2 Estimating Within-Sector and Across-Sector Pricing Interactions in a Ss Model

2.1 Introduction

Largely motivated by previous evidence that prices are maintained fixed for some months[1], many macroeconomic models, including models used for policy analysis, rely on the assumption that prices are sticky. Those models are argued to produce dynamic behavior of aggregate variables consistent with typical Vector Autoregression (VAR) studies. In particular, they reproduce the inertia in inflation and the persistent movements of output in response to monetary policy shocks[2].

Recently, however, the literature using prices on the individual level has challenged the assumption of price stickiness. Several papers analyzing micro price data have shown that individual prices are more flexible than initially thought. In a pioneer work, Bils & Klenow (2004) document much more frequent price changes, in comparison with previous studies: half of the U.S. goods have prices that last less than 4.3 months. Klenow & Kryvtsov (2008) also find that consumer prices in the U.S. change every 4–7 months, depending on the treatment of sales[3]. Similar results of price flexibility are also found in Europe[4] and in other countries. On the other hand, the recent literature has established new interesting facts about sectoral prices responses to different types of shocks. Among those facts, one seems particularly important for models of price adjustment: sectoral prices respond faster to "idiosyncratic" shocks than to "aggregate" shocks (see Boivin et al. (2009) and Maćkowiak et al. (2009)).

In an effort to incorporate some of the new findings into the models of price adjustments, Golosov & Lucas (2007) have recently constructed a state-dependent pricing model that makes use of both cross-sectional and time series evidence on prices at the individual level. Their main finding is that, in the context of state-dependent pricing, monetary shocks generate much stronger effects on the

[1]The conventional view was that prices adjusted once a year. See Carlton (1986), Kashyap (1995), Levy et al. (1997) among others.

[2]See, for example, Christiano et al. (2005), Smets & Wouters (2007).

[3]Nakamura & Steinsson (2008) suggest that sales have an important role in generating price flexibility. Their median frequency of price changes decreases from 19%–20% to 9%–12% per month when they exclude sale prices.

[4]See Dhyne et al. (2006) and other studies from the Eurosystem Inflation Persistence Network.

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price level and much weaker effects on real output. In contrast, Gertler & Leahy (2008) argue that, since Ball & Romer (1990), it is well known that to produce reasonable degree of monetary non-neutrality in macroeconomic models is crucial to introduce strategic complementarities in price setting[5]—known in the literature as "real rigidities". Gertler & Leahy (2008) show that by incorporating real rigidities into a state-dependent pricing model, calibrated to mach the Klenow-Kryvtsov evidence, they can obtain significant monetary non-neutrality.

Several other recent papers have emphasized the importance of real rigidities to generate monetary non-neutrality and explored the mechanisms through which they act. In this respect, there has been a movement in the direction of incorporating heterogeneities into the models. Multi-sector models calibrated using the new evidence on price changes are argued to amplify monetary non-neutrality by a factor between 3 and 4.5 relative to single-sector models (see Carvalho (2006), Nakamura & Steinsson (2010), Shamloo & Silverman (2010)).

Heterogeneities in other aspects of the model are also showed to produce differential responses of prices to sectoral and aggregate shocks. Shamloo & Silverman (2010) explore the potential of heterogeneity in the use of inputs in production (interpreted as production chains) in adding to real rigidities and show that their model predicts much better the relative speed of sectoral responses to monetary shocks. In the same line, Carvalho & Lee (2010) develop a multi-sector model that endogenously generates responses of prices to aggregate and sectoral shocks compatible with Boivin, Giannoni and Mihov's findings. Carvalho & Lee (2010) emphasize the pricing interactions within and across sectors in producing such results. In particular, their model delivers strategic complementarities across all pricing decisions and strategic substitutabilities in interactions within sectors.

Therefore, it has become well understood that not only pricing interactions—real rigidities—matter for monetary policy effects, but also that the intensity and the type of strategic pricing interaction within and across sectors are fundamental to explain the new empirical evidence about the response of sectoral prices to different types of shocks. Consequently, there has been a large effort in the literature to produce empirical estimates of the degree of real rigidities[6]. The difficulty of this task is that strategic complementarities are related to firms' optimal prices, which are non-observable, and to marginal costs and markups, variables for which hardly there is empirical measure. These difficulties explain why the literature has basically used indirect methods to estimate real rigidities. Following this effort, in the first chapter we have proposed a direct methodology to estimate the parameter

[5]See Kimball (1995), Basu (1995), Dotsey & King (2006), among others.

[6]See Klenow & Willis (2006), Burstein & Hellwig (2007), Kryvtsov & Midrigan (2009), Bils et al. (2009), Gopinath & Itskhoki (2010) etc.

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of strategic complementarities in pricing decisions. In contrast to previous studies, we found evidence of significant strategic complementarities in the data.

Notice, however, that these estimates refer to "overall" strategic complementarities, i.e., the dependence of marginal cost on the aggregate price level. The literature has never tried to disentangle the different mechanisms within and across sector in its estimations. For instance, how do these pricing interactions really work in practice? Which mechanisms are more important in the real world, within or across sectors? Are within-sector pricing interactions strategic complementary or substitute? Aiming to answer these questions, in the present paper we derive a state-dependent pricing model with within- and across-sector pricing interactions and use the methodology developed in the first chapter to estimate these real rigidities in the data.

In special, our model combines ingredients from Shamloo & Silverman (2010) and Carvalho & Lee (2010). The result is a model with multiple sectors with heterogeneity in the intensity of the use of inputs in production. The use of the features present in Carvalho & Lee (2010) generate within- and across-sector pricing interactions. The introduction of production chains from Shamloo & Silverman (2010) makes the intensity of these mechanisms to differ among sectors. We depart from both by making the assumption of firm-specific labor market, which can deliver more general results for pricing interactions inside each sector—which can be either substitute or complementary depending on parameter values—and by assuming state-dependent pricing.

In estimations we use a very rich data set of individual prices underlying the Consumer Price Index of Getulio Vargas Foundation (CPI-FGV) in Brazil. The data cover approximately 11 years, from 1996 to 2006. Our results suggest the existence of strong real rigidities in the data, captured by very low parameters measuring the degree of "overall" strategic complementarity in pricing decisions in each sector, confirming our previous findings. In the first chapter we documented that in our estimations the parameter of strategic complementarities for the aggregate economy varies between 0.03 and 0.11. Similarly, here the average of our estimated parameters across sectors is 0.11. Additionally, we find that in general not only pricing interactions *across* sectors are strategic complementary, but also the pricing interactions *within* sector act in the same direction, reinforcing strategic complementarities. Of course, behind this "general" result there is significant heterogeneity, with sectors presenting different combinations involving complementarity and/or substitutability in within- and across-sector pricing interactions, in line with the papers which argue that these different mechanisms can generate differential response of prices to sectoral and aggregate shocks.

The paper is organized as follows. In section 2.2 we present the model,

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derive the firm's frictionless optimal price and discuss the way we measure pricing interactions within and across sectors. In section 2.3 we derive the econometric framework used to estimate the model and the strategy to estimate the parameters of interest. Section 2.4 describes the data set and section 2.5 presents the estimation results. We conclude in section 2.6.

2.2 The Model

The model is a version of the standard state-dependent pricing model with four modifications: (i) intermediate inputs in production; (ii) multiple sectors with heterogeneity in the production function (different sectors in the economy use inputs with varying intensity); (iii) firm-specific labor markets; and (iv) idiosyncratic and aggregate shocks.

There is a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$ supplying differentiated goods that are used for consumption and as intermediate inputs. The economy has a finite number of sectors indexed by $k \in \{1, 2, ...K\}$, and each firm operates in one sector. What differentiates the sectors is the intensity by which firms use inputs, as will become clear ahead. We use the notation $\mathcal{I}_k$ to identify the set containing the indices of firms in sector k, so that $\bigcup_{k=1}^{K} \mathcal{I}_k = [0, 1]$. Each of these sets has measure (mass of firms in sector k) equal to n_k and $\sum_{k=1}^{K} n_k = 1$. Finally, we will refer to firm i in sector k as "firm ik".

2.2.1 Households

The economy is populated by identical, infinitely-lived households of measure one. The representative household obtains utility from consumption, supplies distinct types of labor to firms in different sectors and has access to stochastic payoffs of securities. Households also own the firms, which means that they receive the profits earned by them. Subject to the budget constraint specified below, the representative household maximizes the following lifetime utility function:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \Xi_t \left(\log(C_t) - \sum_{k=1}^{K} \int_{\mathcal{I}_k} \omega_k(i) \frac{H_{kt}(i)^{1+\varphi}}{1+\varphi} di \right) \right],$$

where $\mathbb{E}_0$ denotes the expectations operator conditional on information known at time zero, C_t represents the consumption of the composite consumption good, $H_{kt}(i)$ is the labor services of type ik supplied by the household, Ξ_t is a vector of aggregate shocks and the parameters β, φ and $\omega_k(i)$ are, respectively, the discount factor, the inverse of the (Frisch) elasticity of labor supply and the relative disutilities of supplying labor of type ik. Observe that the labor is firm-specific and that the household simultaneously supplies all types of labor.

Markets are complete and the household's budget constraint can be written as

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$$P_tC_t + \mathbb{E}_t[Q_{t,t+1}B_{t+1}] = B_t + \sum_{k=1}^{K}\int_{\mathcal{I}_k} W_{kt}(i)H_{kt}(i)di + \sum_{k=1}^{K}\int_{\mathcal{I}_k}\Pi_{kt}(i)di,$$

where P_t is the aggregate price index to be defined later, $W_{kt}(i)$ is the wage rate of labor of type ik and $\Pi_{kt}(i)$ represents profits of firm ik. $Q_{t,t+1}$ denotes the nominal stochastic discount factor and B_{t+1} is the stochastic payoff of one-period state-contingent nominal securities.

The household's composite consumption good is an aggregator over the variety of all sectoral consumption composite to be defined below,

$$C_t = \left(\sum_{k=1}^{K}(n_k)^{1/\eta}C_{kt}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between the sectoral consumption composites, C_{kt}. The aggregate price level in period t is defined as

$$P_t = \left(\sum_{k=1}^{K}n_kP_{kt}^{1-\eta}\right)^{1/(1-\eta)},$$

where P_{kt} is the sectoral price index associated with C_{kt}. Given the aggregate consumption C_t, the aggregate price level P_t and the sectoral price P_{kt}, the expenditure-minimization problem of the household implies that the household's demand for goods produced by sector k is given by

$$C_{kt} = n_k\left(\frac{P_{kt}}{P_t}\right)^{-\eta}C_t.$$

In turn, sectoral consumption composites are defined as

$$C_{kt} = \left(\left(\frac{1}{n_k}\right)^{1/\theta}\int_{\mathcal{I}_k}C_{kt}(i)^{(\theta-1)/\theta}di\right)^{\theta/(\theta-1)},$$

whose prices are

$$P_{kt} = \left(\left(\frac{1}{n_k}\right)\int_{\mathcal{I}_k}P_{kt}(i)^{1-\theta}di\right)^{1/(\theta-1)},$$

where θ is the elasticity of substitution between goods inside the same sector. Given C_{kt}, P_{kt} and $P_{kt}(i)$, the household's optimal demand for variety ik is:

$$C_{kt}(i) = \frac{1}{n_k}\left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}C_{kt}.$$

Once the household has decided the composition of its consumption basket,

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given the aggregate consumption, it chooses the optimal consumption expenditure and labor supply. The first order conditions are:

$$Q_{t,t+1} = \beta \left(\frac{\Xi_t}{\Xi_{t+1}}\right)\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right),$$

$$\omega_{kt}(i)H_{kt}(i)^{\varphi}C_t = \frac{W_{kt}(i)}{P_t}.$$

2.2.2 Firms

The differentiated goods in the economy are produced by one of the monopolistically competitive firms. Firms use firm-specific labor and other goods as intermediate inputs in production. The intensity in the use of inputs across sectors is different but the same inside each sector. The representative firm in sector k has the following production technology:

$$Y_{kt}(i) = A_t A_{kt}(i) H_{kt}(i)^{1-\delta_k} Z_{kt}(i)^{\delta_k},$$

where $Y_{kt}(i)$ denotes the product of firm ik, A_t is an aggregate productivity factor and $A_{kt}(i)$ is the firm-specific productivity. $H_{kt}(i)$ represents the quantity of firm-specific labor employed in the production process and $Z_{kt}(i)$ is the usage of intermediate inputs. Finally, δ_k denotes the share of other goods used as intermediate inputs in sector k.

The intermediate inputs are obtained as follows. The good varieties are combined through a Dixit-Stiglitz index to form the sectoral intermediate inputs. In turn, these sectoral intermediate inputs are again combined into the composite intermediate inputs that are used in production. The sectoral intermediate input is defined as

$$Z_{k,k',t}(i) = \left(\left(\frac{1}{n'_k}\right)^{1/\theta} \int_{\mathcal{I}'_k} Z_{k,k',t}(i,i')^{(\theta-1)/\theta} di'\right)^{\theta/(\theta-1)},$$

where $Z_{k,k',t}(i,i')$ represents the quantity of goods that a firm ik, that is, a firm i in sector k, purchases from another firm i' in sector k', and $Z_{k,k',t}(i)$ denotes the amount of goods that firm ik uses from sector k' as a whole. In turn, the composite intermediate input is given by

$$Z_{kt}(i) = \left(\sum_{k=1}^{K} (n'_k)^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}.$$

Observe that we are assuming for intermediate input usage the same across-sector and across-good elasticities of substitution (respectively, η and θ) as in

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consumption usage. This means that the price elasticity of demand does not depend on its use. In addition, notice that we have a round-about model of intermediate inputs in which all goods can potentially be used as an intermediate input in different sectors.

Taking as given the prices P_t, $P_{k't}$ and $P_{k't}(i)$, and wages $W_{kt}(i)$, the firm's cost minimization problem gives the following optimal quantities of labor and intermediate inputs:

$$Z_{kt}(i) = \frac{\delta_k}{1-\delta_k}\frac{W_{kt}(i)}{P_t}H_{kt}(i), \tag{2-1}$$

$$Z_{k,k',t}(i) = n_{k'}\left(\frac{P_{k't}}{P_t}\right)^{-\eta} Z_{kt}(i),$$

$$Z_{k,k',t}(i,i') = \frac{1}{n_{k'}}\left(\frac{P_{k't}(i')}{P_{k't}}\right)^{-\theta} Z_{k,k',t}(i),$$

and the following real marginal cost function:

$$MC_{kt}(i) = \frac{\lambda_k}{A_t A_{kt}(i)}\left(\frac{W_{kt}(i)}{P_t}\right)^{1-\delta_k} \tag{2-2}$$

where $\lambda_k = \frac{1}{1-\delta_k}\left(\frac{\delta_k}{1-\delta_k}\right)^{-\delta_k}$.

2.2.3 Frictionless Optimal Price

With a theory of marginal supply cost, we can now derive the firm's optimal price in the case of perfectly flexible prices. The frictionless optimal price equation will make the mechanisms of strategic complementarities and the way we measure them clearer. In the case of flexible prices the supplier of each good is free to choose a price for it each period, not constrained by the price chosen in the past and with full information about current demand and cost conditions. Thus, the firm chooses its price maximizing the following real profit function:

$$\begin{aligned}\Pi_{kt}^{real}(i) = {} & \frac{P_{kt}(i)}{P_t}\left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}\left(\frac{P_{kt}}{P_t}\right)^{-\eta} Y_t \\ & - \frac{\lambda_k}{A_t A_{kt}(i)}\left(\frac{W_{kt}(i)}{P_t}\right)^{1-\delta_k}\left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}\left(\frac{P_{kt}}{P_t}\right)^{-\eta} Y_t. \end{aligned} \tag{2-3}$$

The first order condition to this problem yields the standard result:

$$\frac{P_{kt}^*(i)}{P_t} = \mu MC_{kt}(i), \tag{2-4}$$

where $P_{kt}^*(i)$ is the frictionless optimal price and $\mu \equiv \theta/(\theta-1)$ is the firm's desired markup. Therefore, the frictionless optimal price is a markup over the marginal cost.

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To see more clearly the different mechanisms driving the firms' pricing interactions we log-linearize the previous equation around the non-stochastic zero-inflation steady-state equilibrium. In general, a deterministic steady-state is not symmetric. In particular it depends on the firms' productivity level, $A_t(i)$, $i \in [0,1]$, and on the firm-specific parameters measuring households' relative disutilities of supplying labor, $\omega_k(i)$, $i \in [0,1]$. We make assumptions that simplifies the steady-state characterization. Appendix A provides a detailed derivation of the steady-state equilibrium and the log-linearization. We should note here that, because the production functions of firms in different sectors differ in the intensity with which they use inputs, the combination of factors of production (labor and intermediate inputs) that they use will be different even in steady state. Firms within sectors will be identical in steady state. A first-order log-linearization of equation (2-4) leads to the following expression for the frictionless optimal price[7,8]:

$$
\begin{aligned}
p^*_{ik,t} &= \frac{(1-\delta_k)[1+\varphi-\varphi\psi]}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}\mathcal{Y}_t + \frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}p_{k,t} + \\
&\quad + \left[1 - \frac{(1-\delta_k)[1+\varphi-\varphi\psi]}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi} - \frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}\right] p_t + \\
&\quad + \frac{(1-\delta_k)\varphi\psi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}z_t - \frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_t \\
&\quad - \frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_{ik,t} \\
&= \zeta_{k1}\mathcal{Y}_t + \zeta_{k2}p_{k,t} + (1-\zeta_{k1}-\zeta_{k2})p_t + \chi_k z_t + \tilde{a}_{k,t} + \tilde{a}_{ik,t}, \qquad (2\text{-}5)
\end{aligned}
$$

where $\tilde{a}_{k,t} = -\frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_t$ and $\tilde{a}_{ik,t} = -\frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_{ik,t}$ are shocks, $\mathcal{Y}_t$ is the nominal expenditure and the other variables are self-explained.

2.2.4 Pricing Interactions

The previous equation makes explicit the different mechanisms driving pricing interactions in our model, within and across sectors. The features that produce these interactions are input-output linkages, labor market segmentation and sectoral use of inputs with varying intensity.

Input-output linkages generate real rigidities through complementarities across all pricing decisions. Because the "round-about" production, the other prices in the economy matter for pricing decisions of each firm. This mechanism is captured by the elasticity of the frictionless optimal price to the aggregate price index.

[7]Lowercase means that the variable is in log deviation from the steady state. See Appendix for a detailed derivation of this equation.

[8]To make the notation easier, from this point on we include the firm identification (index i) into the subscript.

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To see more clearly, assume for a moment linear disutility of labor ($\varphi = 0$). This assumption eliminates the effect of labor market segmentation and equalizes wages across sectors. Thus, equation (2-5) becomes:

$$p^*_{ik,t} = (1 - \delta_k)\mathcal{Y}_t + \delta_k p_t - a_t - a_{ik,t}.$$

In this case, only the aggregate price index appears in the frictionless optimal price equation. In Carvalho and Lee's (2010) model, these strategic complementarities are "uniform". Here the strength in each sector depends on the share of intermediate inputs used in production.

Labor market segmentation generates a different type of pricing interaction. To see more clearly the effect, abstract from the use of intermediate goods in production (assuming $\delta_k = 0$). Under this assumption equation (2-5) becomes:

$$\begin{aligned} p^*_{ik,t} &= \frac{1+\varphi-\varphi\psi}{1+\theta\varphi}\mathcal{Y}_t + \frac{\varphi(\theta+\psi-1)}{1+\theta\varphi}p_t + \frac{\varphi(\theta-\eta)}{1+\theta\varphi}(p_{k,t} - p_t) \\ &\quad + \frac{\varphi\psi}{1+\theta\varphi}z_t - \frac{1+\varphi}{1+\theta\varphi}(a_t + a_{ik,t}). \end{aligned}$$

Notice by the third term that, as long as the elasticities of substitution between goods within (θ) and across (η) sectors differ, there is an additional mechanism of pricing interaction. The intensity and direction of those interactions are captured by the parameter of the sectoral price index. Carvalho & Lee (2010) argue that this parameter should be negative in their model, implying strategic substitutability in pricing decisions within sectors. In our model, because because labor is firm-specific, within-sector pricing interactions can be either complementary or substitute. If $\theta > \eta$, the model delivers within-sector complementarity. The reason is the following. If elasticity of substitution between goods within sector is larger than between goods across sectors, when sectoral price increases, if the individual firm maintains its price constant, the demand for its good will increase (because θ is large, consumers prefer to substitute goods inside the same sector), which will pressure the firm's labor market and consequently the marginal cost, through wages. Thus, the firm also increases its price. On the other hand, if $\theta < \eta$, when sectoral price increases, households and firms substitute their consumption for other sectors' goods. In this case, the firm puts lower pressure on the labor market. Wages and marginal cost decrease and it will be optimal to reduce prices[9].

The combination of segmented labor market, intermediate inputs and heterogeneity in the intensity of use of other goods in production generates the various

[9]For expositional purposes we describe the effects sequentially. However, in a general equilibrium environment things happen simultaneously.

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configuration of within-sector and across-sector pricing interactions represented in equation (2-5). Even though confusing at first sight, the interpretation of those different mechanisms can be easily summarized in the following way. The parameter ζ_{k1} measures the "overall" degree of strategic complementarity in sector k[10]. The lower the parameter ζ_{k1}, the larger is the degree of "overall" strategic complementarities. If $\zeta_{k1} < 1$, pricing interactions are "overall" complementary, even if there is substitutability in pricing decisions within or across sectors. In this case, if for instance within-sector pricing decisions are substitute, necessarily the complementarity in across-sector pricing interactions more than compensate this substitutability, generating "net" complementarities. Therefore, the degree of real rigidities in sector k is measured by the parameter ζ_{k1}. In addition, one can easily show that this parameter is decreasing in the intensity of intermediate inputs usage, δ_k, which means that sectors with more intensive use of other goods in production should have larger real rigidities—thus, for instance, strategic complementarities should be stronger in sectors of goods than in sectors of services[11].

Regarding within- and across-sector pricing interactions, they are complementary or substitute if, respectively, the parameters ζ_{k2} and $(1 - \zeta_{k1} - \zeta_{k2})$ are positive or negative. The larger each parameter, the larger is the respective degree of strategic complementarities.

2.2.5 Firms' Price Setting

We assume that firms face state-dependent price rigidity. Specifically, in each period we allow firms reoptimize over their price changes: they can optimally either keep their old prices or pay a fixed cost F and change prices. We assume that cost F is independent of the sector.

In this environment is not optimal for the firms to charge $p^*_{ik,t}$ at all times. However, letting their actual prices drift away from their frictionless optimal prices also imposes them profit losses. We do not derive here the firms' optimal pricing policies in each sector[12]. For our purposes it is enough to realize that, in this case, firms in sector k will follow Ss pricing rules, like the one showed in Figure 2.1 below.

Let $y^*_{ik,t}$ be the unobservable difference between the frictionless optimal price and the firm's actual price. The graph shows one possible trajectory for $y^*_{ik,t} \equiv p^*_{ik,t} - p_{ik,\tau_t}$. τ_t represents the time in which the firm repriced for the

[10]Observe that the "overall" effect is the sum of within- and across-sector pricing interactions, which is $\zeta_{k2} + (1 - \zeta_{k1} - \zeta_{k2}) = 1 - \zeta_{k1}$.

[11]Notice that $\frac{\partial \zeta_{k1}}{\partial \delta_k} = -\frac{(1+\varphi-\varphi\psi)(1+\varphi)}{[1+\delta_k\varphi+(1-\delta_k)\theta\varphi]^2} < 0$. In addition, $\frac{\partial \zeta_{k2}}{\partial \delta_k} = -\frac{\varphi(\theta-\eta)(1+\varphi)}{[1+\delta_k\varphi+(1-\delta_k)\theta\varphi]^2}$, that is negative if $\theta > \eta$, and positive otherwise.

[12]The parameters of the pricing rule can be optimally determined and they depend, in particular, on the magnitude of the cost F and the variance of shocks in each sector.

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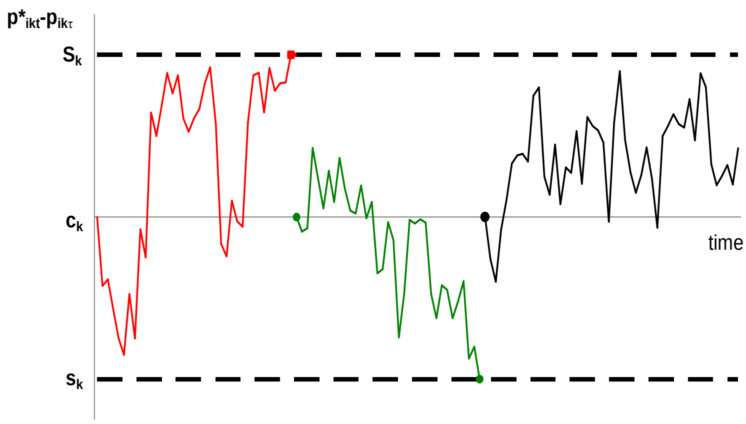
Figure 2.1: State-Dependent Pricing Rule



last time, choosing the price p_{ik,τ_t}. τ_t is indexed by t because it refers to the last price adjustment when we are considering the time t. The parameters S_k, s_k and c_k determine the pricing policy in sector k and the firm ik acts in the following way. While the variable $y^*_{ik,t}$ is inside the range (s_k, S_k), the firm maintains its price fixed. When $y^*_{ik,t}$ reaches the threshold S_k, however, $p^*_{ik,t}$ is sufficiently above the actual price and it is optimal for the firm to pay the cost F and increase its price. Likewise, when the threshold s_k is reached, $p^*_{ik,t}$ is sufficiently below $p_{ik,t}$ and the firm decreases its price. In case of price change, the variable $y^*_{ik,t}$ is set equal to c_k. Notice that the parameter c_k is not necessarily equal to zero, which means that, when the firm reprices, it does not necessarily set $p_{ik,t}$ equal to $p^*_{ik,t}$.

2.3 Econometric Framework

With knowledge that firms in each sector will follow Ss pricing policies and with the frictionless optimal price equation derived from microfoundations, we can estimate the parameters measuring within- and across-sector interactions in firms' pricing behaviour. We will follow the methodology developed in the first chapter, which consists in deriving an ordered probit model from the pricing policy followed by firms, and then mapping the parameters of the probit model into the structural parameters of interest.

Aiming to make the notation easier, in which follows we will write the equation (2-5) as:

$$
\begin{aligned}
p^*_{ik,t} &= \zeta_{k1}\mathcal{Y}_t + \zeta_{k2}p_{k,t} + (1-\zeta_{k1}-\zeta_{k2})p_t + \chi_k z_t + \tilde{a}_{k,t} + \tilde{a}_{ik,t} \\
&= \boldsymbol{x}'_{k,t}\boldsymbol{\beta}_k + \tilde{a}_{k,t} + \tilde{a}_{ik,t}, \qquad (2\text{-}6)
\end{aligned}
$$

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where $\boldsymbol{\beta}_k = [\zeta_{k1}, \zeta_{k2}, (1-\zeta_{k1}-\zeta_{k2}), \chi_k]'$ and $\boldsymbol{x}_{k,t} = (\mathcal{Y}_t, p_{k,t}, p_t, z_t)'$.

After defining the unobserved $y^*_{ik,t} \equiv p^*_{ik,t} - p_{ik,\tau_t}$, let the observable variable $y_{ik,t}$ be

$$y_{ik,t} = \begin{cases} 1, & if \quad p_{ik,t} > p_{ik,t-1} \\ 0, & if \quad p_{ik,t} = p_{ik,t-1} \\ -1, & if \quad p_{ik,t} < p_{ik,t-1} \end{cases}. \tag{2-7}$$

Using the equation for $p^*_{ik,t}$ and the pricing rule, which says that at the moment τ_t, $p^*_{ik,\tau_t} - p_{ik,\tau_t} = c_k$, we can write:

$$\begin{aligned} y^*_{ik,t} \equiv p^*_{ik,t} - p_{ik,\tau_t} &= (\boldsymbol{x}'_{k,t}\boldsymbol{\beta}_k + \tilde{a}_{k,t} + \tilde{a}_{ik,t}) - (\boldsymbol{x}'_{k,\tau_t}\boldsymbol{\beta}_k + \tilde{a}_{k,\tau_t} + \tilde{a}_{ik,\tau_t} - c_k) \\ &= (\boldsymbol{x}_{k,t} - \boldsymbol{x}_{k,\tau_t})'\boldsymbol{\beta}_k + c_k + (\tilde{a}_{k,t} - \tilde{a}_{k,\tau_t}) + (\tilde{a}_{ik,t} - \tilde{a}_{ik,\tau_t}) \\ &= \boldsymbol{z}'_{ik,t}\boldsymbol{\beta}_k + c_k + (\tilde{a}_{k,t} - \tilde{a}_{k,\tau_t}) + (\tilde{a}_{ik,t} - \tilde{a}_{ik,\tau_t}), \end{aligned} \tag{2-8}$$

where $\boldsymbol{z}_{ik,t} = \boldsymbol{x}_{k,t} - \boldsymbol{x}_{k,\tau_t}$. Observe that only aggregate variables and the sectoral price index are included in vector $\boldsymbol{x}_k$. But because the length of price spells vary among firms and along the time (once firms adjust their prices in different moments in time), at time t the difference $(\boldsymbol{x}_{k,t} - \boldsymbol{x}_{k,\tau_t})$ is not the same among firms. That is why we include a subscript i in $\boldsymbol{z}_{ik,t}$.

Notice that the shock $\tilde{a}_{k,t}$ results from the aggregate shock. Following the recent literature, such as Boivin et al. (2009) and others, which shows that the aggregate component of inflation is persistent, we assume that $\tilde{a}_{k,t}$ is a random walk:

$$\tilde{a}_{k,t} = \tilde{a}_{k,t-1} + v_{k,t}, \quad v_{k,t} \sim iid(0, \sigma^2_{v,k}). \tag{2-9}$$

We also assume that the idiosyncratic shock is a combination of an individual fixed component and an individual shock, $\tilde{a}_{ik,t} = \kappa_{ik} + a_{ik,t}$, where κ_{ik} is the individual fixed effect. In addition, the individual shock is given by

$$a_{ik,t} = \eta_k + a_{ik,t-1} + \varepsilon_{ik,t}, \quad \varepsilon_{ik,t} \sim \mathbb{N}(0, \sigma^2_k). \tag{2-10}$$

Under these assumptions we can rewrite equation (2-8) as[13]

$$\begin{aligned} y^*_{ik,t} \equiv p^*_{ik,t} - p_{ik,\tau_t} &= \boldsymbol{z}'_{ik,t}\boldsymbol{\beta}_k + c_k + (\tilde{a}_{k,t} - \tilde{a}_{k,\tau_t}) + (a_{ik,t} - a_{ik,\tau_t}) \\ &= \eta_k\delta_{ik,t} + \boldsymbol{z}'_{ik,t}\boldsymbol{\beta}_k + c_k + \sum_{j=1}^{T}\gamma_{kj}d_{ikj,t} + u_{ik,t}, \end{aligned} \tag{2-11}$$

where $\delta_{ik,t}$ is the time interval between t and τ_t,

[13] To obtain equation (2-11) we iterate backward $a_{ik,t}$ and $\tilde{a}_{k,t}$ until the moment of the last price adjustment. For details see the first chapter.

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$$u_{ik,t} = \varepsilon_{ik,t} + ... + \varepsilon_{ik,t-\delta_{ik,t}+1} \sim \mathbb{N}(0, \delta_{ik,t}\sigma_k^2)$$

and

$$d_{ikj,t} = \begin{cases} 1, & if \ \ j \in [t, t-\delta_{ik,t}+1] \\ 0, & if \ \ otherwise \end{cases} , \qquad j = 1, ..., T. \tag{2-12}$$

Two points are worth noting here. The first is that $u_{ik,t}$ is naturally autocorrelated and heteroscedastic, and given our assumptions this heteroskedasticity is known. It depends on the number of periods since the last price adjustment—$u_{ik,t}$ is a moving average $MA(\delta_{ik,t}+1)$ process— and we make use of this information in the estimation procedure. Second, the dummy variables in equation (2-11) control for the common shocks hitting the firm ik since the last time it repriced. For example, two firms that kept their prices unchanged during a period overlapping the time interval between τ_t and t have the respective dummy variables assuming value 1 during this period. The objective of controlling for these common shocks is isolating the strategic interaction among the firms, making sure that possible joint movements of prices are simply not because they were affected by the same shock.

Define $\boldsymbol{w}_{ik,t} = (\delta_{ik,t}, \boldsymbol{z}'_{ik,t}, \boldsymbol{d}'_{ik,t})'$, where $\boldsymbol{d}_{ik,t} = (d_{ik1,t}, ..., d_{ikT,t})'$. Then, the probability of a price increase is given by

$$\begin{aligned}
Pr[y_{ik,t} = 1 | \boldsymbol{w}_{ik,t}] &= Pr[y^*_{ik,t} \geq S_k | \boldsymbol{w}_{ik,t}] \\
&= Pr[\eta_k \delta_{ik,t} + \boldsymbol{z}'_{ik,t}\boldsymbol{\beta}_k + c_k + \sum_{j=1}^{T} \gamma_{kj} d_{ikj,t} + u_{ik,t} \geq S_k | \boldsymbol{w}_{ik,t}] \\
&= Pr\left[\frac{u_{ik,t}}{\sqrt{\delta_{ik,t}}\sigma_k} \geq \frac{S_k - c_k - \eta_k \delta_{ik,t} - \boldsymbol{z}'_{ik,t}\boldsymbol{\beta}_k - \sum_{j=1}^{T} \gamma_{kj} d_{ikj,t}}{\sqrt{\delta_{ik,t}}\sigma_k} \right] \\
&= 1 - \Phi\left(\frac{S_k - c_k}{\sigma_k} \frac{1}{\sqrt{\delta_{ik,t}}} - \frac{\eta_k}{\sigma_k}\sqrt{\delta_{ik,t}} - \frac{\boldsymbol{z}'_{ik,t}}{\sqrt{\delta_{ik,t}}} \frac{\boldsymbol{\beta}_k}{\sigma_k} - \sum_{j=1}^{T} \frac{\gamma_{kj}}{\sigma_k} \frac{d_{ikj,t}}{\sqrt{\delta_{ik,t}}} \right) \\
&= 1 - \Phi\left(\pi_{k1} \ddot{1}_{ik,t} - \tilde{\eta}_k \ddot{\delta}_{ik,t} - \ddot{\boldsymbol{z}}'_{ik,t}\tilde{\boldsymbol{\beta}}_k - \sum_{j=1}^{T} \tilde{\gamma}_{kj} \ddot{d}_{ikj,t} \right)
\end{aligned}$$

where the variables with two dots represent the variables divided by $\sqrt{\delta_{ik,t}}$, the parameters with tilde means that the parameters is scaled by σ_k, $\Phi(.)$ is the cumulative distribution function of a standard normal variable and $\pi_{k1} = (S_k - c_k)/\sigma_k$. In the third line we have used the fact that $u_{ik,t}$ is independent of $\boldsymbol{w}_{ik,t}$.

We can also derive the probability of observing the other two possible outcomes for $y_{ik,t}$. Thus, we obtain the following ordered probit model for price changes:

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$$
\begin{aligned}
Pr[y_{ik,t}=1|\boldsymbol{w}_{ik,t}] &= 1-\Phi\left(\frac{S_k-c_k}{\sigma_k}\frac{1}{\sqrt{\delta_{ik,t}}}-\frac{\eta_k}{\sigma_k}\sqrt{\delta_{ik,t}}-\frac{\boldsymbol{z}'_{ik,t}}{\sqrt{\delta_{ik,t}}}\frac{\boldsymbol{\beta}_k}{\sigma_k}-\sum_{j=1}^{T}\frac{\gamma_{kj}}{\sigma_k}\frac{d_{ikj,t}}{\sqrt{\delta_{ik,t}}}\right)\\
&= 1-\Phi\left(\pi_{k1}\ddot{1}_{ik,t}-\tilde{\eta}_k\ddot{\delta}_{ik,t}-\ddot{\boldsymbol{z}}'_{ik,t}\tilde{\boldsymbol{\beta}}_k-\sum_{j=1}^{T}\tilde{\gamma}_{kj}\ddot{d}_{ikj,t}\right)
\end{aligned}
$$

$$
\begin{aligned}
Pr[y_{ik,t}=0|\boldsymbol{w}_{ik,t}] = \Phi\left(\pi_{k1}\ddot{1}_{ik,t}-\tilde{\eta}_k\ddot{\delta}_{ik,t}-\ddot{\boldsymbol{z}}'_{ik,t}\tilde{\boldsymbol{\beta}}_k-\sum_{j=1}^{T}\tilde{\gamma}_{kj}\ddot{d}_{ikj,t}\right)- \\
\Phi\left(\pi_{k0}\ddot{1}_{ik,t}-\tilde{\eta}_k\ddot{\delta}_{ik,t}-\ddot{\boldsymbol{z}}'_{ik,t}\tilde{\boldsymbol{\beta}}_k-\sum_{j=1}^{T}\tilde{\gamma}_{kj}\ddot{d}_{ikj,t}\right) \quad \text{(2-13)}
\end{aligned}
$$

$$
\begin{aligned}
Pr[y_{ik,t}=-1|\boldsymbol{w}_{ik,t}] &= \Phi\left(\frac{s_k-c_k}{\sigma_k}\frac{1}{\sqrt{\delta_{ik,t}}}-\frac{\eta_k}{\sigma_k}\sqrt{\delta_{ik,t}}-\frac{\boldsymbol{z}'_{ik,t}}{\sqrt{\delta_{ik,t}}}\frac{\boldsymbol{\beta}_k}{\sigma_k}-\sum_{j=1}^{T}\frac{\gamma_{kj}}{\sigma_k}\frac{d_{ikj,t}}{\sqrt{\delta_{ik,t}}}\right)\\
&= \Phi\left(\pi_{k0}\ddot{1}_{ik,t}-\tilde{\eta}_k\ddot{\delta}_{ik,t}-\ddot{\boldsymbol{z}}'_{ik,t}\tilde{\boldsymbol{\beta}}_k-\sum_{j=1}^{T}\tilde{\gamma}_{kj}\ddot{d}_{ikj,t}\right)
\end{aligned}
$$

We estimate the model by quasi-maximum likelihood method, which guarantees consistency. Inference is carried out using a robust variance-covariance matrix for autocorrelation and heteroskedasticity. For details see the first chapter.

2.3.1 Estimation of Pricing Interactions

The estimation of parameters measuring within- and across-sector pricing interactions is carried out through the estimated probit models parameters. If the ordered probit model estimation is consistent, we can use the Slutsky's Theorem of convergence to consistently estimate ζ_{k1} and ζ_{k2}.

First, note that from equations (2-6) and (2-13) we have:

$$
\tilde{\beta}_{k1}=\frac{\zeta_{k1}}{\sigma_k} \quad , \quad \tilde{\beta}_{k2}=\frac{\zeta_{k2}}{\sigma_k} \quad \text{and} \quad \tilde{\beta}_{k3}=\frac{1-\zeta_{k1}-\zeta_{k2}}{\sigma_k}.
$$

From these equations we can obtain:

$$
\zeta_{k1}=\frac{\tilde{\beta}_{k1}}{\tilde{\beta}_{k1}+\tilde{\beta}_{k2}+\tilde{\beta}_{k3}}, \quad \text{(2-14)}
$$

$$
\zeta_{k2}=\frac{\tilde{\beta}_{k2}}{\tilde{\beta}_{k1}+\tilde{\beta}_{k2}+\tilde{\beta}_{k3}}, \quad \text{(2-15)}
$$

$$
\sigma_k=\frac{1}{\tilde{\beta}_{k1}+\tilde{\beta}_{k2}+\tilde{\beta}_{k3}}. \quad \text{(2-16)}
$$

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Additionally, we can construct confidence intervals for these parameter from the probit model parameters using the Delta Method.

We can also obtain estimations of the widths of the top and bottom bands of the pricing rules using the standard deviation of shocks and the intercepts of the probit model estimations:

$$S_k - c_k = \pi_{k1}\sigma_k \quad \text{and} \quad s_k - c_k = \pi_{k0}\sigma_k. \tag{2-17}$$

2.4 Data

2.4.1 Data Description

The data we use in this paper comprise individual price quotes of products collected by the Brazilian Institute of Economics of the Getulio Vargas Foundation (IBRE-FGV) to compute the Consumer Price Index (CPI-FGV). FGV calculates the CPI-FGV since 1944, being one of the oldest price indices in Brazil. We use disaggregated data that come from the electronic data set that stores the primary information underlying the computation of the index since 1996[14].

The CPI-FGV has a wide coverage. It measures the price change of a basket of goods and services that mirrors the composition of the budget spent by families receiving income up to 33 minimum wages per month, obtained from a household consumption survey—Pesquisa de Orcamento Familiar (POF)—also conducted by FGV. Currently, the CPI-FGV comprises 456 goods and services grouped into seven different groups: Food; Education and Recreation; Housing; Medical and Personal Care; Transportation; Apparel; and Other Goods and Services. The coverage has changed over time, but for most of our sample period prices were collected in the seven largest metropolitan regions of Brazil[15].

Prices are collected systematically by employees of FGV. The prices of some products are collected every ten days, while the remaining are collected on a monthly basis. The identification through codes makes it possible to ascertain which specific product or service is being collected, when and where. We refer to the most disaggregated level of the data as an *item*. Items are identified with a set of very specific characteristics, including brand, size, model, packaging, neighborhood and city where they are sold etc[16].

[14]The first papers to work with this data set were Gouvea (2007) and Barros et al. (2009).

[15]Up to December 2000 CPI-FGV comprised only the metropolitan regions of the two largest cities in Brazil: São Paulo and Rio de Janeiro. After January 2001, ten other cities were included: Belo Horizonte, Brasilia, Porto Alegre, Recife, Salvador, Belém, Curitiba, Florianopolis, Fortaleza and Goiania. At the beginning of 2005 the last five cities were dropped.

[16]For example, For example, type I black beans of the Combrasil brand, sold in a 1kg package in the outlet number 16,352, in Belém.

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The number of items whose prices are collected are not constant in each month. There is variation due to item exclusions and inclusions over time. But one feature of FGV collection is that there is never substitution of an item by another similar belonging to the same category. If a price of one item is not collected anymore, the price trajectory continues with missing values. Therefore, it is possible to follow the same item along the time. In addition, we can be assured that each price change registered is due to an actual price change.

We have a very representative sample of the overall CPI-FGV—around 85%—containing 243 categories of goods and services, with more than 7.4 million observations and more than 120 thousand price trajectories. *Price trajectory* is a sequence of price quotes of an item collected over the sample period. Our sample spans approximately 11 years, ranging from 1996 to 2006. The original seven sectors classified by IBRE-FGV are very well represented. The sector with the lowest representation has products comprising around 78% of its weight in the CPI-FGV.

2.4.2 Data Treatment

The original data sample was treated in order to have a set of information more suited to our goals. First, because our main objective is to estimate pricing interactions among firms, we have chosen not to work with products that have been regulated by either executive governments or regulatory agencies throughout the sample period. Second, we wanted to work with monthly data. Thus, for products whose prices are collected every 10 days, in the aggregation we choose to keep always the first price quote in each month.

Third, we excluded very short price trajectories and items with too many missing prices. Trajectories with less than 18 observations or with more than 30% of observations missing were dropped. We also treated items with consecutive missing information. Gaps with only one observation missing were filled using the price collected in the immediately preceding month. Gaps with up to three observations missing, when preceded and succeeded by the same price, were filled with the last price available. For those trajectories with four or more observations missing, we decided to maintain the longest uninterrupted piece of the price trajectory.

Fourth, price quotes classified as sale or promotion were also treated. Retail prices are characterized by a significant number of these temporary price decreases. They are used as instrument either to attract costumers or to control inventories. Of course, these episodes can have impacts on the estimated price setting behavior and, therefore, is a sensible issue (for discussions about sales in this environment see, for example, Nakamura & Steinsson (2008) and Silva (2009)). But following the recent literature, such as Midrigan (2006), Golosov & Lucas (2007) among others, we treated the observations identified as sales. Because IBRE-FGV does not label

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products that are "on sale" when they collect prices, we identify these temporary price changes using a "sale filter". The filter works in the following way: if a price decrease was large enough and quickly reversed to levels near the one prevailing before the change, we classified those observations as sales[17]. In this case, sale prices were replaced by the price collected in the immediately preceding period. The filter was repeated three times to capture "V-shape" promotions. Furthermore, "outlier" values were substituted by the previous observed price record. Outlier is defined here as an observation that is 10 times larger or 10 times smaller than the previous one.

Dealing with censoring is another important issue. To avoid the problem of left-censored spells, for all price trajectories we dropped the observations before the first observed price change. Right-censored spells are not problems for our estimations.

This treatment results in a final sample with a total number of more than 2.6 million price quotes and 58 thousand price trajectories. Table (2.1) below shows details of these data. Because our main goal is to estimate firms's pricing interaction within and across sectors, we proceed to a finer division of our sample in sectors. We use the same classification proposed in Barros et al. (2009) (hereafter BBCM) and disaggregate the original seven groups of the CPI-FGV into 15 sectors: 9 sectors of goods and 6 sectors of services. We will use this classification in our estimations in the next section. Table (2.1) shows that after the treatment each sector still have enough number of observations to carry out the estimations of the model derived in the previous sections. The sector with the smallest number of observations (Educational Services) still have more than 13 thousand price quotes.

2.5 Estimation

In this section we use the data set of microdata combined with aggregate data to estimate the probit model derived in section 2.3. We follow the strategy outlined in subsection 2.3.1 to estimate, through the probit model parameters, the pricing interaction within and across sectors.

2.5.1 Estimated Models

The theoretical model suggests that the parameters measuring pricing interactions should vary among sectors. Thus, we estimate one probit model for each sector. An important issue in this respect is the definition of "sectors" to which we should look. In this classification, each group of prices must really represent an eco-

[17]Formally, if $\frac{(p_{t-1}-p_t)}{p_{t-1}} > 25\%$ and $p_{t+1} \geq p_t\left(1+\frac{p_{t-1}-p_t}{p_{t-1}}\right)$. One can always argue that 25% is an arbitrary value, and/or that it is high for some sectors and/or low for some others. But we keep a single rule to minimize arbitrariness. Changing this value does not change our main results, however.

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Table 2.1: Number of Observations in the Data Sample

Sector	Original Data Set		Treated Data Set	
	# of obs	# of trajectories	# of obs	# of trajectories
Goods				
Vehicles and Equipments	20,211	791	13,884	386
Raw Food	1,010,894	7,213	306,116	4,913
Processed Food	3,051,026	28,777	916,621	17,594
Apparel	444,228	18,393	279,808	8,425
Personal Care Goods	651,437	6,297	225,584	4,384
House Maintenance Goods	862,530	18,003	305,976	7,780
Fuel	70,079	2,253	54,310	1,285
Education and Recreational Goods	193,969	6,388	120,820	3,439
Other Goods	95,775	4,569	51,332	1,437
Services				
House Maintenance Services	74,112	2,901	44,926	1,246
Transportation	24,591	638	15,225	378
Food Away From Home	202,272	4,179	93,166	2,310
Personal and Recreational Services	248,871	8,703	150,124	3,356
Educational Services	40,675	2,439	13,878	317
Medical Care Services	84,744	1,888	54,004	1,158
Total	7,075,414	113,432	2,645,774	58,408

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nomic sector. As already pointed out, we decided to follow the same classification adopted in BBCM (see Table 2.1). We split the individual prices of the CPI-FGV into 16 sectors: 9 sectors of goods, 6 sectors of services and one group of regulated prices (either by Federal or State governments and agencies). We do not consider the regulated prices in our estimations. Therefore, we estimate one probit model for each of the 15 sectors of goods and services.

For each sector we estimate two specifications of the probit model. The first formulation, which we name "simple specification", do not include the dummy variables that control for the aggregate shocks into the model. In the second formulation, called "complete specification", we estimate the model exactly as it appears in equations (2-13) and, therefore, includes the full bunch of dummy variables. Obviously, the absence of these variables in the simple specification can lead to bias in the estimated parameters, but the estimation of this specification can provide a measure of magnitude and direction of this possible bias. Aware of this issue, our analysis is mainly based on the results of the complete specification.

In estimations we combine the data set of microdata with aggregate variables. The dependent variable in the probit model is the observed $y_{ik,t}$ defined in equation (2-7), which is constructed for each individual price through the information in the microdata. Based on the theoretical model, the simple specification has six explanatory variables. The complete specification has the same six regressors, in addition to the whole set of dummies variables (constructed for each item in our data set according to equation (2-12)). The first two variables are, respectively, the square root of the time interval between t and τ_t, and its inverse. Those variables are obtained for each item in the data set using the history of its price spells. The other four variables are, respectively, the accumulated variation since the last price change (i.e. between t and τ_t) in the nominal expenditure, $\Delta\mathcal{Y}_{i,t} = \log(\mathcal{Y}_t/\mathcal{Y}_{\tau_t})$, in the sectoral price level, $\Delta p_{ik,t} = \log(P_{k,t}/P_{k,\tau_t})$, in the aggregate price level, $\Delta p_{i,t} = \log(P_t/P_{\tau_t})$, and in the intermediate input usage, $\Delta z_{i,t} = \log(Z_t/Z_{\tau_t})$, divided by the square root of the time interval between t and τ_t, $\sqrt{\delta_{i,t}}$. We calculate the accumulated changes in these variables for each item in our data set in every period t. This variability across individuals (because the length of price spells differs across individuals) is what allows us to estimate the models using basically aggregate variables as regressors. As previously explained, we divide the variables by $\sqrt{\delta_{i,t}}$ to take into account the heteroskedasticity in the residuals that naturally appears in the theoretical model.

We use the monthly nominal GDP series calculated by the Central Bank of Brazil from the quarterly national accounts of the Brazilian Institute of Geography and Statistics – IBGE as our measure of nominal expenditure. Aggregate prices are measured by the Consumer Price Index – CPI-FGV computed by FGV using the microdata described in the previous section. Likewise, we use the microdata to compute the sectoral price indices. Regarding the utilization of intermediate inputs, observe that what appears as regressor in the probit model is not the use of inputs in each sector, but only the aggregate use of intermediate input by the whole economy. Thus, we do not need disaggregated measurement, such as information about the input-output matrix of the economy. We use the quantum of intermediate goods produced by the industrial sector[18] as our measure of the utilization of intermediate inputs.

Before examining the results regarding pricing interactions we provide some information about the fit of the models. We compare the probability of price change predicted by the estimated model in each sector with the frequency of price chance computed directly from the data. The probability of price chance is the sum of the probability of price increase, $Pr[y_{ik,t} = 1|\bar{\boldsymbol{w}}_{ik,t}]$, with the probability of price decrease, $Pr[y_{ik,t} = -1|\bar{\boldsymbol{w}}_{ik,t}]$, both evaluated in the average of the explanatory variables in our data set. We use the simple specification in this exercise. The reason is that it is much easier to calculate those probabilities without worrying with the dummy variables. Moreover, the simple specification provides a lower bound to the fit of the complete specification (once the introduction of more explanatory variables into the model should improve its fit, or at least should not worse it) and, therefore, can be very informative about the ability of the model in explaining the data. We do not compute the frequencies of price change in each sector. Instead, we borrow the values estimated by BBCM. They calculate frequency, duration and

[18]Production indicator by category of use – intermediate goods, measured by IBGE.

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size of price changes in Brazil, among many other statistics, using the same set of microdata that we use here. Results are reported in Table (2.3) in the Appendix B.

We do not have confidence intervals for the probabilities of price changes, but only point estimations. Using the difference between the point estimates of the probabilities and the frequencies of price changes as criterium, we conclude that the performance of the estimated models seems good, especially if we consider the simplicity of the theoretical model. First, the estimated probabilities of price changes produce an ordering of the sectors regarding the degree of price rigidity very similar to that obtained using the frequency estimated from the data. Of course, the ordering based on the results of the estimated models is not perfect, but notice that the most flexible sectors (Vehicles and Equipments, Raw Food, and Processed Food, into the group of goods; and House Maintenance Services, into the group of services) have higher predicted probabilities of price changes, while the most rigid sectors (Education and Recreational Goods, and Other Goods, inside the group of goods) have lowest probabilities of price changes. The performance was worse for sectors inside the group of services. Second, the comparison of the probabilities of price changes to the values of frequencies shows that for some sectors the prediction is very good. For instance, while the frequencies of price changes in Vehicles and Equipments and in Education and Recreational Goods sectors are, respectively, 55% and 32%, the predicted probabilities are, respectively, 59% and 33%. For some other sectors, in particular for sectors inside the group of services, the point estimates are not very good.

However, despite the relative good performance, it is worth mentioning that our objective is not constructing a model with accurate predictions. Instead, we want a model with good adherence to the data and that can capture the mechanisms used in our strategy of estimating pricing interactions. Remember that our methodology infers the parameters measuring within- and across-sector pricing interactions from the relationship between the frictionless optimal price and the macroeconomic variables derived from the microfounded model. In this respect, our estimations also seem good. Tables (2.5) and (2.6) in Appendix show that most of estimated parameters of the probit models are statistically significant in any of the usual significance levels. In general, they also have the correct expected sign, based on the theoretical model. The significance of the probit models parameters provides evidence about the state-dependent nature of pricing decisions in practice, and this dependence must come from the relationship between the frictionless optimal price and the macroeconomic variables.

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2.5.2 Estimated Pricing Interactions

The previous subsection suggests that the estimated probit models capture the relationship through which our methodology estimate the parameters measuring pricing interactions within and across sectors. Thus, we now use the estimated parameters of the probit models in the way outlined in the subsection 2.3.1 to obtain estimates of the parameters ζ_{k1} and ζ_{k2} in each sector. Table (2.2) below reports summarized results from complete specification. It presents the parameters of the probit models involved in estimation, estimates of the parameters of pricing interactions and 95%-confidence intervals for them. Detailed results are in Appendix B. The simple specification results are also reported in Appendix B.

The first piece of evidence that emerges from the results is that there seems to be a significant degree of "overall" strategic complementarities in each sector. As emphasized in subsection 2.2.4, pricing decisions are overall strategic complementary in sector k if $\zeta_{k1} < 1$. Conversely, they are strategic substitute if $\zeta_{k1} > 1$. The estimates suggest that in every sectors the parameter ζ_{k1} is close to zero. For instance, none of the upper bounds of confidence intervals is greater than 0.35. The sector with the largest point estimate is Personal and Recreational Services, with estimated ζ_{k1} equal to 0.25. On the other hand, the theoretical primitive parameters composing ζ_{k1} reveal that it cannot be negative. In our estimations we decided not imposing any restriction, but estimating the parameters freely. For three sectors the point estimates are negative, but notice from the confidence intervals that in general we cannot reject the null hypothesis that they are positive and close to zero. Using the simple specification we find similar results suggesting strong strategic complementarities in every sectors.

Those results confirm the evidence of strong real rigidities found in the first chapter. The parameter measuring strategic complementarities in pricing decisions in the estimations of the first chapter varies between 0.03 and 0.11, for the aggregate economy. If we take the average value of our estimates of ζ_{k1} among all sectors we will find 0.11, therefore inside the same range. In addition, if we consider the average value of estimates among sectors in each group, we will find that "overall" strategic complementarity is stronger in sectors of goods than in sectors of services, with average values respectively equal to 0.07 and 0.17. This is an interesting result and corroborates the intuition based on the theoretical model: remember that $\partial\zeta_{k1}/\partial\delta_k < 0$, and goods sectors are more intensive in the use of intermediate inputs than services sectors, which are more intensive in the labor usage.

Regarding pricing interaction within sectors, the results of estimations suggest that in general the parameter is positive. As emphasized in subsection 2.2.4, Carvalho & Lee (2010) argue that parameter ζ_{k2} should be negative in their model,

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Table 2.2: Pricing Interactions, Complete Specification

Sectors	y_t	p_t	p_{kt}	ζ_1	ζ_2	$1-\zeta_1-\zeta_2$	Confidence interval for ζ_1	Confidence interval for ζ_2
Goods								
Vehicles and Equipments	-1.06	22.27	13.51	-0.03	0.39	0.64	-0.10 - 0.04	0.32 - 0.46
	(1.249)	(1.615)	(1.211)	(0.037)	(0.037)			
Raw Food	1.25	3.44	3.49	0.15	0.43	0.42	0.10 - 0.21	0.39 - 0.46
	(0.272)	(0.198)	(0.087)	(0.029)	(0.017)			
Processed Food	-0.24	-0.35	7.22	-0.04	1.09	-0.05	-0.08 - 0.01	1.01 - 1.16
	(0.145)	(0.213)	(0.154)	(0.022)	(0.038)			
Apparel	0.42	4.26	2.18	0.06	0.32	0.62	-0.01 - 0.13	0.27 - 0.37
	(0.269)	(0.203)	(0.159)	(0.037)	(0.025)			
Personal Care Goods	-0.69	5.88	3.77	-0.08	0.42	0.66	-0.14 - -0.02	0.35 - 0.49
	(0.263)	(0.333)	(0.308)	(0.031)	(0.034)			
House Maintenance Goods	0.44	4.21	5.68	0.04	0.55	0.41	0.00 - 0.08	0.49 - 0.61
	(0.231)	(0.338)	(0.302)	(0.022)	(0.030)			
Fuel	2.69	-5.04	14.85	0.22	1.19	-0.40	0.14 - 0.29	1.06 - 1.31
	(0.577)	(0.558)	(0.185)	(0.040)	(0.064)			
Education and Recreational Goods	3.47	17.48	-1.02	0.17	-0.05	0.88	0.15 - 0.20	-0.07 - -0.03
	(0.314)	(0.225)	(0.181)	(0.013)	(0.009)			
Other Goods	1.74	0.93	18.70	0.08	0.87	0.04	0.04 - 0.12	0.83 - 0.92
	(0.501)	(0.311)	(0.250)	(0.022)	(0.023)			
Services								
House Maintenance Services	9.95	40.24	5.15	0.18	0.09	0.73	0.16 - 0.20	0.08 - 0.10
	(0.582)	(0.228)	(0.331)	(0.009)	(0.006)			
Transportation	0.67	6.58	1.63	0.08	0.18	0.74	-0.10 - 0.25	0.08 - 0.29
	(0.830)	(0.416)	(0.443)	(0.087)	(0.052)			
Food Away From Home	2.55	4.81	13.73	0.12	0.65	0.23	0.09 - 0.15	0.61 - 0.69
	(0.377)	(0.354)	(0.384)	(0.016)	(0.020)			
Personal and Recreational Services	3.53	4.33	6.18	0.25	0.44	0.31	0.22 - 0.28	0.39 - 0.49
	(0.266)	(0.228)	(0.380)	(0.016)	(0.025)			
Educational Services	4.01	9.44	3.68	0.23	0.21	0.55	0.15 - 0.32	0.14 - 0.29
	(0.921)	(0.505)	(0.578)	(0.043)	(0.036)			
Medical Care Services	3.84	2.43	15.14	0.18	0.71	0.11	0.14 - 0.22	0.67 - 0.74
	(0.448)	(0.157)	(0.441)	(0.018)	(0.018)			

Note: These results are obtained using the complete specification. Frictionless equation: $p^*_{ik,t} = \zeta_{k1}y_t + (1-\zeta_{k1}-\zeta_{k2})p_t + \zeta_{k2}p_{k,t} + \chi_k z_t + \tilde{a}_{k,t} + \tilde{a}_{ik,t}$.

Robust standard deviations are in parenthesis.

The confidence interval is 95% of confidence. Standard deviations of ζ_1 and ζ_2 were obtained by the Delta method.

which would imply that pricing decisions of firms inside the same sector are strategic substitute. This mechanism would help to speed up the response of sectoral prices to sectoral shocks. In our model they can be either positive or negative. But the estimations suggest that for most sectors the parameter is positive. The only exception is the sector of Education and Recreational Goods, with estimate equal to -0.05. In fact, in some sectors the value is not only positive but also large—in the sectors of Processed Food and Fuel, for instance, the parameter is larger than 1. Similar results are found in the simple specification.

Notice that the larger the parameter ζ_{k2}, the stronger is strategic complementarity within sectors. Interestingly, the estimates suggests that within-sector strategic complementarities are on average stronger in sectors of goods than in sectors of services. The average value among sectors inside the group of goods is 0.58, while the average value for services is 0.38. The interpretation of this result is more difficult because $\partial\zeta_{k2}/\partial\delta_k$ depends on the magnitude of θ relative to η. Remember that if

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$\theta > \eta$, $\partial\zeta_{k2}/\partial\delta_k < 0$. Otherwise, ζ_{k2} is increasing in δ_k. However, once the estimates of ζ_{k2} are in general positive, the evidence suggests that the first case is valid. Therefore, in light of these intricate mechanisms, we should expect an opposite result regarding the intensity of within-sector pricing interactions in the sectors of goods and services than that indicated by the average values. We have to be careful with these interpretations, however. In special because there is great heterogeneity in the estimated parameters among sectors, which means that the average may not be representative. What seems more robust is the evidence that within-sector pricing interactions are in general strategic complementary, rather than substitute.

Results also suggest that pricing interactions across sectors are strategic complementary. Except for Processed Food and Fuel, the parameter of aggregate price index is positive in every sectors. Even in the Processed Food sector we cannot reject the null hypothesis that it is positive. Only in the Fuel sector we really estimate a negative parameter. In this sector, however, the complementarity in pricing decisions within sector is strong enough to compensate the substitutability in pricing interactions across sectors. The result is "overall" strategic complementarity.

2.6 Conclusions

In this paper we develop a multi-sector state-dependent pricing model with intermediate goods, heterogeneity in the intensity of intermediate inputs usage and segmented labor market with firm-specific labor. Those ingredients generate within- and across-sector interactions in firms' pricing decisions. In addition, the intensity of these mechanisms can be different in each sector. In the literature these pricing interactions are argued to increase monetary non-neutrality and generate differential responses of prices to aggregate and sectoral shocks. We estimate the pricing interactions using the data set of individual prices underlying the Consumer Price Index (CPI-FGV) in Brazil. We carry out estimations for 15 sectors: 9 sectors of goods and 6 sectors of services.

In general, the results that emerge from the data is that pricing decisions are overall complementary rather than substitute. Even in sectors where pricing decisions within sectors (across sectors) are strategic substitute, the degree of strategic complementarity in decisions across (within) sectors more than compensates substitutability. In most sectors, however, pricing interactions within and across sectors are both complementary. Thus, strategic complementarities in pricing interactions within sectors reinforce the degree of real rigidities. But there is also great heterogeneity in the combination of these sectoral and aggregate effects among sectors.

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2.7 Appendix A: Derivation of the Main Equations

In this appendix we derive in a more detailed way the equations presented in the main text.

Profit Function

To obtain the firm's profit function, first observe that the total demand for output of firm ik is given by the total demand for consumption and the total demand by other firms (to be used as intermediate inputs):

$$Y_{kt}(i) = C_{kt}(i) + \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',k,t}(i',i)di'. \tag{2-18}$$

Using the optimal demand of households and firms developed in the main text we can write this equation as

$$\begin{aligned} Y_{kt}(i) &= \left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}\left(\frac{P_{kt}}{P_t}\right)^{-\eta} C_t + \sum_{k'=1}^{K}\int_{\mathcal{I}_{k'}}\left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}\left(\frac{P_{kt}}{P_t}\right)^{-\eta} Z_{k't}(i')di' \\ &= \left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}\left(\frac{P_{kt}}{P_t}\right)^{-\eta}\left\{C_t + \underbrace{\sum_{k'=1}^{K}\int_{\mathcal{I}_{k'}} Z_{k't}(i')di'}_{\equiv Z_t}\right\} \\ &= \left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta}\left(\frac{P_{kt}}{P_t}\right)^{-\eta} Y_t, \end{aligned} \tag{2-19}$$

where $Y_t = C_t + Z_t$.

Besides, combining production function with equation (2-1) yields the following relationship:

$$H_{kt}(i) = \left(\frac{\delta_k}{1-\delta_k}\right)^{-\delta_k}\left(\frac{W_{kt}(i)}{P_t}\right)^{-\delta_k}\frac{Y_{kt}(i)}{A_t A_{kt}(i)}. \tag{2-20}$$

The real profit of firm ik is defined as:

$$\Pi_{kt}^{real}(i) \equiv \frac{P_{kt}(i)}{P_t}Y_{kt}(i) - \frac{W_{kt}(i)}{P_t}H_{kt}(i) - \frac{P_t Z_{kt}(i)}{P_t}.$$

Substituting equation (2-1) from the main text:

$$\Pi_{kt}^{real}(i) = \frac{P_{kt}(i)}{P_t}Y_{kt}(i) - \frac{1}{1-\delta_k}\frac{W_{kt}(i)}{P_t}H_{kt}(i). \tag{2-21}$$

Finally, substituting equations (2-19) and (2-20) into equation (2-21) provides the real profit function (2-3) reported in the main text:

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$$
\begin{aligned}
\Pi_{kt}^{real}(i) &= \frac{P_{kt}(i)}{P_t} Y_{kt}(i) - \lambda_k \left(\frac{W_{kt}(i)}{P_t}\right)^{1-\delta_k} \frac{Y_{kt}(i)}{A_t A_{kt}(i)} \\
&= \frac{P_{kt}(i)}{P_t} \left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta} \left(\frac{P_{kt}}{P_t}\right)^{-\eta} Y_t \\
&\qquad - \frac{\lambda_k}{A_t A_{kt}(i)} \left(\frac{W_{kt}(i)}{P_t}\right)^{1-\delta_k} \left(\frac{P_{kt}(i)}{P_{kt}}\right)^{-\theta} \left(\frac{P_{kt}}{P_t}\right)^{-\eta} Y_t,
\end{aligned}
$$

where $\lambda_k = \frac{1}{1-\delta_k} \left(\frac{\delta_k}{1-\delta_k}\right)^{-\delta_k}$.

Frictionless Optimal Price

If prices are perfectly flexible, the firm's optimal price is obtained maximizing the previous real profit function each period. The first order condition of this problem is:

$$
(1-\theta) P_{kt}^*(i)^{-(1+\theta)} \left(\frac{1}{P_{kt}}\right)^{-\theta} \left(\frac{P_{kt}}{P_t}\right)^{-\eta} Y_t \left[\frac{P_{kt}^*(i)}{P_t} - \frac{\theta}{1-\theta} \underbrace{\lambda_k \left(\frac{W_{kt}(i)}{P_t}\right)^{1-\delta_k} \frac{1}{A_t A_{kt}(i)}}_{MC_{kt}(i)} \right] = 0,
$$

which implies the equation (2-4) in the main text.

Steady State

As mentioned in the main text, we log-linearize the frictionless optimal price equation around a non-stochastic steady state, which is detailed here. In general, a non-stochastic steady state is not symmetric. In particular, the steady state depends on the firms' productivity level, $A_t(i)$, $i \in [0,1]$, and the firm-specific parameters measuring households' relative disutilities of supplying labor, $\omega_k(i)$, $i \in [0,1]$. But we make two assumptions that simplifies the steady-state characterization: i) $\omega_k(i) = n_k^{-\varphi}$, for all i and k; ii) without loss of generality, we set the aggregate productivity level equal to $A = 1$, and in each sector $A_k(i) = A_k$, $i \in \mathcal{I}_k$, and A_k is such that equalizes steady-state marginal cost among sectors. Therefore, the first assumption relates the relative disutilities of working to the size of the sectors and makes the real wage payed by each firm equal in steady state, which without loss of generality we set in $W/P = \Delta$. However, because production function is sector-specific, different sectors use inputs with varying intensity even in the steady state.

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But the second assumption equalizes firms' steady-state marginal cost in different sectors.

We solve for $\{Y_k, C_k, Z_k, H_k, A_k, \frac{\Pi_k}{P}\}$: the steady state values of sectoral gross output, value-added output, intermediate input usage, hours of working, productivity and real profits. Once we obtain sectoral values, we can use symmetry inside sectors to calculate the micro variables, i.e., $Y_k = n_k Y_k(i)$, $C_k = n_k C_k(i)$, $Z_k = n_k Z_k(i)$, $H_k = n_k H_k(i)$, $\Pi_k = n_k \Pi_k(i)$. Aggregate variables are given by $Y = \sum_{k=1}^{K} Y_k$, $C = \sum_{k=1}^{K} C_k$, $Z = \sum_{k=1}^{K} Z_k$, $H = \sum_{k=1}^{K} H_k$, $\Pi = \sum_{k=1}^{K} \Pi_k$. Additionally, $W_k(i) = W_k = W$, $\frac{W}{P} = \Delta$ and $\frac{P_k(i)}{P} = \frac{P_k}{P} = 1$.

With previous assumptions, the system os equilibrium conditions can be reduced to seven equations:

$$\sum_{k=1}^{K} C_k = \frac{W}{P}\sum_{k=1}^{K} H_k + \frac{1}{P}\sum_{k=1}^{K}\Pi_k \tag{2-22}$$

$$\frac{W}{P} = \omega_{ik} H_k^{\varphi}\sum_{k=1}^{K} C_k \tag{2-23}$$

$$Y_k = A_k H_k^{1-\delta_k} Z_k^{\delta_k} \tag{2-24}$$

$$Y_k = C_k + Z_k \tag{2-25}$$

$$\frac{\Pi_k}{P} = Y_k - \frac{W}{P}H_k - Z_k \tag{2-26}$$

$$Z_k = \frac{\delta_k}{1-\delta_k}\frac{W}{P}H_k \tag{2-27}$$

$$1 = \frac{\theta}{\theta-1}\frac{\lambda_k}{A_k}\left(\frac{W}{P}\right)^{1-\delta_k} \tag{2-28}$$

First, from (2-28) we solve for sectoral productivity:

$$A_k = \frac{\theta}{\theta-1}\lambda_k\left(\frac{W}{P}\right)^{1-\delta_k}.$$

Second, substituting out Z_k in equations (2-24) and (2-26) by (2-27) yields:

$$H_k = \left(\frac{\delta_k}{1-\delta_k}\right)^{-\delta_k}\left(\frac{W}{P}\right)^{-\delta_k}\frac{Y_k}{A_k},$$

$$\frac{\Pi_k}{P} = Y_k - \left(\frac{1}{1-\delta_k}\right)\left(\frac{W}{P}\right)H_k.$$

Combining the two previous equation gives:

$$\frac{\Pi_k}{P} = \left[1 - \frac{\lambda_k}{A_k}\left(\frac{W}{P}\right)^{1-\delta_k}\right]Y_k.$$

But from (2-28), $\frac{\lambda_k}{A_k}\left(\frac{W}{P}\right)^{1-\delta_k} = \frac{\theta-1}{\theta}$. Consequently,

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$$\frac{\Pi_k}{P} = \frac{1}{\theta} Y_k.$$

Third, from equations (2-24) and (2-27) we can write:

$$Z_k = \left(\frac{\delta_k}{1-\delta_k} \frac{W}{P} \right)^{1-\delta_k} \frac{Y_k}{A_k}.$$

But $\lambda_k = \frac{1}{1-\delta_k} \left(\frac{\delta_k}{1-\delta_k} \right)^{-\delta_k}$ and from (2-28) $\frac{W}{P} = \left[\frac{A_k}{\lambda_k} \frac{\theta-1}{\theta} \right]^{1/1-\delta_k}$. Then, we obtain sectoral intermediate input usage:

$$Z_k = \delta_k \frac{\theta-1}{\theta} Y_k.$$

Next, from equation (2-25) we know that $C_k = Y_k - Z_k$. Then,

$$\begin{aligned} C_k &= Y_k - \delta_k \frac{\theta-1}{\theta} Y_k \\ &= \left(1 - \delta_k \frac{\theta-1}{\theta} \right) Y_k. \end{aligned}$$

Finally, substituting the sectoral intermediate input usage into (2-27) yields:

$$H_k = \left(\frac{W}{P} \right)^{-1} (1-\delta_k) \left(\frac{\theta-1}{\theta} \right) Y_k.$$

But from (2-28), $\left(\frac{W}{P} \right)^{-1} = \left(\frac{A_k}{\lambda_k} \frac{\theta-1}{\theta} \right)^{-1/1-\delta_k}$. Consequently, sectoral labor supply is given by:

$$H_k = A_k^{\frac{-1}{1-\delta_k}} \left[\delta_k \left(\frac{\theta-1}{\theta} \right) \right]^{\frac{-\delta_k}{1-\delta_k}} Y_k.$$

So far we have expressed the steady-state values of the other variables in terms of Y_k. In turn, the value of Y_k can be implicitly determined using (2-23):

$$\begin{aligned} \frac{W}{P} &= \omega_{ik} H_k^{\varphi} \sum_{k=1}^{K} C_k \\ \left(\frac{A_k}{\lambda_k} \frac{\theta-1}{\theta} \right)^{\frac{1}{1-\delta_k}} &= n_k^{-\varphi} \left[A_k^{\frac{-1}{1-\delta_k}} \left[\delta_k \left(\frac{\theta-1}{\theta} \right) \right]^{\frac{-\delta_k}{1-\delta_k}} Y_k \right]^{\varphi} \sum_{k=1}^{K} \left(1 - \delta_k \frac{\theta-1}{\theta} \right) Y_k \\ \left(\frac{1}{\lambda_k} \frac{\theta-1}{\theta} \right)^{\frac{1}{1-\delta_k}} A_k^{\frac{1+\varphi}{1-\delta_k}} &= n_k^{-\varphi} Y_k^{\varphi} \left[\delta_k \left(\frac{\theta-1}{\theta} \right) \right]^{\frac{-\varphi\delta_k}{1-\delta_k}} \sum_{k=1}^{K} \left(1 - \delta_k \frac{\theta-1}{\theta} \right) Y_k \end{aligned}$$

In the special case in which $A_k = 1$ and $\delta_k = \delta$, $\forall k$, we have a symmetric equilibrium with $Y_k = n_k Y_k(i) = n_k Y$ and the previous expression simplifies to

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that presented in Carvalho & Lee (2010):

$$\begin{aligned}\left(\frac{1}{\lambda}\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\delta}} &= n_k^{-\varphi}(n_kY)^{\varphi}\left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{-\varphi\delta}{1-\delta}}\left(1-\delta\frac{\theta-1}{\theta}\right)Y\left(\sum_{k=1}^{K}n_k\right)\\ &\implies Y=\left\{\left(\frac{1}{\lambda}\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\delta}}\left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{\varphi\delta}{1-\delta}}\left(1-\delta\frac{\theta-1}{\theta}\right)^{-1}\right\}^{\frac{1}{1+\varphi}}\end{aligned}$$

If in addition $\delta_k = 0, \forall k$, it simplifies to:

$$\begin{aligned}\left(\frac{\theta-1}{\theta}\right) &= n_k^{-\varphi}(n_kY)^{\varphi}\left(Y\sum_{k=1}^{K}n_k\right)\\ &\implies Y=\left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1+\varphi}},\end{aligned}$$

which is the standard result in models without intermediate inputs.

Loglinear Approximation

Here we follow the steps to obtain the log-linearized frictionless optimal price equation presented in the main text. We do not present the full set of log-linearized equations, but only those necessary for our task. First, note that the demand functions in the log-linear form are:

$$y_{kt}(i) - y_{kt} = -\theta[p_{kt}(i) - p_{kt}], \tag{2-29}$$

$$y_{kt} - y_t = -\eta[p_{kt} - p_t], \tag{2-30}$$

$$c_{kt}(i) - c_{kt} = -\theta[p_{kt}(i) - p_{kt}], \tag{2-31}$$

$$c_{kt} - c_t = -\eta[p_{kt} - p_t], \tag{2-32}$$

$$z_{k,k',t}(i,i') - z_{k,k',t}(i) = -\theta[p_{k',t}(i) - p_{k',t}],$$

$$z_{k,k',t}(i) - z_{k,t}(i) = -\eta[p_{k',t} - p_t].$$

Household's labor supply and firm's demand for intermediate inputs are:

$$w_{kt}(i) - p_t = \varphi h_{kt}(i) + c_t, \tag{2-33}$$

$$w_{kt}(i) - p_t = z_{kt}(i) + h_{kt}(i). \tag{2-34}$$

The firm's production function is

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$$y_{kt}(i) = a_t + a_{kt}(i) + (1-\delta_k)h_{kt}(i) + \delta_k z_{kt}(i), \tag{2-35}$$

and the real marginal cost:

$$mc_{kt}(i) = (1-\delta_k)[w_{kt}(i) - p_t] + a_t + a_{kt}(i). \tag{2-36}$$

Then, log-linearization of the equation (2-4) in the main text gives:

$$\begin{aligned} p^*_{kt}(i) - p_t &= mc_{kt}(i) \\ &= (1-\delta_k)[w_{kt}(i) - p_t] + a_t + a_{kt}(i). \end{aligned} \tag{2-37}$$

Also, note that from equations (2-33) and (2-34) we get:

$$z_{kt}(i) = (1+\varphi)h_{kt}(i) + c_t.$$

Substituting into the production function:

$$\begin{aligned} y_{kt}(i) &= a_t + a_{kt}(i) + (1-\delta_k)h_{kt}(i) + \delta_k z_{kt}(i) \\ &= a_t + a_{kt}(i) + (1-\delta_k)h_{kt}(i) + \delta_k[(1+\varphi)h_{kt}(i) + c_t] \\ &= a_t + a_{kt}(i) + (1+\delta_k\varphi)h_{kt}(i) + \delta_k c_t, \end{aligned}$$

and isolating $h_{kt}(i)$, we obtain:

$$h_{kt}(i) = \frac{1}{1+\delta_k\varphi}y_{kt}(i) - \frac{\delta_k}{1+\delta_k\varphi}c_t - \frac{1}{1+\delta_k\varphi}a_t - \frac{1}{1+\delta_k\varphi}a_{kt}(i).$$

So, from labor supply, we get:

$$w_{kt}(i) - p_t = \frac{\varphi}{1+\delta_k\varphi}y_{kt}(i) - \frac{\delta_k\varphi}{1+\delta_k\varphi}c_t - \frac{\varphi}{1+\delta_k\varphi}a_t - \frac{\varphi}{1+\delta_k\varphi}a_{kt}(i) + c_t$$

Therefore, we can write equation (2-37) as

$$\begin{aligned} p^*_{kt}(i) &= (1-\delta_k)\left[\frac{\varphi}{1+\delta_k\varphi}y_{kt}(i) - \frac{\delta_k\varphi}{1+\delta_k\varphi}c_t - \frac{\varphi}{1+\delta_k\varphi}a_t - \frac{\varphi}{1+\delta_k\varphi}a_{kt}(i) + c_t\right] + \\ &\quad + a_t + a_{kt}(i) + p_t \\ &= \frac{(1-\delta_k)\varphi}{1+\delta_k\varphi}y_{kt}(i) + \frac{(1-\delta_k)}{1+\delta_k\varphi}c_t - \frac{1+\varphi}{1+\delta_k\varphi}a_t - \frac{1+\varphi}{1+\delta_k\varphi}a_{kt}(i) + p_t. \end{aligned} \tag{2-38}$$

But, from optimal demand for good ik, $y_{kt}(i) = y_{kt} - \theta(p^*_{kt}(i) - p_{kt})$. Then,

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$$
\begin{aligned}
p^*_{kt}(i) &= \frac{(1-\delta_k)\varphi}{1+\delta_k\varphi}[y_{kt} - \theta(p^*_{kt}(i) - p_{kt})] + \frac{(1-\delta_k)}{1+\delta_k\varphi}c_t - \frac{1+\varphi}{1+\delta_k\varphi}a_t - \frac{1+\varphi}{1+\delta_k\varphi}a_{kt}(i) + p_t \\
&= \frac{\frac{(1-\delta_k)\varphi}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}y_{kt} + \frac{\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}p_{kt} + \frac{\frac{(1-\delta_k)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}c_t + \frac{1}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}p_t - \\
&\qquad \frac{\frac{(1+\varphi)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}a_t - \frac{\frac{(1+\varphi)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}a_{kt}(i). \qquad (2\text{-}39)
\end{aligned}
$$

Likewise, $y_{kt} = y_t - \eta(p_{kt} - p_t)$. Substituting into the previous equation yields:

$$
\begin{aligned}
p^*_{kt}(i) &= \frac{1+\frac{(1-\delta_k)\eta\varphi}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}p_t + \frac{\frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}p_{kt} + \frac{\frac{(1-\delta_k)\varphi}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}y_t + \frac{\frac{(1-\delta_k)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}c_t - \\
&\qquad \frac{\frac{(1+\varphi)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}a_t - \frac{\frac{(1+\varphi)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}a_{kt}(i). \qquad (2\text{-}40)
\end{aligned}
$$

In addition, aggregate output is used for consumption and as intermediate input, $Y_t = C_t + Z_t$. In the log-linear form this equation is $y_t = (1-\psi)c_t + \psi z_t$, where $\psi = \sum_k Z_k / \sum_k Y_k$. Then,

$$
\begin{aligned}
p^*_{kt}(i) &= \frac{1+\frac{(1-\delta_k)\eta\varphi}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}p_t + \frac{\frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}p_{kt} + \frac{\frac{(1-\delta_k)[1+\varphi-\varphi\psi]}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}c_t + \frac{\frac{(1-\delta_k)\varphi\psi}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}z_t - \\
&\qquad \frac{\frac{(1+\varphi)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}a_t - \frac{\frac{(1+\varphi)}{1+\delta_k\varphi}}{1+\frac{(1-\delta_k)\theta\varphi}{1+\delta_k\varphi}}a_{kt}(i).
\end{aligned}
$$

This equation can be written as

$$
\begin{aligned}
p^*_{kt}(i) &= \left[1 - \frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}\right]p_t + \frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}p_{kt} + \\
&\qquad + \frac{(1-\delta_k)[1+\varphi-\varphi\psi]}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}c_t + \frac{(1-\delta_k)\varphi\psi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}z_t \\
&\qquad - \frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_t - \frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_{kt}(i).
\end{aligned}
$$

Finally, we assume an exogenous stochastic process for nominal expenditure, $\mathcal{Y}_t = p_t + c_t$. Then, substituting $c_t = \mathcal{Y}_t - p_t$ into the previous equation gives the frictionless optimal price equation presented in the main text:

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$$
\begin{aligned}
p^*_{kt}(i) &= \frac{(1-\delta_k)[1+\varphi-\varphi\psi]}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}\mathcal{Y}_t + \frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}p_{kt} + \\
&\qquad + \left[1 - \frac{(1-\delta_k)[1+\varphi-\varphi\psi]}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi} - \frac{(1-\delta_k)\varphi(\theta-\eta)}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}\right] p_t + \\
&\qquad + \frac{(1-\delta_k)\varphi\psi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}z_t - \frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_t \\
&\qquad - \frac{1+\varphi}{1+\delta_k\varphi+(1-\delta_k)\theta\varphi}a_{kt}(i) \\
&= \zeta_{k1}\mathcal{Y}_t + \zeta_{k2}p_{kt} + (1-\zeta_{k1}-\zeta_{k2})p_t + \chi_k z_t - \tilde{a}_{kt} - \tilde{a}_{kt}(i).
\end{aligned}
$$

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2.8 Appendix B: Additional Estimation Results

Table 2.3: Fit of the Models, Simple Specification

Sectors	Predicted Probability of Price Change[1)]	Frequency of Price Change[2)]
Goods		
Vehicles and Equipments	0.59	0.55
Raw Food	0.68	0.51
Processed Food	0.64	0.46
Apparel	0.63	0.46
Personal Care Goods	0.57	0.40
House Maintenance Goods	0.58	0.38
Fuel	0.56	0.37
Education and Recreational Goods	0.33	0.32
Other Goods	0.34	0.19
Services		
House Maintenance Services	0.44	0.45
Transportation	0.34	0.26
Food Away From Home	0.16	0.15
Personal and Recreational Services	0.36	0.12
Educational Services	0.34	0.10
Medical Care Services	0.30	0.06

Notes: 1) Predicted probability of price change is calculated using the average of the explanatory variables in the probit models.
2) Frequency of price change is computed by Barros, Bonomo, Carvalho and Matos (2009).

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Table 2.4: Pricing Interactions, Simple Specification

Sectors	y_t	p_t	p_{kt}	ζ_1	ζ_2	$1-\zeta_1-\zeta_2$	Confidence interval for ζ_1	Confidence interval for ζ_2
Goods								
Vehicles and Equipments	1.09	16.15	12.81	0.04	0.43	0.54	0.00 - 0.07	0.38 - 0.47
	(0.609)	(1.015)	(0.627)	(0.019)	(0.023)			
Raw Food	1.62	4.25	2.41	0.20	0.29	0.51	0.17 - 0.22	0.27 - 0.31
	(0.125)	(0.158)	(0.047)	(0.013)	(0.008)			
Processed Food	0.27	2.03	4.80	0.04	0.68	0.29	0.02 - 0.06	0.64 - 0.71
	(0.071)	(0.150)	(0.093)	(0.009)	(0.019)			
Apparel	0.68	3.11	2.42	0.11	0.39	0.50	0.07 - 0.15	0.36 - 0.42
	(0.141)	(0.152)	(0.087)	(0.020)	(0.017)			
Personal Care Goods	-0.28	4.68	3.47	-0.04	0.44	0.59	-0.07 - 0.00	0.39 - 0.49
	(0.139)	(0.226)	(0.198)	(0.018)	(0.025)			
House Maintenance Goods	0.44	4.27	3.88	0.05	0.45	0.50	0.02 - 0.08	0.41 - 0.50
	(0.126)	(0.229)	(0.186)	(0.014)	(0.023)			
Fuel	3.44	-3.67	16.29	0.21	1.01	-0.23	0.18 - 0.25	0.95 - 1.07
	(0.319)	(0.450)	(0.121)	(0.018)	(0.031)			
Education and Recreational Goods	5.66	14.24	-0.56	0.29	-0.03	0.74	0.28 - 0.31	-0.04 - -0.02
	(0.199)	(0.168)	(0.128)	(0.008)	(0.007)			
Other Goods	2.03	6.01	12.20	0.10	0.60	0.30	0.08 - 0.13	0.58 - 0.62
	(0.280)	(0.218)	(0.163)	(0.013)	(0.011)			
Services								
House Maintenance Services	13.29	26.40	5.21	0.30	0.12	0.59	0.28 - 0.31	0.11 - 0.13
	(0.365)	(0.180)	(0.221)	(0.006)	(0.005)			
Transportation	-0.40	5.32	2.42	-0.05	0.33	0.72	-0.20 - 0.09	0.25 - 0.41
	(0.504)	(0.310)	(0.278)	(0.072)	(0.043)			
Food Away From Home	1.99	8.10	8.10	0.12	0.39	0.49	0.10 - 0.14	0.36 - 0.43
	(0.215)	(0.286)	(0.294)	(0.012)	(0.017)			
Personal and Recreational Services	2.94	3.45	6.84	0.22	0.52	0.26	0.20 - 0.24	0.48 - 0.55
	(0.167)	(0.162)	(0.259)	(0.011)	(0.018)			
Educational Services	2.25	6.58	5.86	0.15	0.40	0.45	0.09 - 0.22	0.34 - 0.46
	(0.557)	(0.377)	(0.362)	(0.0330	(0.030)			
Medical Care Services	2.52	3.70	9.74	0.16	0.61	0.23	0.13 - 0.19	0.58 - 0.64
	(0.279)	(0.119)	(0.296)	(0.016)	(0.016)			

Note: These results are obtained using the complete specification. Frictionless equation: $p^*_{ik,t} = \zeta_{k1}y_t + (1-\zeta_{k1}-\zeta_{k2})p_t + \zeta_{k2}p_{k,t} + \chi_k z_t + \tilde{a}_{k,t} + \tilde{a}_{ik,t}$.
Robust standard deviations are in parenthesis.
The confidence interval is 95% of confidence. Standard deviations of ζ_1 and ζ_2 were obtained by the Delta method.

Table 2.5: Detailed Results of Probit Models, Complete Specification

Sector: Vehicles and Equipments
Likelihood: **Number of obs:** 13884

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.17	0.049	3.44	0.07	0.26
(s-c)/σ	-1.30	0.048	-27.18	-1.39	-1.20
$(\delta_{i,t})$^(0.5)	-0.29	0.046	-6.41	-0.38	-0.20
$\mathcal{Y}_t$	-1.06	1.249	-0.85	-3.51	1.39
p_{kt}	13.51	1.211	11.16	11.13	15.88
p_t	22.27	1.615	13.79	19.11	25.43
z_t	2.08	1.132	1.84	-0.14	4.30
ζ_1	-0.03	0.037	-	-0.10	0.04
ζ_2	0.39	0.037	-	0.32	0.46
σ	0.03	0.001	-	0.03	0.03

Sector: Raw Food
Likelihood: 398822.00 **Number of obs:** 306116

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.45	0.020	22.75	0.41	0.49
(s-c)/σ	-0.55	0.020	-27.55	-0.59	-0.51
$(\delta_{i,t})$^(0.5)	-0.01	0.987	-0.01	-1.94	1.93
$\mathcal{Y}_t$	1.25	0.272	4.60	0.72	1.78
p_{kt}	3.49	0.087	39.95	3.31	3.66
p_t	3.44	0.198	17.37	3.05	3.82
z_t	-0.10	0.237	-0.44	-0.57	0.36
ζ_1	0.15	0.029	-	0.10	0.21
ζ_2	0.43	0.017	-	0.39	0.46
σ	0.12	0.004	-	0.11	0.13

Sector: Processed Food
Likelihood: 1209100.00 **Number of obs:** 916621

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.44	0.014	32.43	0.42	0.47
(s-c)/σ	-0.72	0.014	-53.46	-0.75	-0.70
$(\delta_{i,t})$^(0.5)	0.02	1.055	0.02	-2.04	2.09
$\mathcal{Y}_t$	-0.24	0.145	-1.63	-0.52	0.05
p_{kt}	7.22	0.154	46.73	6.91	7.52
p_t	-0.35	0.213	-1.64	-0.77	0.07
z_t	-0.08	0.126	-0.62	-0.32	0.17
ζ_1	-0.04	0.022	-	-0.08	0.01
ζ_2	1.09	0.038	-	1.01	1.16
σ	0.15	0.004	-	0.14	0.16

Sector: Apparel
Likelihood: 367693.00 **Number of obs:** 279808

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.55	0.056	9.83	0.44	0.65
(s-c)/σ	-0.64	0.056	-11.45	-0.75	-0.53
$(\delta_{i,t})$^(0.5)	-2.05	16.870	-0.12	-35.11	31.02
$\mathcal{Y}_t$	0.42	0.269	1.57	-0.10	0.95
p_{kt}	2.18	0.159	13.72	1.87	2.50
p_t	4.26	0.203	20.98	3.86	4.65
z_t	1.03	0.250	4.12	0.54	1.52
ζ_1	0.06	0.037	-	-0.01	0.13
ζ_2	0.32	0.025	-	0.27	0.37
σ	0.15	0.007	-	0.13	0.16

Sector: Personal Care Goods
Likelihood: 300415.00 **Number of obs:** 225584

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.57	0.029	19.39	0.51	0.63
(s-c)/σ	-0.90	0.029	-31.06	-0.96	-0.84
$(\delta_{i,t})$^(0.5)	-0.13	1.680	-0.08	-3.42	3.16
$\mathcal{Y}_t$	-0.69	0.263	-2.61	-1.20	-0.17
p_{kt}	3.77	0.308	12.26	3.17	4.37
p_t	5.88	0.333	17.69	5.23	6.54
z_t	-0.03	0.220	-0.12	-0.46	0.41
ζ_1	-0.08	0.031	-	-0.14	-0.02
ζ_2	0.42	0.034	-	0.35	0.49
σ	0.11	0.004	-	0.10	0.12

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Sector: House Maintenance Goods

Likelihood: 401513.00 **Number of obs:** 305976

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	0.51	0.024	20.90	0.46	0.56
$(s-c)/\sigma$	-0.90	0.024	-37.29	-0.95	-0.85
$(\delta_{i,t})^{\wedge}(0.5)$	-0.40	3.276	-0.12	-6.82	6.02
y_t	0.44	0.231	1.90	-0.01	0.89
p_{kt}	5.68	0.302	18.82	5.09	6.27
p_t	4.21	0.338	12.45	3.54	4.87
z_t	0.19	0.203	0.93	-0.21	0.59
ζ_1	0.04	0.022	-	0.00	0.08
ζ_2	0.55	0.030	-	0.49	0.61
σ	0.10	0.002	-	0.09	0.10

Sector: Fuel

Likelihood: **Number of obs:** 54310

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	0.57	0.015	38.55	0.54	0.60
$(s-c)/\sigma$	-0.93	0.015	-64.11	-0.96	-0.90
$(\delta_{i,t})^{\wedge}(0.5)$	-0.06	1.360	-0.05	-2.73	2.60
y_t	2.69	0.577	4.66	1.56	3.82
p_{kt}	14.85	0.185	80.36	14.49	15.21
p_t	-5.04	0.558	-9.05	-6.14	-3.95
z_t	0.19	0.481	0.39	-0.75	1.13
ζ_1	0.22	0.040	-	0.14	0.29
ζ_2	1.19	0.064	-	1.06	1.31
σ	0.08	0.004	-	0.07	0.09

Sector: Education and Recreational Goods

Likelihood: **Number of obs:** 120820

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	1.44	0.022	64.70	1.39	1.48
$(s-c)/\sigma$	-2.16	0.022	-99.19	-2.20	-2.12
$(\delta_{i,t})^{\wedge}(0.5)$	-5.31	6.840	-0.78	-18.71	8.10
y_t	3.47	0.314	11.05	2.86	4.09
p_{kt}	-1.02	0.181	-5.64	-1.37	-0.67
p_t	17.48	0.225	77.58	17.03	17.92
z_t	3.57	0.242	14.73	3.10	4.05
ζ_1	0.17	0.013	-	0.15	0.20
ζ_2	-0.05	0.009	-	-0.07	-0.03
σ	0.05	0.001	-	0.05	0.05

Sector: Other Goods

Likelihood: **Number of obs:** 51332

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	1.34	0.021	64.32	1.30	1.38
$(s-c)/\sigma$	-2.14	0.020	-107.65	-2.18	-2.10
$(\delta_{i,t})^{\wedge}(0.5)$	-20.67	0.913	-22.64	-22.46	-18.88
y_t	1.74	0.501	3.48	0.76	2.72
p_{kt}	18.70	0.250	74.85	18.21	19.19
p_t	0.93	0.311	2.99	0.32	1.54
z_t	12.00	0.355	33.78	11.30	12.69
ζ_1	0.08	0.022	-	0.04	0.12
ζ_2	0.87	0.023	-	0.83	0.92
σ	0.05	0.001	-	0.04	0.05

Sector: House Maintenance Services

Likelihood: **Number of obs:** 44926

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	2.13	0.026	82.90	2.08	2.18
$(s-c)/\sigma$	-4.33	0.024	-178.62	-4.38	-4.28
$(\delta_{i,t})^{\wedge}(0.5)$	-0.36	0.034	-10.58	-0.43	-0.29
y_t	9.95	0.582	17.09	8.81	11.09
p_{kt}	5.15	0.331	15.56	4.50	5.80
p_t	40.24	0.228	176.23	39.79	40.69
z_t	7.72	0.480	16.09	6.78	8.66
ζ_1	0.18	0.009	-	0.16	0.20
ζ_2	0.09	0.006	-	0.08	0.10
σ	0.02	0.000	-	0.02	0.02

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Sector: Transportation

Likelihood: **Number of obs:** 15225

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	1.89	0.042	44.81	1.81	1.97
(s-c)/σ	-2.16	0.043	-50.45	-2.24	-2.07
$(\delta_{i,t})$^(0.5)	-1.55	2.604	-0.60	-6.66	3.55
$\mathcal{Y}_t$	0.67	0.830	0.81	-0.95	2.30
p_{kt}	1.63	0.443	3.68	0.76	2.50
p_t	6.58	0.416	15.81	5.77	7.40
z_t	0.28	0.612	0.45	-0.92	1.48
ζ_1	0.08	0.087	-	-0.10	0.25
ζ_2	0.18	0.052	-	0.08	0.29
σ	0.11	0.010	-	0.09	0.13

Sector: Food Away From Home

Likelihood: **Number of obs:** 93166

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	1.08	0.024	44.70	1.03	1.12
(s-c)/σ	-1.47	0.024	-61.41	-1.52	-1.43
$(\delta_{i,t})$^(0.5)	0.01	1.948	0.00	-3.81	3.83
$\mathcal{Y}_t$	2.55	0.377	6.76	1.81	3.29
p_{kt}	13.73	0.384	35.80	12.98	14.49
p_t	4.81	0.354	13.59	4.11	5.50
z_t	-0.07	0.303	-0.23	-0.66	0.52
ζ_1	0.12	0.016	-	0.09	0.15
ζ_2	0.65	0.020	-	0.61	0.69
σ	0.05	0.008	-	0.03	0.06

Sector: Personal and Recreational Services

Likelihood: **Number of obs:** 150124

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	1.67	0.023	73.69	1.63	1.72
(s-c)/σ	-1.98	0.023	-86.64	-2.02	-1.93
$(\delta_{i,t})$^(0.5)	0.12	4.217	0.03	-8.14	8.39
$\mathcal{Y}_t$	3.53	0.266	13.27	3.01	4.05
p_{kt}	6.18	0.380	16.27	5.44	6.92
p_t	4.33	0.228	19.00	3.88	4.77
z_t	-0.03	0.197	-0.14	-0.41	0.36
ζ_1	0.25	0.016	-	0.22	0.28
ζ_2	0.44	0.025	-	0.39	0.49
σ	0.07	0.001	-	0.07	0.07

Sector: Educational Services

Likelihood: **Number of obs:** 13878

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	1.58	0.039	41.04	1.51	1.66
(s-c)/σ	-2.11	0.038	-54.86	-2.18	-2.03
$(\delta_{i,t})$^(0.5)	0.72	3.422	0.21	-5.99	7.43
$\mathcal{Y}_t$	4.01	0.921	4.36	2.21	5.82
p_{kt}	3.68	0.578	6.37	2.55	4.82
p_t	9.44	0.505	18.68	8.45	10.43
z_t	0.38	0.677	0.56	-0.95	1.71
ζ_1	0.23	0.043	-	0.15	0.32
ζ_2	0.21	0.036	-	0.14	0.29
σ	0.06	0.003	-	0.05	0.06

Sector: Medical Care Services

Likelihood: **Number of obs:** 54004

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	2.31	0.026	87.88	2.26	2.36
(s-c)/σ	-2.65	0.026	-100.27	-2.70	-2.60
$(\delta_{i,t})$^(0.5)	-0.27	3.130	-0.09	-6.40	5.87
$\mathcal{Y}_t$	3.84	0.448	8.56	2.96	4.72
p_{kt}	15.14	0.441	34.36	14.28	16.00
p_t	2.43	0.157	15.44	2.12	2.74
z_t	-0.04	0.318	-0.12	-0.66	0.59
ζ_1	0.18	0.018	-	0.14	0.22
ζ_2	0.71	0.018	-	0.67	0.74
σ	0.05	0.001	-	0.04	0.05

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Table 2.6: Detailed Results of Probit Models, Simple Specification

Sector: Vehicles and Equipments
Likelihood: **Number of obs:** 13884

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.14	0.048	2.89	0.04	0.23
(s-c)/σ	-1.30	0.047	-28.01	-1.40	-1.21
$(\delta_{i,t})$^(0.5)	-0.27	0.045	-6.10	-0.36	-0.19
$\mathcal{Y}_t$	1.09	0.609	1.78	-0.11	2.28
p_{kt}	12.81	0.627	20.42	11.58	14.04
p_t	16.15	1.015	15.91	14.16	18.14
z_t	1.61	0.615	2.62	0.41	2.82
ζ_1	0.04	0.020	-	0.00	0.07
ζ_2	0.43	0.023	-	0.38	0.47
σ	0.03	0.001	-	0.03	0.04

Sector: Raw Food
Likelihood: **Number of obs:** 306116

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.45	0.020	22.25	0.41	0.49
(s-c)/σ	-0.55	0.020	-27.25	-0.59	-0.51
$(\delta_{i,t})$^(0.5)	-0.04	0.020	-1.80	-0.07	0.00
$\mathcal{Y}_t$	1.62	0.125	12.97	1.38	1.87
p_{kt}	2.41	0.047	51.39	2.32	2.51
p_t	4.25	0.158	26.95	3.94	4.56
z_t	-1.11	0.123	-8.99	-1.35	-0.86
ζ_1	0.20	0.013	-	0.17	0.22
ζ_2	0.29	0.008	-	0.27	0.31
σ	0.12	0.003	-	0.12	0.13

Sector: Processed Food
Likelihood: **Number of obs:** 916621

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.44	0.012	37.72	0.42	0.47
(s-c)/σ	-0.72	0.012	-61.78	-0.74	-0.70
$(\delta_{i,t})$^(0.5)	-0.06	0.012	-4.75	-0.08	-0.03
$\mathcal{Y}_t$	0.27	0.071	3.81	0.13	0.41
p_{kt}	4.80	0.093	51.40	4.62	4.99
p_t	2.03	0.150	13.49	1.73	2.32
z_t	-0.43	0.068	-6.37	-0.57	-0.30
ζ_1	0.04	0.010	-	0.02	0.06
ζ_2	0.68	0.019	-	0.64	0.71
σ	0.14	0.002	-	0.14	0.15

Sector: Apparel
Likelihood: **Number of obs:** 279808

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.54	0.048	11.16	0.44	0.63
(s-c)/σ	-0.65	0.048	-13.43	-0.74	-0.55
$(\delta_{i,t})$^(0.5)	-0.04	0.048	-0.86	-0.14	0.05
$\mathcal{Y}_t$	0.68	0.141	4.83	0.40	0.95
p_{kt}	2.42	0.087	27.69	2.25	2.59
p_t	3.11	0.152	20.42	2.81	3.40
z_t	0.38	0.138	2.74	0.11	0.65
ζ_1	0.11	0.020	-	0.07	0.15
ζ_2	0.39	0.017	-	0.36	0.42
σ	0.16	0.005	-	0.15	0.17

Sector: Personal Care Goods
Likelihood: **Number of obs:** 225584

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.56	0.023	24.51	0.52	0.61
(s-c)/σ	-0.90	0.023	-40.13	-0.95	-0.86
$(\delta_{i,t})$^(0.5)	-0.07	0.023	-3.07	-0.11	-0.02
$\mathcal{Y}_t$	-0.28	0.139	-2.01	-0.55	-0.01
p_{kt}	3.47	0.198	17.55	3.08	3.85
p_t	4.68	0.226	20.66	4.23	5.12
z_t	-0.08	0.126	-0.65	-0.33	0.16
ζ_1	-0.04	0.018	-	-0.07	0.00
ζ_2	0.44	0.025	-	0.39	0.49
σ	0.13	0.003	-	0.12	0.13

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Sector: House Maintenance Goods

Likelihood: **Number of obs:** 305976

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S\text{-}c)/\sigma$	0.51	0.020	25.52	0.47	0.54
$(s\text{-}c)/\sigma$	-0.90	0.020	-46.15	-0.94	-0.87
$(\delta_{i,t})$^(0.5)	-0.08	0.020	-3.98	-0.12	-0.04
y_t	0.44	0.126	3.47	0.19	0.69
p_{kt}	3.88	0.186	20.89	3.52	4.25
p_t	4.27	0.229	18.65	3.82	4.71
z_t	0.52	0.119	4.32	0.28	0.75
ζ_1	0.05	0.014	-	0.02	0.08
ζ_2	0.45	0.023	-	0.41	0.50
σ	0.12	0.002	-	0.11	0.12

Sector: Fuel

Likelihood: **Number of obs:** 54310

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S\text{-}c)/\sigma$	0.56	0.013	44.71	0.54	0.59
$(s\text{-}c)/\sigma$	-0.90	0.012	-72.52	-0.93	-0.88
$(\delta_{i,t})$^(0.5)	-0.07	0.010	-6.56	-0.09	-0.05
y_t	3.44	0.320	10.74	2.81	4.06
p_{kt}	16.29	0.121	134.14	16.05	16.52
p_t	-3.67	0.450	-8.16	-4.55	-2.79
z_t	-0.01	0.281	-0.04	-0.56	0.54
ζ_1	0.21	0.018	-	0.18	0.25
ζ_2	1.01	0.031	-	0.95	1.07
σ	0.06	0.002	-	0.06	0.07

Sector: Education and Recreational Goods

Likelihood: **Number of obs:** 120820

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S\text{-}c)/\sigma$	1.40	0.012	116.50	1.38	1.42
$(s\text{-}c)/\sigma$	-2.17	0.012	-187.43	-2.19	-2.15
$(\delta_{i,t})$^(0.5)	-0.14	0.011	-13.02	-0.16	-0.12
y_t	5.66	0.199	28.45	5.27	6.05
p_{kt}	-0.56	0.128	-4.38	-0.81	-0.31
p_t	14.24	0.168	84.86	13.91	14.57
z_t	1.67	0.153	10.90	1.37	1.97
ζ_1	0.29	0.008	-	0.28	0.31
ζ_2	-0.03	0.007	-	-0.04	-0.02
σ	0.05	0.001	-	0.05	0.05

Sector: Other Goods

Likelihood: **Number of obs:** 51332

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S\text{-}c)/\sigma$	1.33	0.013	104.78	1.30	1.35
$(s\text{-}c)/\sigma$	-2.03	0.012	-169.91	-2.06	-2.01
$(\delta_{i,t})$^(0.5)	-0.13	0.009	-13.72	-0.14	-0.11
y_t	2.03	0.279	7.27	1.48	2.58
p_{kt}	12.20	0.163	74.93	11.88	12.52
p_t	6.01	0.218	27.58	5.58	6.44
z_t	3.56	0.098	36.33	3.37	3.76
ζ_1	0.10	0.013	-	0.08	0.13
ζ_2	0.60	0.011	-	0.58	0.62
σ	0.05	0.001	-	0.05	0.05

Sector: House Maintenance Services

Likelihood: **Number of obs:** 44926

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S\text{-}c)/\sigma$	1.76	0.017	101.58	1.73	1.79
$(s\text{-}c)/\sigma$	-4.61	0.015	-313.31	-4.64	-4.58
$(\delta_{i,t})$^(0.5)	-0.40	0.011	-36.17	-0.42	-0.38
y_t	13.29	0.365	36.39	12.57	14.00
p_{kt}	5.21	0.221	23.54	4.77	5.64
p_t	26.40	0.180	146.40	26.04	26.75
z_t	4.98	0.294	16.96	4.41	5.56
ζ_1	0.30	0.006	-	0.28	0.31
ζ_2	0.12	0.005	-	0.11	0.13
σ	0.02	0.000	-	0.02	0.02

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Sector: Transportation
Likelihood: **Number of obs:** 15225

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	1.85	0.054	34.22	1.74	1.95
$(s-c)/\sigma$	-2.12	0.055	-38.70	-2.23	-2.01
$(\delta_{i,t})$^(0.5)	-0.06	0.053	-1.13	-0.16	0.04
y_t	-0.40	0.504	-0.80	-1.39	0.59
p_{kt}	2.42	0.279	8.68	1.88	2.97
p_t	5.32	0.310	17.16	4.71	5.93
z_t	0.12	0.394	0.30	-0.66	0.89
ζ_1	-0.05	0.072	-	-0.20	0.09
ζ_2	0.33	0.043	-	0.25	0.41
σ	0.14	0.009	-	0.12	0.15

Sector: Food Away From Home
Likelihood: **Number of obs:** 93166

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	1.06	0.013	78.89	1.03	1.08
$(s-c)/\sigma$	-1.47	0.013	-110.39	-1.50	-1.45
$(\delta_{i,t})$^(0.5)	-0.10	0.012	-8.08	-0.12	-0.08
y_t	1.99	0.215	9.24	1.57	2.41
p_{kt}	6.58	0.294	22.41	6.00	7.15
p_t	8.10	0.286	28.27	7.54	8.66
z_t	0.01	0.183	0.07	-0.34	0.37
ζ_1	0.12	0.012	-	0.10	0.14
ζ_2	0.39	0.017	-	0.36	0.43
σ	0.06	0.001	-	0.06	0.06

Sector: Personal and Recreational Services
Likelihood: **Number of obs:** 150124

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	1.67	0.014	117.68	1.64	1.70
$(s-c)/\sigma$	-1.97	0.014	-138.09	-2.00	-1.94
$(\delta_{i,t})$^(0.5)	-0.06	0.014	-4.61	-0.09	-0.04
y_t	2.94	0.167	17.64	2.62	3.27
p_{kt}	6.84	0.260	26.35	6.33	7.35
p_t	3.45	0.162	21.30	3.14	3.77
z_t	-0.02	0.131	-0.14	-0.27	0.24
ζ_1	0.22	0.011	-	0.20	0.24
ζ_2	0.52	0.018	-	0.48	0.55
σ	0.08	0.001	-	0.07	0.08

Sector: Educational Services
Likelihood: **Number of obs:** 13878

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	1.48	0.044	33.86	1.39	1.56
$(s-c)/\sigma$	-2.09	0.043	-48.12	-2.17	-2.00
$(\delta_{i,t})$^(0.5)	-0.10	0.041	-2.40	-0.18	-0.02
y_t	2.25	0.557	4.04	1.16	3.34
p_{kt}	5.86	0.362	16.19	5.15	6.57
p_t	6.58	0.377	17.47	5.84	7.32
z_t	0.09	0.437	0.21	-0.76	0.95
ζ_1	0.15	0.033	-	0.09	0.22
ζ_2	0.40	0.030	-	0.34	0.46
σ	0.07	0.003	-	0.06	0.07

Sector: Medical Care Services
Likelihood: **Number of obs:** 54004

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
$(S-c)/\sigma$	2.27	0.018	122.96	2.23	2.31
$(s-c)/\sigma$	-2.64	0.019	-142.66	-2.68	-2.60
$(\delta_{i,t})$^(0.5)	-0.06	0.017	-3.75	-0.10	-0.03
y_t	2.52	0.278	9.06	1.98	3.07
p_{kt}	9.74	0.296	32.93	9.16	10.32
p_t	3.70	0.119	31.03	3.46	3.93
z_t	-0.18	0.221	-0.80	-0.61	0.26
ζ_1	0.16	0.016	-	0.13	0.19
ζ_2	0.61	0.016	-	0.58	0.64
σ	0.06	0.001	-	0.06	0.07

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