problems where the cardinality of the number of variables assuming one in a given master solution is constant, as is the case in all problems solved in this thesis. In this case, we use only the variables that are equal to one in the disjunctions and treat the other variables as continuous. We believe that this procedure proposed by Balas can further improve the quality of the cuts obtained by making use of the integrality of the variables not present in the disjunctions.

- **Test and compare a new cut generation procedure**: The infeasible master solution can be projected into the feasible region of the subproblem and a cut can be obtained by the hyperplane perpendicular to the projection direction and passing through the projected integer point. This idea is similar to the one presented in a paper by Sawaya and Grossmann [Saw05].
- **Extend the Hull Relaxation Formulation to more than two classes of tankers**: For this purpose, two directions can be pursued. We can use our algorithm to determine the convex hull of points for knapsack inequalities with two variables, and verify whether or not there exists a closed form expression for lifting the obtained cuts. Alternatively, we can use a program such as PORTA [Por99] or QHull [Bar96] to calculate the convex hull of knapsack inequalities with more than two integer variables and use these cuts as our new formulation. In both alternatives we need to test whether or not the additional work to obtain a better formulation pays off in term of overall computational time.
- **Backlog in the terminals**: In this case, the inventory reformulation proposed in Chapter 6 gives rise to cascading knapsack inequalities with one continuous variable in each inequality. These inequalities are known in the literature as Mixing Sets (see, for instance, the book from Pochet and Wolsey [Poc06]) and the same study presented in this thesis for one and two classes of tankers can be performed.