3 Credit Default and Business Cycles: An Empirical Investigation of Brazilian Retail Loans[1]

3.1 Introduction

Credit default plays an important role in credit decisions of financial institutions and is also crucial for financial regulatory issues. The importance of credit default has led to a recent surge in the interest for issues related to credit risk, which has resulted in several interesting fields of research. In particular, the 2004 reform on Banking Supervision approved by the Basel Committee, known as Basel II Accord, has brought renewed interest in the relationship between credit risk and macroeconomic conditions[2]. The Basel II Accord introduced a menu of approaches for determining capital requirements, including the internal ratings-based (IRB) approach that allows banks to compute the capital charges based on their estimates of probability of default and loss given default. Under the internal ratings-based approach of Basel II Accord, capital requirements are an increasing function in the probability of default and loss given default parameters.

As a result of this risk-sensitiveness of regulatory capital, a recent widespread concern is that the Basel II Accord might amplify fluctuations in the business cycles. For example, in periods of recession, when the probabilities of default and correlations among risk ratings might increase, capital requirements of banking institutions should also be increased, which eventually may lead to an increase in capital costs and reduction in credit supply. These effects may ultimately further amplify the economic downturn. The opposite effect might occur in periods of economic expansion (see, for example, Kashyap & Stein (2000), Saurina & Trucharte (2007), Repullo & Suarez (2008), Repullo et al. (2009)).

Following this reasoning, one proposal to mitigate the procyclical effects of the Basel II Accord has been discussed by the Committee on Banking Supervision: the construction of capital buffers above the minimum regulatory capital of the banking sector during periods of large economic growth[3]. These buffers could be

[1] This is joint work with Jaqueline Terra Moura Marins, Myrian Beatriz Eiras das Neves and Antonio Carlos Magalhães da Silva, from the Research Department of the Central Bank of Brazil.

[2] For a first overview on this relationship see Caouette et al. (1998), BIS (2001), and Allen & Saunders (2002).

[3] This issue is referred to in the literature as procyclicality of capital requirements and countercyclical regime of capital buffers.

used in periods of economic distress to achieve the key macro-prudential goal of protecting the banking system during difficulties[4].

The present paper aims to contribute to this literature providing more evidence about the relationship between credit defaults and business cycles using a very rich dataset of microdata of loan transactions. In particular, we are interested in the first part of the reasoning previously explained, i.e., whether recessions really increase credit defaults and what are their impacts on the losses in portfolios of lender institutions. However, we do not study in this paper the second part of the argument, i.e., if this increase in credit defaults, in the losses of portfolios and the consequent recomposition of capital requirements really cause a reduction in credit supply. This would require separating the effects of supply and demand for credit, and the difficulty of this task is enough to deserve a separate treatment in another paper.

We add to the current literature in three different ways. First, we explore both time series and cross-sectional variations in the data. The advantage of using time series data is that more information about the dynamics over the business cycle can be extracted. The microdata, on the other hand, allow detailed analysis on the individual level. In particular, they allow estimating the effect of the business cycles on defaults controlling for the borrower's quality through a probit model. For example, by not controlling for the borrower's rating and/or the size of the local market in which the credit was granted, we could obtain an increasing probability of default only because the lender institution may begin to lend to worse borrowers in saturated markets, when the economy experiences a strong growth period.

Second, by carrying out our cross-sectional analysis, differently from other papers in the literature, we take into account the unobserved individual effects that can bias the parameter estimation. Obviously, control for individual effects in probit models without making additional assumptions is very hard. In this paper we assume that, conditional on the observable variables, the unobservable individual component is normally distributed, i.e., we use random effects probit models. Third, we use data from the retail sector in our analysis. Despite its importance, the difficulty of obtaining data from this segment of market may possibly explain the complete inexistence of studies about procyclicality for the retail sector. Our paper fills this gap in the literature by using information on retail transactions in Brazil in two credit modalities—Consumer Credit and Vehicle Financing[5]—obtained from the Credit Information System of the Central Bank of Brazil (SCR).

[4]For a detailed discussion about procyclicality, capital buffers and macro-prudential policies, see the documents "Basel III: A global regulatory framework for more resilient banks and banking systems", (BIS (2010a)); "Basel III: International framework for liquidity risk measurement, standards and monitoring", (BIS (2010b)); and "Guidance for national authorities operating the countercyclical capital buffer", (BIS (2010c)).

[5]Automobile Vehicles Financing.

Our results also provide evidence of a negative relationship between business cycles and credit defaults, but less strong than suggested in previous studies. After a positive shock in the unemployment rate, identified in a VAR model, credit defaults increase, achieving a peak after 4 or 5 months and then starting to decrease. However, the increase is modest. Similar results of negative relationship are also found in the cross-sectional analysis. After controlling for the effect of different variables, the probability of default slightly increases when the economy goes into a recession. Moreover, default correlations estimated among retail transactions are low and very dispersed. Value at Risk experiments using a simulated portfolio based on the credit transactions of two large Brazilian financial institutions showed that losses in recessions are around 14% higher in the Consumer Credit modality and only 4% higher in Vehicle Financing modality, when compared to the losses during booming periods. These losses are much lower than those found in the literature that uses corporate data.

These lower losses, smaller correlations and less strong relationship between credit default and business cycles than those found in previous papers may possibly be explained by the fact that, in the retail sector, loans are given to a large number of individuals, which may help to diversify the influence of default events. We also find that, in general, women default less than men and the older the borrower the lower is the probability of delinquency.

The rest of the paper proceeds as follows. Section 3.2 reviews the literature on the relationship between credit default, default correlations and business cycles. Section 3.3 explores the time series variation and section 3.4 presents our dataset of microdata and explores the cross-sectional evidence on the relationship between credit delinquency and business cycles. In section 3.5 we estimate transition probabilities and default correlations in retail transactions. In section 3.6 we go further on the relationship between credit risk and business cycles through Value at Risk (VaR) experiments. Section 3.7 concludes.

3.2 Literature Review

Macroeconomic conditions can be a reason for systematic changes that are very important for credit risk. Despite this obvious importance, the literature focusing on the relationship between credit default and macroeconomic environment is rather sparse. The first group of papers explores the link between rating changes and macroeconomic conditions. Older studies on this issue that use cross-sectional or panel data methods include Bangia et al. (2002), Carpenter et al. (2001), and Kavvathas (2001). The first two papers use GDP growth to classify the different phases of the business cycle and compute separate default and rating transition probabilities for each of these regimes. Kavvathas (2001) applies a duration model for rating transitions and incorporates macroeconomic variables to capture systematic effects on transition probabilities. Papers that use time series techniques include Koopman & Lucas (2005) and Koopman et al. (2005). They use a multivariate unobserved components framework to study cyclical co-movements between GDP and business failures. All these papers find evidence supporting the relationship between credit risk and macroeconomic variables.

Another branch of this sparse literature relates default correlations to macroeconomic conditions. Default correlation is a measure of interdependence among risks, and its own concept already embodies the idea that common events (such as business cycles) might lead default events to happen in bunches or clusters. Nagpal & Bahar (2001), for example, calculate default correlations and conclude that data support the idea that credit events are correlated and caused by common economic conditions. Servigny & Renault (2002) calculate default correlation empirically and find higher coefficients for recessionary periods using data of U.S. companies. Cowan and Cowan (2004) use a large portfolio of residential subprime loans to show that default correlation is substantial in the data and that regulators and lenders would be well served to develop more sophisticated credit measurement techniques. They also suggest that the impact of changes in the business cycle on the portfolio losses should be considered in the measurement of credit risk. Trück & Rachev (2005), using Value at Risk experiment based on a loan portfolio of a large European bank, find that the losses are much higher in recessions than in booming periods.

More recently, after widespread concerns about the possible procyclical effects of the Basel II Accord on the economy, there has been a considerable flurry of activity around this theme. Koopman et al. (2005) find a cyclical behavior in default rates using a time series approach based on unobserved components and highlight the main effects of this behavior in a credit risk experiment, addressing the issue of procyclicality in ratings and capital buffer formation. Repullo & Suarez (2008) show that banks have an incentive to maintain capital buffers, but that these buffers maintained in expansions are typically insufficient to prevent a contraction in the supply of credit in recessions. Repullo et al. (2009) compare alternative methods to mitigate the possible procyclical effects of the Basel II Accord. As a consequence of concerns about this issue, the Committee on Banking Supervision has begun to discuss the idea of capital buffers above the minimum regulatory capital of the banking sector during periods of large economic growth. This discussion is presented in the three documents cited in footnote 3.

Aiming to study the procyclicality issue from another point of view, we also analyze the impact of business cycles on potential losses in portfolios of lender institutions. To estimate these losses we use Value at Risk experiments. First, however, we need estimates of the transition probabilities and default correlation matrices in our data. The analytical modeling commonly employed in the literature to estimate default correlations within a portfolio is based on the model developed by Merton (1974) for the joint distribution of borrowing firms' asset values. In this type of modeling, by assumption, transitions between risk ratings are defined by a stochastic process that describes the asset values as a function of systematic and idiosyncratic risk factors. When these values fall below certain critical levels, transitions occur. The correlations between systematic risk factors define the correlations between the asset values and, consequently, the transitions between different risk ratings—known in the literature as asset correlation. The Basel II Accord uses this risk factor structure. But such modeling requires assumptions about the relationship between the equity prices and the default events. Additionally, equity prices for borrowers must exist, which makes it impossible to use this method in our context, once there is no equity price for individual borrowers in the retail sector. Alternatively, we will infer transition probabilities and default correlations from historical data using a methodology developed by Servigny & Renault (2002).

3.3 Evidence From Time Series

In this section we explore the time series evidence about the relationship between credit default and business cycles. We begin by plotting a monthly series of credit defaults together with the seasonally adjusted aggregate unemployment rate in Brazil from 2001:10 to 2010:10. We decided to use here unemployment rate as the variable measuring business cycles instead of the traditional GDP or output gap because we only have information about these two variables quarterly, which would significantly reduce our number of observations.

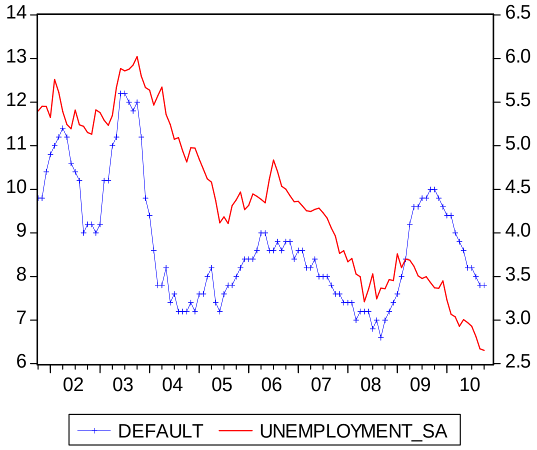
The default measure used here is quite general, including lending, financing, advances and leasing transactions granted by Brazilian financial institutions, and is calculated by the Central Bank of Brazil using the same database of microdata that we will use in the next sections. The unemployment rate is measured by the Brazilian Institute of Geography and Statistics (IBGE) considering six large metropolitan regions of Brazil[6].

Figure 3.1 shows an impressive co-movement of these two series along the period considered. The graph shows that they both initially decrease and then start to increase until roughly the beginning of 2004. After that, they consistently decrease, having a rapid increase until the middle of 2006, and again begin to decrease throughout 2007 and 2008. Another common cycle is observed after the end of 2008. This visual impression of co-movement is also confirmed by a correlation

[6]The metropolitan regions are Recife, Salvador, Belo Horizonte, Rio de Janeiro, São Paulo and Porto Alegre.

coefficient between the two series of 0.53. If we consider the 2003-2008 period, the correlation between the defaults and unemployment series is of 0.73.

Figure 3.1: Default and Unemployment Rate, 2001:10 - 2010:10

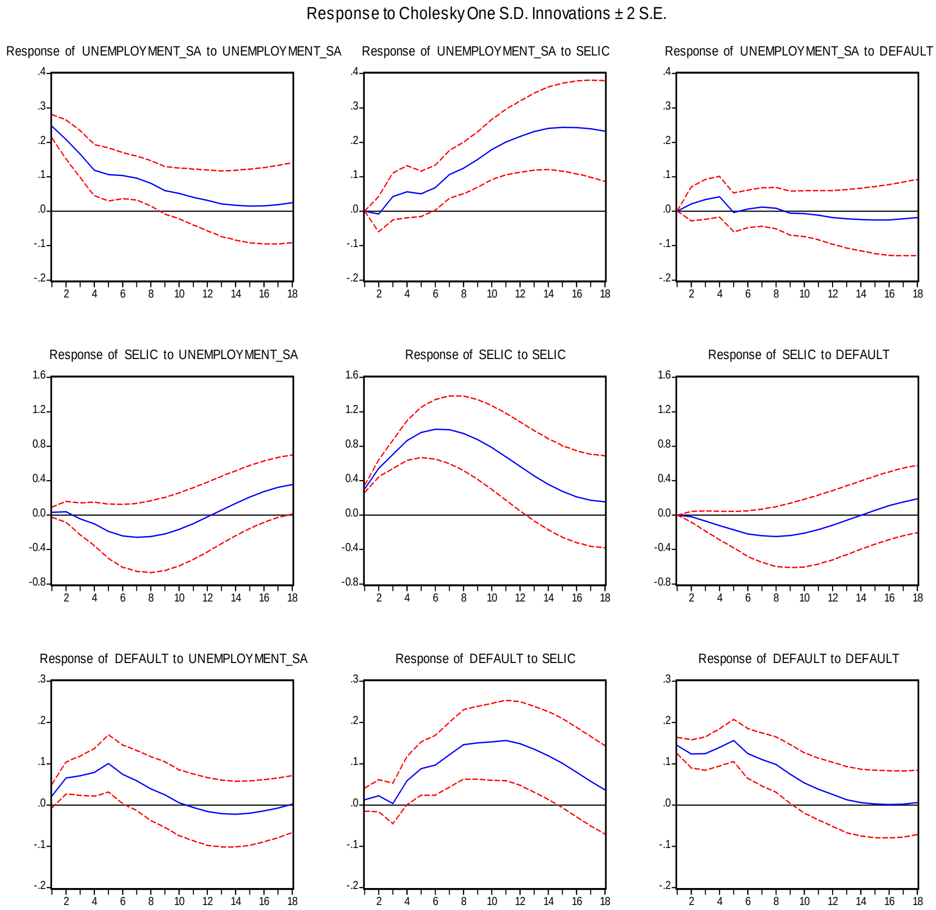


To address the issue more formally we estimate a monthly Vector Autoregressive (VAR) model with three variables: default, unemployment and interest rate. We do not carry out a cointegration analysis because two of our variables (unemployment and default) are “rates”, which, by definition, are limited between zero and 100% and conceptually cannot be non-stationary. Even though tests for stationarity may indicate that these variables are I(1), this result would be a sample phenomenon. The two series previously described measure default and unemployment, and interest rate is given by the monthly Selic rate annualized. The lag structure of the VAR model was chosen using AIC information criterium and has 5 lags. In addition, LM tests were carried out in the residual to guarantee that they were not autocorrelated.

We estimate impulse response functions of shocks with this 3-dimensional VAR(5) model using Cholesky decomposition with the following ordering: unemployment, Selic and default. This ordering was chosen based on the facts that (i) in an inflation-targeting regime the interest rate decision is affected by the economic activity level and (ii) by economic reasons default events might be affected by both interest rate and the level of activity.

Figure 3.2 below plots these impulse response functions following an one-standard-deviation shock for a horizon of 18 months, and confidence intervals (± 2 standard errors) for these responses. First of all, as the first graph at the bottom row shows, it really seems to have a relationship between business cycles and credit defaults, here captured by a positive relationship between unemployment and default rate. After a positive shock in the unemployment rate, the defaults start to smoothly increase, achieving a peak after 4 or 5 months, and then starting to decrease. Therefore, despite the fact that the defaults response is not very strong, the time series evidence captured by a VAR model seems to support the idea of a movement in credit defaults along the business cycles.

Figure 3.2: Impulse Response Functions in a VAR(5)



The other impulse responses are also interesting. Default rate also increases after a positive shock in the interest rate, but this movement takes time. Initially the delinquency in credit transactions does not react, but after approximately 3 months it starts to slowly increase. As expected, Selic rate is reduced after a shock in unemployment or in default events, but the reaction is very small and slow. Unemployment does not react to default. Finally, unemployment strongly respond to interest rate, but the reaction is slow (the peak is achieved after one year or more), evidencing that this channel of monetary policy has a long delay.

3.4 Evidence From Microdata

We now turn to individual data. After exploring the cross-sectional variation to examine the relationship between credit delinquencies and business cycles we will estimate default correlations and transition probabilities using the historical method and the traditional segmentation of transactions based on risk ratings. These correlations and probabilities are used to calculate the potential losses in a portfolio composed of retail loans through Value at Risk experiments. The next subsection presents the data set and the other two subsections present the probit model and carry out the analysis.

3.4.1 Data Set of Microdata

The microdata for this paper come from a retail credit database consisting of transactions registered in the Credit Information System of the Central Bank of Brazil – SCR from January 2003 through July 2008. The Credit Information System of the Central Bank of Brazil (hereafter SCR) is the database that registers information of individual commercial loans whose total obligation exceed 5 thousand Brazilian Reais (R$), reported by Brazilian financial institutions to the Central Bank of Brazil. The data, reported monthly by the institutions, contain detailed information about the loans, including some characteristics of the borrowers and the transactions, and their ratings. The level of disaggregation allows analyzing credit risk considering the heterogeneity existing among debtors.

Because of the lack of studies in the literature focusing on retail transactions, we restrict our analysis in this paper to the retail sector. Retail transactions were defined as those transactions in which the total obligations of each borrower in the financial system fall between 5 thousand and 50 thousand Brazilian Reais at the date of contracting.

Considering the richness of the data set, in our analysis the individuals are credits, i.e., transactions, instead of people or firms. Each transaction has a corresponding credit rating by month, and a respective group of characteristics, including borrower's and transaction's characteristics. Because the number of transactions registered at the SCR is really very large (amounting around 64 million transactions in July 2008), we decided to select the two largest credit modalities in number of transactions during the sample period: Consumer Credit and Vehicle Financing modalities. In addition, we have chosen two financial institutions with relevant loan volumes into these two modalities to compose our sample. This screening process was necessary to make the number of observations treatable.

To ensure the anonymity of the two selected institutions, we will avoid to present disaggregated statistics when this can give any information about them and we will call the institutions simply as Institution A and Institution B. Together, these two institutions represented approximately 31% of the Consumer credits and 38% of the Vehicle Financing credits in the whole system during the period of study. Additionally, their transactions in Consumer Credit and in Vehicle Financing modality represented, respectively, 16% and 23% of the total financial volume in the Brazilian financial system in January 2003. The percentages are similar if we consider the number of transactions instead of financial volume.

The sample of retail loans considered in this paper is composed almost entirely by loans granted to individuals. Very few are loans for firms. In Vehicle Financing, this percentage is approximately 91% and in Consumer Credit, by the nature of the modality, it is virtually 100%.

As noted by Jarrow & Turnbull (2000), the time horizon commonly used in the literature to measure credit risk issues is one year. For example, Servigny & Renault (2002), which developed the methodology we use to calculate the empirical correlations in this paper, also assume one year as the time horizon. Despite the richness of our data, we had to consider time intervals of less than one year, given the small number of years covered by our database (from 2003 to 2008). Therefore, we calculate transition and correlation matrices based on semiannual credit risk rating migrations, amounting to 11 six-month observation periods.

We classified the loan transactions according to the risk ratings reported by the lender institutions to the SCR. These risk ratings are based on the National Monetary Council (CMN) Resolution 2682/99, which defines nine possible ratings (AA, A, B, C, D, E, F, G and H), varying according to the period of delinquency. Specifically, we use the following definition of default in this paper: a transaction is in default if it receives from the lender institution a grade equal to D or worse. Therefore, credit transactions with risk ratings ranging from D to H were considered as being in default. We should mention that the legislation establishes that a transaction in delinquency for 60 days or more must be rated by the lender at least as C (or worse). But of course the lender institution can classify as C even a non-delinquent transaction, if it wants, based on its classification method. In addition, each institution is responsible for classifying its transactions based on their own criteria, and each institution actually has different criteria, as we will see ahead. Of course, these classifications have a direct impact on the amount of provision that institutions have to maintain.

Despite all those facts, we decided to use the lender classification instead of the actual time of delinquency as the criterium defining defaults, once are those classifications which really affect the provisions of the financial institutions. Transactions that were written-off because of a long period of delinquency (rating HH) were also considered in our estimations, but we removed from the sample loans that stayed in this state for more than one semester[7].

Considering the two institutions together, our data sample has a total number of 730 thousand transactions in the Consumer Credit modality and 2.55 million

[7]Proceeding this way we avoid that an HH credit transaction is considered a new transaction every semester.

transactions in the Vehicle Financing modality. To calculate the percentage of default in our data set we use the following procedure. First, in each semester we calculate the ratio between the number of transactions that migrate to default and the total number of transactions in that semester. Then, we obtain the average of these ratios weighted by the number of transactions in each semester. Using this criterium, the average percentage of default is approximately 12% and 6% in Consumer Credit and Vehicle Financing modalities, respectively. The percentage of defaults in the Consumer Credit modality is much higher than those in the literature[8]. These results, however, come from the criteria that institutions we have chosen to compose our data use to classify their transactions according to the risk ratings. In particular, one of the two institutions seems to use tough criteria. But, by the arguments expressed in the previous paragraph, we decided to maintain the criteria previously outlined to define default events. After all, instead of looking only to the level of default, we should also verify if these criteria do what they were supposed to do: capture the intrinsic risks of each transaction. And as will be shown ahead, they really seem to capture these risks: in our data, in both modality, when the risk classification gets worse, the percentage of defaults increases.

Our data set has information about the following characteristics of the borrower: gender, age, geographic region of residence and type of occupation. Figures 3.4 and 3.5 in Appendix summarize these information. If we had to provide a general profile in our data sample, we would say that in the Consumer Credit modality the representative borrower is male (around 61%), aged between 35 and 60 years old (around 62%), living in the Southeastern region (approximately 70%) and working in the private sector (private sector employees, self-employed and company owner sum up around 60%). In the Vehicle Financing modality we have almost the same profile: borrowers are mostly male (67%), in the middle-aged groups, employed in the private sector and living particularly in the Southeastern (60%) and Southern (18%) regions of Brazil. In this last modality, however, there is a large proportion of borrowers whose occupations were not informed (around 43%).

Figure 3.3 in Appendix A shows the default rates calculated in our data sample of microdata for each modality along the time. Observe that both series (Consumer Credit and Vehicle Financing defaults) have roughly a similar temporal behavior than that of the more general default rates presented in the previous section. The series decrease from 2003 until approximately 2004 (there is a difference in the turning point of the series here), then increase throughout 2005 and 2006 and again start to decrease. There is also a difference in the level of the two series—the percentage of defaults in Consumer Credit modality is larger. This difference

[8]Cowan & Cowan (2004) estimate the percentage of default in subprime transactions in the U.S. around 6% in some semesters, when they use 90 or more days delinquent as criterium to default. Observe that our criteria are even more stringent (60 days).

may possibly be explained by the existence of collateral in Vehicle Financing transactions.

3.4.2 Probit Model with Unobserved Individual Component

To examine the relationship between credit defaults and business cycles in the microdata we use probit models. First, as already pointed out, some previous works argue that historical rates of default support the idea that credit episodes are correlated and this correlation comes from common components, which might include macroeconomic and/or sectoral events. In addition, the literature has provided evidence that default events might depend on the personal characteristics of the borrower and the characteristics of the transaction. Therefore, the econometric formulation of the probit models can be thought as coming from the following economic model.

Assume that the borrower, who receives a given risk rating from the lender institution, when apply for a loan, mainly intends to use the money to implement a project. The return of the project should depend on (i) the borrower's personal characteristics and the transaction's characteristics, (ii) the macroeconomic environment (in particular, the phase of the business cycle) and (iii) other control variables, which may include possibly the risk rating[9].

The dependence of the project's return on the macroeconomic environment/business cycle can be rationalized by the existence of common factors in credit risk and/or by the interdependence of existing projects in the economy. For instance, if the economy goes into a recession, there may be a reduction in the returns of other projects and a increase in defaults of these loans (inside and/or outside the same sector) and, through a cross effect, reduce the return of the individual borrower's project considered. The same would occur if we think in terms of potential wages: the economic recession reduces the potential wage of the borrower.

Thus, we can write:

$$y^*_{i,j,t} = \boldsymbol{x}'_i\boldsymbol{\beta} + \boldsymbol{m}'_{i,t}\boldsymbol{\gamma} + \boldsymbol{z}'_{i,t}\boldsymbol{\theta} + c_i + d_j + u_{i,j,t}, \tag{3-1}$$

where i represents the borrower, j is the bank and t is the time. Therefore, $y^*_{i,j,t}$ is the unobserved return of the borrower i's project (or his/her potential wage), who took credit at the bank j, at time t. In addition, $\boldsymbol{x}_i$ is a vector with observable personal characteristics of the borrower i; $\boldsymbol{m}_{i,t}$ are macroeconomic and/or sectoral variables at time t (there is an index i in $\boldsymbol{m}$ because sectoral variables change across individuals from different sectors); $\boldsymbol{z}_{i,t}$ are control variable that can change

[9]Instead of thinking in terms of the return of a project, once we are dealing with retail transactions, we could also think in terms of the potential wage received by the individual who is asking for credit. In this case, the potential wage would depend on personal characteristics, the macroeconomic environment and other variables, including the credit transaction's characteristics. The probability of default, in this case, would depend on the wage.

over individuals i and over time t; $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\theta}$ are vectors with parameters, and $u_{i,j,t}$ is a shock affecting the project's return (or potential wage). c_i is an unobserved individual effect of the borrower and d_j is an individual effect of financial institution.

In order to repay the loan, the borrower must obtain a minimum return equal to α in its project (or a minimum wage). Otherwise, the borrower will default. But $y^*_{i,j,t}$ is an unobservable variable—only the borrower observes it. What we observe is the following variable:

$$y_{i,j,t} = \begin{cases} 1, & if \quad y^*_{i,j,t} \leq \alpha \\ 0, & if \quad otherwise \end{cases},$$

that is,

$$y_{i,j,t} = \begin{cases} 1, & if \quad default \\ 0, & if \quad otherwise \end{cases}. \tag{3-2}$$

Assume that $u_{i,j,t} \sim \mathbb{N}(0,1)$. Write $\boldsymbol{w}_{i,j,t} = (\boldsymbol{x}'_i, \boldsymbol{m}'_{i,t}, \boldsymbol{z}'_{i,t}, d_j)'$ and $\boldsymbol{w}_{i,j} = (\boldsymbol{w}_{i,j,1}, ..., \boldsymbol{w}_{i,j,T})'$. In the context of models for binary outcomes, the presence of unobserved individual effects introduces many complications and makes the estimation very complicated and computationally demanding. First, because of the presence of c_i, the $y_{i,j,t}$ are dependent across t conditional only on $\boldsymbol{w}_{i,j,t}$. In that environment is standard to assume two assumptions: (i) $\boldsymbol{w}_{i,j,t}$ is strictly exogenous[10] conditional on c_i and (ii) $y_{i,j,1}, ..., y_{i,j,T}$ are independent conditional on $(\boldsymbol{w}_{i,j}, c_i)$.

Under these assumptions we can derive a probit model for default probability:

$$\begin{aligned} Pr[y_{i,j,t} = 1 | \boldsymbol{w}_{i,j,t}, c_i] &= Pr[y^*_{i,j,t} \leq \alpha | \boldsymbol{w}_{i,j,t}, c_i] \\ &= Pr\left[\boldsymbol{x}'_i\boldsymbol{\beta} + \boldsymbol{m}'_{i,t}\boldsymbol{\gamma} + \boldsymbol{z}'_{i,t}\boldsymbol{\theta} + c_i + d_j + u_{i,j,t} \leq \alpha | \boldsymbol{w}_{i,j,t}, c_i\right] \\ &= Pr\left[u_{i,j,t} \leq \alpha - \boldsymbol{x}'_i\boldsymbol{\beta} - \boldsymbol{m}'_{i,t}\boldsymbol{\gamma} - \boldsymbol{z}'_{i,t}\boldsymbol{\theta} - c_i - d_j\right] \\ &= \Phi\left(\alpha - \boldsymbol{x}'_i\boldsymbol{\beta} - \boldsymbol{m}'_{i,t}\boldsymbol{\gamma} - \boldsymbol{z}'_{i,t}\boldsymbol{\theta} - c_i - d_j\right), \end{aligned} \tag{3-3}$$

where in the third line we have used the fact that $u_{i,j,t}$ is independent of $\boldsymbol{w}_{i,j,t}$ and c_i. $\Phi(.)$ is the standard normal cumulative distribution function. The unobserved effect of financial institutions d_j can be controlled for through dummy variables of banks. Remember that we have two banks in our data.

Likewise, we have:

[10]Strict exogeneity means that, once $\boldsymbol{w}_{i,j,t}$ and c_i are controlled for, $\boldsymbol{w}_{i,j,s}$ has no partial effect on $y_{i,j,t}$ for $s \neq t$. This requires that, for example, future movements of explanatory variables cannot depend on current or past values of $y_{i,j}$. Even though we recognize that, by the procyclicality argument, movements in the aggregate default rate may affect macroeconomic variables in the future, it does not seem that an individual default can affect macroeconomic conditions, especially in the retail sector. Then, strict exogeneity seems reasonable in this context.

$$Pr[y_{i,j,t} = 0|\boldsymbol{w}_{i,j,t}, c_i] = 1 - \Phi\left(\alpha - \boldsymbol{x}_i'\boldsymbol{\beta} - \boldsymbol{m}_{i,t}'\boldsymbol{\gamma} - \boldsymbol{z}_{i,t}'\boldsymbol{\theta} - c_i - d_j\right). \quad (3\text{-}4)$$

The density of $(y_{i,j,1}, ...y_{i,j,T})$ conditional on $(\boldsymbol{w}_{i,j,t}, c_i)$ is

$$f(y_{i,j,1}, ...y_{i,j,T}|\boldsymbol{w}_{i,j}, c_i; .) = \prod_{t=1}^{T} f(y_{i,j,t}|\boldsymbol{w}_{i,j,t}, c_i; .), \quad (3\text{-}5)$$

where $f(y_{i,j,t}|\boldsymbol{w}_{i,j,t}, c_i; .) = \Phi(.)^{y_{i,j,t}}[1 - \Phi(.)]^{1-y_{i,j,t}}$ and $\Phi(.)$ is defined in equation (3-3).

Observe that the parameters c_i appear in equation (3-5), but they are unobserved and cannot appear in the likelihood function. This imply that take into account the unobserved individual effects in probit models without making additional assumptions, in particular without restricting the relationship between c_i and the $\boldsymbol{w}_{i,j,t}$, is very hard. One approach is to assume a particular correlation structure and then use full conditional maximum likelihood (FCML). However, the calculation of FCML is computationally very difficult even if you have only moderate time periods in the sample.

In this paper we follow the random effects probit model approach. As usual in that methodology, we assume that

$$c_i|\boldsymbol{w}_{i,j,t} \sim \mathbb{N}(0, \sigma_c^2), \quad (3\text{-}6)$$

which implies that c_i and $\boldsymbol{w}_{i,j,t}$ are independent. Using this assumption together with the two previous one, we can derive the maximum likelihood function to consistently estimate the parameters $\boldsymbol{\Psi}' = (\alpha, \boldsymbol{\beta}', \boldsymbol{\gamma}', \boldsymbol{\theta}', d_j, \sigma_c^2)'$.

To find the joint distribution of $(y_{i,j,1}, ..., y_{i,j,T})$ conditional only on $\boldsymbol{w}_{i,j}$ we have to integrate c_i out. We use the fact that c_i is normally distributed to write the likelihood function for each i as:

$$f(y_{i,j,1}, ...y_{i,j,T}|\boldsymbol{w}_{i,j}; \boldsymbol{\Psi}) = \int_{-\infty}^{+\infty} \left\{ \left[\prod_{t=1}^{T} f(y_{i,j,t}|\boldsymbol{w}_{i,j,t}, c_i; .)\right] \left(\frac{1}{\sigma_c}\right) \phi\left(\frac{c}{\sigma_c}\right) \right\} dc, \quad (3\text{-}7)$$

where $\phi(.)$ is the density function of the standard normal distribution. The log-likelihood function for the entire sample can now be maximized to consistently estimate the parameters $\boldsymbol{\Psi}$ using numerical methods to approximate the integral in (3-7). For details, see Wooldridge (2002).

In spite of being very useful, we have always to keep in mind that assumption (3-6) can be restrictive. We should also be sure about what we can estimate by using this random effects probit model. In this context, consistent estimation of $\boldsymbol{\Psi}$ means that we can consistently estimate the partial effects of the elements of $\boldsymbol{w}_{i,j,t}$ on the response probability $Pr[y_{i,j,t} = 1|\boldsymbol{w}_{i,j,t}, c_i]$ at the average value of c_i in the population, $c_i = 0$.

The application of this model to our data described in the previous subsection is very straightforward. The dependent variable defined in equation (3-2) is easily obtained from the microdata, once we observe the history of each transaction along the time. Following the previous model, the explanatory variables we use in estimations include borrower's and transaction's characteristics, variables measuring the business cycles and other controls. As already pointed out in the data set description, the information contained in the data allows us to control for the following borrower's characteristics: gender, age, type of occupation and the geographic localization in which the borrower lives. The age information is introduced in the model through five dummy variables, which are defined as (not including the upper bounds): less than 25 years old (baseline dummy), between 25 and 35, from 35 to 45, from 45 to 60, and more than 60 years old. There are also six dummy variables to control for the borrower's type of occupation: private sector (baseline dummy), public sector and military, self-employed, company owner, pensioner, and other occupations.

Transaction's characteristics include the risk rating of the loan and the identification of the financial institution that granted the credit. We use the information about the bank to take into account possible individual fixed effects of financial institution, including in the models a dummy variable for one of the banks (remember that we have two banks in our data). Ideally, we would like to introduce variables measuring the borrower's income, but we do not have this information in the data. Instead, we include the transaction's risk rating as an explanatory variable. In fact, the risk rating contains much information about the borrower and the transaction, in particular information about the borrower's income and his capacity of paying the loan, and can be viewed more generally as an important variable summarizing many critical factors that determine credit risk. In our estimations, rating AA is the baseline dummy. We use the average interest rate of each modality to control for interest rate.

There are two additional factors that must be controlled for in order we can isolate the effect of the business cycles on default events: the borrower's quality and the size of the market in which the credit transaction is made. Otherwise, by not controlling for the borrower's quality and/or the local market size, we can obtain an increasing default probability only because the lender institution may begin to lend to worse borrowers in saturated markets, when the economy experiences a strong growth period. In addition to the reasons explained in the previous paragraph, we introduce the transaction's risk rating in our models also to control for the borrower's quality. We use the information about the geographic localization in which the credit was given contained in our data set to take the size of the local market into account in our estimations. The literature generally uses variables such as local GDP per capita or local population to measure market size. Even though the ideal information would be the city, in our data set the more disaggregated data about geographic localization is the State where the borrower lives. Therefore, to capture market size in our estimations we use the population of the State. We decided in favor of this variable, instead of per GDP capita, because the last variable is also influenced by the business cycles.

In our estimations we measure business cycles through three different variables. First, aiming to have a more disaggregated measure of the economic activity, we use the unemployment rate in the Geographic Region in which the borrower lives[11]. For each Region, this variable is a mean of the seasonally adjusted unemployment rates calculated by the Brazilian Institute of Geography and Statistics (IBGE) in the metropolitan regions of the State capital cities[12], weighted by the population of each city. Second, we employ the seasonally adjusted aggregate unemployment rate that we used in the vector autoregressive estimation of section 3.3. The last variable we use to measure the business cycles is the seasonally adjusted aggregate GDP growth rate.

3.4.3 Results

We estimate four specifications of this probit model for each modality to analyze the relationship between credit defaults and business cycles. The difference between the specifications is the use of the variables measuring business cycles. Marginal effects on the probability of default, evaluated on the average of explanatory variables, are reported in Table 3.1 and Table 3.2 below. Specification (1) includes, in addition to all controls, only the regional unemployment rate. Model (2) includes only the aggregate unemployment rate. Model (3) has both measures of unemployment, and specification (4) includes these two variables and the GDP growth rate. For comparison, we also estimate this complete specification using a linear probability model with unobserved individual effect through the random effect estimation (model (5)).

To measure the models performance in explaining the data, we calculate in each model, for each modality, the percentage of observations correctly predicted in the three groups of observations: total, observations in default and observations not in default. We use the cut off of 50% to define the result predicted by the model, i.e., if the predicted probability is more than 50% we consider that the model is predicting default of that operation. Considering the total number of observations, all models correctly predict more than 83% of the results. If only transactions that

[11]Brazil has five Geographic Regions: North, Northeast, Central-West, Southeast and South.
[12]But IBGE does not calculate unemployment rate in each State capital city. See footnote 6.

resulted in default are considered, more than 70% of the results in the Consumer Credit modality and more than 55% of the results in the Vehicle Financing modality are correctly predicted. Therefore, in terms of goodness of fit all models do a good job.

Table 3.1: Marginal Effect on Default Probability – Consumer Credit Modality

	(1)	(2)	(3)	(4)	(5)
Regional unemployment	0.0107***		-0.0003	-0.0003	-0.0004
	(0.0004)		(0.0005)	(0.0005)	(0.0003)
Aggregate unemployment		0.0330***	0.0337***	0.0389***	0.0100***
		(0.0006)	(0.0008)	(0.0010)	(0.0004)
GDP				-0.0071***	-0.0023***
				(0.0007)	(0.0004)
Rating A	0.1944***	0.2151***	0.2109***	0.2101***	0.0140***
	(0.0092)	(0.0089)	(0.0092)	(0.0092)	(0.0011)
Rating B	0.5041***	0.5257***	0.5182***	0.5173***	0.1653***
	(0.0092)	(0.0085)	(0.0090)	(0.0090)	(0.0014)
Rating C	0.6426***	0.6477***	0.6476***	0.6470***	0.2941***
	(0.0060)	(0.0053)	(0.0057)	(0.0057)	(0.0022)
Rating D	0.9285***	0.9318***	0.9312***	0.9308***	0.6126***
	(0.0018)	(0.0016)	(0.0017)	(0.0017)	(0.0015)
Male	0.0149***	0.0143***	0.0151***	0.0151***	0.0083***
	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0007)
Age from 25 to 35	0.0268***	0.0287***	0.0300***	0.0296***	0.0133***
	(0.0033)	(0.0032)	(0.0033)	(0.0033)	(0.0019)
Age from 35 to 45	-0.0046	-0.0042	-0.0021	-0.0024	-0.0038***
	(0.0031)	(0.0030)	(0.0031)	(0.0031)	(0.0019)
Age from 45 to 60	-0.0375***	-0.0378***	-0.0351***	-0.0355***	-0.0213***
	(0.0030)	(0.0029)	(0.0030)	(0.0030)	(0.0019)
Age more than 60	-0.0672***	-0.0670***	-0.0639***	-0.0642***	-0.0378***
	(0.0031)	(0.0029)	(0.0031)	(0.0031)	(0.0020)
Population	-0.0086***	-0.0066***	-0.0098***	-0.0096***	-0.0051***
	(0.0009)	(0.0007)	(0.0009)	(0.0009)	(0.0005)
σ_c	0.6285***	0.6111***	0.6067***	0.6039***	0.1888
	(0.0055)	(0.0052)	(0.0055)	(0.0055)	
ρ	0.2832***	0.2719***	0.2690***	0.2672***	0.4356
	(0.0035)	(0.0033)	(0.0035)	(0.0035)	
Percent correctly predicted - Total	83.77	88.81	83.78	83.78	83.78
Percent correctly predicted - Default	76.36	73.47	76.36	76.36	76.24
Percent correctly predicted - Non Default	87.84	97.24	87.86	87.86	87.91
Log-likelihood value	-432515.16	-482208.97	-431699.89	-431657.92	-
No. obs.	1406843	1566423	1406843	1406843	1406843
No. groups	655295	728040	655295	655295	655295

Notes: 1) Models (1), (2), (3) and (4) are probits with unobserved individual component, and specification (5) is a linear probability model estimated by random effect estimation.
2) All models also include variables controling for the borrower's occupation, interest rate and unobserved fixed effect of financial institution.
3) Standard errors are in parenthesis. Significance: ***=1%, **=5%, *=10%
4) Probability of more than 50% is the criterium used to define predicted default.

In terms of the relationship between credit defaults and business cycles, the models estimated in the two modalities provide basically the same evidence. However, the effects seem to be stronger in Consumer Credit transactions. Our interpretation is that, because Vehicle Financing loans usually have collateral, the rates of default in this modality are smaller and less responsive to business cycles than Consumer Credit transactions.

The first piece of evidence that emerges from our results is that the effect of regional unemployment on credit delinquencies is very small. When business cycles are measured only by regional unemployment, the estimations indicate that one additional percentage point in the unemployment rate produces an increase in the probability of default in Consumer Credit transactions of approximately one percentage point. This means that, for example, if the average unemployment rate is

10% and the probability of default evaluated on the average of explanatory variables is 6%, if the unemployment rate goes to 11%, the probability of default increases to 7%. If we include the other variables measuring business cycles, this effect becomes statistically insignificant. In the Vehicle Financing modality, despite of being statistically significant, the effect is still smaller.

Table 3.2: Marginal Effect on Default Probability – Vehicle Financing Modality

	(1)	(2)	(3)	(4)	(5)
Regional unemployment	0.0024***		0.0011***	0.0011***	0.0013***
	(0.0001)		(0.0001)	(0.0002)	(0.0001)
Aggregate unemployment		0.0059***	0.0048***	0.0067***	0.0062***
		(0.0001)	(0.0001)	(0.0001)	(0.0002)
GDP				-0.0058***	-0.0061***
				(0.0002)	(0.0002)
Rating A	0.0013***	-0.0008**	-0.0011***	-0.0015***	-0.0054***
	(0.0003)	(0.0004)	(0.0003)	(0.0004)	(0.0005)
Rating B	0.0925***	0.0893***	0.0872***	0.0863***	0.0739***
	(0.0013)	(0.0012)	(0.0013)	(0.0013)	(0.0010)
Rating C	0.2245***	0.2249***	0.2210***	0.2198***	0.1911***
	(0.0021)	(0.0020)	(0.0021)	(0.0021)	(0.0016)
Rating D	0.8106***	0.8105***	0.8112***	0.8124***	0.7427***
	(0.0016)	(0.0015)	(0.0016)	(0.0016)	(0.0015)
Male	0.0024***	0.0023***	0.0024***	0.0024***	0.0032***
	(0.0002)	(0.0003)	(0.0003)	(0.0003)	(0.0003)
Age from 25 to 35	-0.0013***	-0.0007**	-0.0008**	-0.0008**	-0.0005
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0005)
Age from 35 to 45	-0.0038***	-0.0031***	-0.0032***	-0.0033***	-0.0038***
	(0.0003)	(0.0003)	(0.0004)	(0.0004)	(0.0005)
Age from 45 to 60	-0.0069***	-0.0061***	-0.0062***	-0.0063***	-0.0073***
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0005)
Age more than 60	-0.0100***	-0.0089***	-0.0091***	-0.0091***	-0.0116***
	(0.0004)	(0.0005)	(0.0005)	(0.0005)	(0.0007)
Population	0.0001	-0.0013***	-0.0005***	-0.0005***	-0.0006***
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
σ_c	0.2981***	0.2917***	0.2915***	0.2842***	0.0745
	(0.0101)	(0.0096)	(0.0102)	(0.0104)	
ρ	0.0815***	0.0784***	0.0783***	0.0747	0.1655
	(0.0051)	(0.0047)	(0.0050)	(0.0051)	
Percent correctly predicted - Total	87.85	95.88	87.85	87.85	87.85
Percent correctly predicted - Default	57.96	52.8	57.96	57.96	57.96
Percent correctly predicted - Non Default	90.07	99.08	90.07	90.07	90.07
Log-likelihood value	-254211.74	-283792.62	-253573.23	-252951.29	-
No. obs.	1750841	1928644	1750841	1750841	1750841
No. groups	1265684	1392716	1265684	1265684	1265684

Notes: 1) Models (1), (2), (3) and (4) are probits with unobserved individual component, and specification (5) is a linear probability model estimated by random effect estimation.
2) All models also include variables controling for the borrower's occupation, interest rate and unobserved fixed effect of financial institution.
3) Standard errors are in parenthesis. Significance: ***=1%, **=5%, *=10%
4) Probability of more than 50% is the criterium used to define predicted default.

The effect of the aggregate unemployment seems to be larger, in particular in the Consumer Credit modality. One additional percentage point in the aggregate unemployment rate appears to increase the probability of default in 3 or 4 percentage points. But even showing a statistically significant relationship between credit defaults and business cycles, these numbers provide evidence that the effect of economic activity on the probability of default of retail credit transactions is still reduced. In the Vehicle Financing modality, the increase in the probability of default associated to one additional percentage point in the aggregate unemployment is estimated in less than 1 percentage point. These results show that, interestingly, movements in the level of aggregate economic activity have more influence in defaults than regional variables. Not only the impact of the aggregate unemployment is larger than that of the regional unemployment rate, but also, in the Consumer Credit modality, this second effect becomes statistically insignificant when we additionally introduce aggregate variables measuring business cycles in the model.

Similar conclusions about the effect of the business cycle on the probability of default emerge if we use GDP instead of unemployment as measure of economic activity. Our estimations suggest that one additional percentage point in the GDP growth rate reduces the probability of default in less than one percentage point. We should report, however, that more uncertainty is associated with this effect. In some other specifications we estimate, the GDP variable was not statistically significant, even though the point estimations preserve the magnitude of the effect.

Jointly, therefore, our results using data on the individual level show the same evidence obtained in the time series estimations of section 3.3: there is a significant relationship between business cycles and credit defaults, but the impact of the economic activity on delinquency rates in retail sector transactions seems to be limited. The unemployment rate effect, as well as the GDP effect, on the probability of default in these transactions appears to be modest. The magnitude of the impact of business cycles are still smaller in the Vehicle Financing modality.

Besides, our estimations provide some other interesting results. First, as already pointed out, the risk classifications of the banks are consistent and seem to capture the intrinsic risks of each transaction. The worse the risk rating of the transaction, the large is the estimated probability of default. For example, the probit estimations show that, in the Consumer Credit modality, a transaction classified as A has a probability of default around 20 percentage points larger, when compared to a transaction with rating AA; while a transaction rated as D has a probability approximately 90 percentage points larger. Despite the difference in the level, the same conclusion can be obtained from the probit estimations in the Vehicle Financing modality and from the linear probability model.

Second, the results suggest that, controlling for the other variables, in both modalities the probability of default is somewhat higher among males, when compared to females. In Consumer Credit transactions, the estimated probability among males is more than 1 percentage point larger than the value calculated for females. In Vehicle Financing transactions, the difference coming from gender is smaller. Likewise, the estimations also indicate that older borrowers have smaller probability of delinquency in their credit transactions.

Tables 3.1 and 3.2 also report the standard deviation of the unobserved individual effect and the correlation between the composite latent errors, $c_i + u_{i,j,t}$, across any two time periods: $\rho = \sigma_c^2/(1 + \sigma_c^2)$. This correlation is also the ratio of the variance of c_i to the variance of the composite error (remember that the variance of the idiosyncratic error in the latent variable model is unity), and it is useful as a measure of the relative importance of the individual unobserved effect. Our probit estimations suggest that, in the Consumer Credit modality, the individual component accounts for more than 26% of the variance of the composite error. In the Vehicle Financing models this number is around 7%. Moreover, the absence of unobserved individual effects is statistically equivalent to $H_0 : \sigma_c^2 = 0$. Our results show that we cannot reject this hypothesis in any of the usual significance levels, indicating that the presence of unobserved individual components cannot be neglected, as usually the literature on this issue seems to do.

Finally, one can also argue that the effect of these variables measuring business cycles on default events is not contemporaneous. Because of this argument, we carry out some other estimations for robustness check, including these variables lagged one period. We do not present the results here, but they all support the same conclusions just presented and can be provided under request.

3.5 Correlation and Transition Matrices

In this section we estimate default correlations and transition probabilities among risk ratings in our data. Our final goal is to investigate the relationship between credit defaults and business cycles from other perspective. After estimating these parameters, we use them in Value at Risk (VaR) experiments to calculate the losses in portfolios of financial institutions along the business cycles. The idea is verifying if economic recessions, through increases in the rate of default, cause large increases in the losses of banks' portfolios that (in addition to increases in capital requirements) can enlarge capital costs, reduce credit supply and further intensify the business cycles, as usually argued in discussions about procyclical effect of the Basel II Accord.

We adopt the historical approach to estimate default correlations and transition probabilities matrices in our data. By default correlation we mean the correlation between pairs of transactions with different (or equal) risk ratings jointly moving to the state of default. Our estimations are based on the methodology developed by Servigny & Renault (2002). These authors propose a way of extracting information about marginal and joint transitions between risk ratings from historical data without assuming any specific model for transitions. Gómez et al. (2007), argue that this methodology has many important applications. Servigny & Renault (2002) worked with the Standard & Poor's database and selected a sample covering only U.S. companies.

The methodology uses the cohort method, assuming the discrete time Markovian assumption. Specifically, loan transactions are grouped into risk ratings, and correlations between these ratings are calculated through transition probabilities. These transition probabilities are estimated under the assumption that the time series of classifications is a realization of a discrete time Markov chain, with states being the risk ratings. The transition probability from state i to state j is estimated by dividing the number of observed transitions from i to j in a given period by the total number of observations in state i at the beginning of the period.

However, this method uses discrete time and therefore disregards intermediate transitions occurring within each period. Lando & Skodeberg (2002) consider this fact one disadvantage, when compared to other methods that consider continuous time. They report that null estimates for transition probabilities can be mistakenly obtained if the initial rating state is equal to the final state. Furthermore, transitions of transactions that do not stay in the data set during the entire period, either because they were finished before the end of the period or because they were initiated after the beginning of the period, are not considered in the calculation. We hope dealing with this problem in future works, through the use of methods in continuous time, where the frequency of transition observations is minimized.

Keeping all these issues in mind, we proceed to use this method to empirically estimate default correlations. Details are given in which follows. First, marginal or univariate transition matrices are obtained from the frequencies of transitions between the risk ratings. In particular, we are interested in the probability of a given transaction moving from different risk ratings to the state of default. Univariate transition frequencies in one period are calculated by the following method:

$$f_i^k = \frac{T_i^k}{N_i}, \tag{3-8}$$

where:

- f_i^k is the marginal transition frequency from rating i to rating k in one period;
- T_i^k is the total number of transactions moving from rating i at the beginning of the period to rating k at the end of the same period;
- N_i is the total number of transactions belonging to rating i at the beginning of the period.

Similarly, joint or bivariate transition frequencies are estimated by:

$$f_{i,j}^{k,l} = \frac{T_i^k * T_j^l}{N_i * N_j}, \tag{3-9}$$

where:

- $f_{i,j}^{k,l}$ is the joint transition frequency from ratings i and j, respectively to ratings k and l, in one period;
- T_i^k is the total number of transactions moving from rating i at the beginning of the period to rating k at the end of the same period;

- T_j^l is the total number of transactions moving from rating j at the beginning of the period to rating l at the end of the same period;
- N_i is the total number of transactions belonging to rating i at the beginning of the period;
- N_j is the total number of transactions belonging to rating j at the beginning of the period.

Specifically, we are more interested in the probabilities of two given transactions with, say, ratings i and j, jointly moving to the default state d, i.e., $f_{i,j}^{d,d}$.

We obtain these marginal and joint transition frequencies for each period. To obtain a measure of transition probabilities, we aggregate these period frequencies using as weights the number of transactions belonging to a certain rating at the beginning of each period relative to the total number of transactions belonging to that same rating at the beginning of all periods.

Then, the transition correlations between risk ratings are obtained by:

$$\rho_{i,j}^{k,l} = \frac{f_{i,j}^{k,l} - f_i^k * f_j^l}{\sqrt{f_i^k * \left(1 - f_i^k\right) * f_j^l * \left(1 - f_j^l\right)}}, \tag{3-10}$$

where:

- $\rho_{i,j}^{k,l}$ is the correlation coefficient between a pair of loans moving from ratings i and j at the beginning of one period respectively to ratings k and l at the end of the period.

We apply this methodology to our data set of microdata. We estimate marginal transition probabilities, joint transition probabilities and default correlations in retail loans in the two modalities we have been using in this paper, adopting the traditional segmentation used in the literature, based on risk ratings. So, we split the transactions in five ratings (AA, A, B, C and Default) according to classifications of the two financial institutions.

Table 3.3 below presents univariate transition probabilities for both Consumer Credit and Vehicle Financing modalities. First, the table shows that, in general, in both modalities the probability of staying in the same risk rating in which the transaction was in the last period (main diagonal of the matrices) is higher than the probability of changing. Therefore, for instance, the transaction that was rated as AA in a given period tends to continue in this risk rating in the next period. Second, this fact is particularly true for the state of default, which shows that this state is almost absorbing, i.e., once a loan moves to default, it nearly stays there forever. Third, showing the consistency of risk classification, in both modalities when the risk rating of a transaction gets worse, the probability of moving to the state of default also increases—for example, while the probability of an AA transaction moving to default is approximately 3%, the same probability of a C transaction is around 40%. Finally, consistent with Figure 3.3 in Appendix, Table 3.3 also shows that in general the probabilities of moving from any state to default (last column of matrices) are higher in Consumer Credit modality than in Vehicle Financing. This can be explained by the existence of collateral in the last modality, as already argued. Similar results were found in a previous paper of the last three authors[13].

Table 3.3: Univariate Transition Probabilities

		Consumer Credit Final Rating				
		AA	A	B	C	Default
Initial Rating	AA	48.29%	42.74%	2.52%	3.20%	3.24%
	A	1.26%	77.18%	11.61%	2.37%	7.58%
	B	0.07%	8.78%	60.12%	4.46%	26.58%
	C	0.13%	3.05%	8.48%	47.65%	40.69%
	Default	0.01%	0.51%	2.85%	0.79%	95.83%
		Vehicle Financing Final Rating				
		AA	A	B	C	Default
Initial Rating	AA	88.92%	1.81%	3.05%	2.88%	3.33%
	A	9.31%	76.44%	5.95%	4.00%	4.30%
	B	8.68%	18.25%	44.99%	10.74%	17.35%
	C	10.07%	11.51%	6.46%	34.35%	37.61%
	Default	2.75%	3.39%	1.80%	2.59%	89.47%

Note: Average of semi-annual transition frequencies from rating i (initial rating) to rating k (final rating), where i and k = AA, A, B, C, Default. Period: Jan/2003 to Jul/2008.

We also estimate the joint transition probabilities in both modalities. Because there are so many combinations and the results are hard to interpret, we only present them in Appendix. These joint transition probabilities are, however, necessary to obtain the default correlation matrices, as can be seen in equation (3-10). The correlation matrix for Consumer Credit as well as for Vehicle Financing modality are presented in Table 3.4 below.

There is great dispersion in the estimated default correlations in both modalities. Similar results are found in the literature, where empirical papers show default correlations ranging from very negative to high positive values[14]. Default correlations are also found to be generally low in our data, which may be possibly explained by the fact that our data come from the retail sector. In this segment loans are given to a large number of different individuals, which may lead to a diversification effect, thus spreading the influence among default events. Despite of being also generally low, the literature that uses corporate data reports that correlations should increase as ratings decrease, once low-rated companies are more susceptible to problems in the aggregate economy. The reasoning is that, if the economy experience

[13]See Silva et al. (009a).

[14]See, for example, Lucas (1995), Nagpal & Bahar (2001), Rosch (2003) and Servigny & Renault (2002).

a downturn, all companies close to the edge of default are more likely to experience solvency problems, which makes default events more correlated in these groups of transactions. Interestingly, we do not obtain higher correlations among low-rated individuals in our data, except for those already in default.

Table 3.4: Empirical Default Correlation Matrices

	Consumer Credit					Vehicle Financing				
	AA	A	B	C	Default	AA	A	B	C	Default
AA	1.67%	1.03%	2.40%	-0.46%	2.27%	0.75%	0.51%	1.13%	0.47%	-6.82%
A	1.03%	-2.77%	-3.68%	-3.63%	-17.69%	0.51%	0.01%	0.23%	0.96%	-0.83%
B	2.40%	-3.68%	-3.07%	-3.04%	-22.83%	1.13%	0.23%	0.96%	1.12%	-7.42%
C	-0.46%	-3.63%	-3.04%	-6.34%	15.86%	0.47%	0.96%	1.12%	-2.04%	-20.40%
Default	2.27%	-17.69%	-22.83%	15.86%	23.88%	-6.82%	-0.83%	-7.42%	-20.40%	32.86%

Note: Default correlations calculated according to equation (8), using univariate default probabilities of Table 3 and bivariate default probabilities of Table 7 and 8. Period: Jan/2003 to Jul/2008.

We also estimate univariate transition probabilities in different phases of the business cycle. To define the periods of growth and recessions during the period covered by our data we rely on a recent work of the Brazilian Institute of Economics of the Getulio Vargas Foundation (IBRE-FGV), which develops a methodology to identify recessions and booming periods of the Brazilian economy—Committee on Business Cycle Dating, IBRE-FGV[15]. According to the Committee, from January 2003 to July 2008, except for the first semester of 2003, the Brazilian economy experienced a period of growth. Using this classification we split our data set and estimate transition probabilities matrices for recessions and booming periods, which are reported in Table 3.5. As we would expect, in general the probability of moving from any risk rating to the state of default is larger in recessions than during expansions, in both modalities.

Table 3.5: Univariate Transition Probabilities – Recession and Boom

		Consumer Credit - Recession Final Rating						**Vehicle Financing - Recession** Final Rating				
		AA	A	B	C	Default		AA	A	B	C	Default
Intial Rating	AA	40.03%	35.15%	3.35%	17.41%	4.07%	AA	77.40%	12.62%	6.85%	1.66%	1.47%
	A	2.02%	61.06%	14.84%	8.58%	13.50%	A	0.02%	84.40%	6.50%	3.40%	5.68%
	B	0.13%	9.52%	49.76%	6.44%	34.16%	B	0.11%	22.46%	45.64%	7.59%	24.19%
	C	0.04%	0.74%	1.85%	56.45%	40.92%	C	0.03%	23.45%	8.41%	14.82%	53.30%
	Default	0.00%	0.26%	0.68%	0.32%	98.74%	Default	0.01%	4.08%	1.73%	1.66%	92.52%
		Consumer Credit - Booming Final Rating						**Vehicle Financing - Booming** Final Rating				
		AA	A	B	C	Default		AA	A	B	C	Default
Initial Rating	AA	48.97%	43.36%	2.45%	2.04%	3.17%	AA	88.92%	1.81%	3.05%	2.88%	3.33%
	A	1.25%	77.38%	11.57%	2.30%	7.50%	A	10.28%	75.61%	5.89%	4.06%	4.16%
	B	0.07%	8.77%	60.24%	4.44%	26.48%	B	9.29%	17.95%	44.94%	10.96%	16.87%
	C	0.14%	3.24%	9.03%	46.93%	40.67%	C	10.50%	11.01%	6.37%	35.17%	36.95%
	Default	0.01%	0.52%	2.91%	0.80%	95.76%	Default	2.92%	3.35%	1.81%	2.65%	89.28%

Note: Average of semi-annual transition frequencies from rating i (initial rating) to rating k (final rating) in periods of recession and booming. Period: Jan/2003 to Jul/2008.

[15]See Comitê de Datação dos Ciclos Econômicos, IBRE/FGV, May 2009.

3.6 Value at Risk Exercises

To analyze the impact of these increasing default probabilities and transition probabilities during periods of economic recessions on the portfolio losses, in this section we carry out some credit Value at Risk (VaR) simulation experiments. The portfolio we study is composed by retail loans based on the portfolio positions of the two selected institutions in March 2009.

Based on these data, we obtain estimates of credit VaR in two scenarios: periods of economic growth and recessions. The default probabilities and correlations estimated in the previous section are used as input parameters in the simulation of losses in each scenario. Because of the large number of transactions in the portfolio we have in hands, we decided to randomly chose a sample of 50 thousand transactions from this population, stratified by risk ratings, to compose our portfolio.

Then, we assigned a hypothetical unit value to each portfolio exposure. One hundred simulations of this hypothetical portfolio were performed in each simulation run, in a total of five runs. In each simulation, binary variables (default or non-default) were sampled from a Bernoulli distribution, according to the parameters (default probabilities and correlations) estimated empirically for each risk rating.

We used a default-mode version of the CreditMetrics model, as presented in Gordy (2000), to simulate future portfolio losses. By default-mode version we mean that only losses arising from default events are considered in the model, i.e., losses associated with credit quality deterioration of the borrower are not considered in the model. This method is known in the literature as Simplified CreditMetrics. The time horizon used in the simulation experiment is one semester.

The model structure is the following:

$$L = \sum_{i=1}^{N} EAD_i * LGD_i * Y_i, \qquad (3\text{-}11)$$

where:

- L is the total portfolio loss at the end of the period (semester), equal to the sum of individual losses;
- N is the number of transactions in the sample (50,000 in our simulation);
- EAD_i is the exposure at default for the i-th credit transaction (equal to R$1);
- LGD_i is the loss given default for the i-th credit transaction;
- Y_i default indicator variable (Bernoulli) for the i-th credit transaction.

In our exercise LGD_i is a random variable with Beta probability distribution, whose parameters come from Silva et al. (009b). We simulate the portfolio losses distribution for each of the two economic scenarios and obtain the VaR in each scenario for three percentiles (95%, 99% and 99.9%).

Table 3.6 below presents the estimated VaR for recessions and booming phases of the business cycles, for both modalities and the three percentiles considered. In general, the estimated losses in recessions are larger than those in expansionary periods. In Credit Consumer modality the difference is around 14% in every percentile, while in Vehicle Financing the losses are approximately 4% higher. The smaller VaR in Vehicle Financing, compared to that of the Consumer Credit, results from lower estimated probabilities and default correlations.

Table 3.6: Simulated Credit VaR

Consumer Credit			
Percentiles	**95.0%**	**99.0%**	**99.9%**
Booming	18.85%	18.89%	18.91%
Recession	21.55%	21.61%	21.62%
Vehicle Financing			
Percentiles	**95.0%**	**99.0%**	**99.9%**
Booming	12.27%	12.31%	12.32%
Recession	12.82%	12.88%	12.90%

Note: Percentiles of the simulated potential losses distribution. The VaR experiment is based in a portfolio composed by 50 thousand transactions sampled from portfolios of the two banks. Results are based in one hundred simulations in five runs.

Results of larger VaR in recessions are also found in the literature, using different types of data. But our simulations suggest much smaller losses than those of previous papers. For example, Servigny & Renault (2002) simulate a "typical" portfolio of one hundred non-investment grade bonds with unit exposure using the S&P data set. They find a VaR for recession 45% higher than that of periods of growth. Trück & Rachev (2005) simulate the losses in a loan portfolio of a large European bank in two distinct periods of the business cycle. They obtain a VaR in periods of recession six times higher than that of expansionary years, and the VaR in recessions is more than twice the average VaR, considering the whole sample period.

In light of these results Trück & Rachev (2005) conclude that average values of default probabilities and correlations should not be used in models of credit risk, and the effect of the business cycle on these parameters, and hence on the VaR, is quite obvious to be overlooked. Likewise, Cowan and Cowan (2004) claim that, by not considering the impact of changes in the business cycle on the portfolio losses, the measure of credit risk will be underestimated and, consequently, so is the capital required to manage the underlying risks.

We also quote that understanding the impact of the business cycle on credit risks is crucial for both supervisors and lenders, once calculations of regulatory capital take into account parameters such as probabilities of default and default correlations, which can be influenced by the level of economic activity. However, our results show that, at least in the retail sector in Brazil, the difference in the losses along the business cycle seems to be smaller than that reported in the literature. But obviously other studies in this sector, and particularly in other credit modalities, need to be done to further extend our knowledge on this issue.

3.7 Conclusions

In this paper we analyze the relationship between credit defaults and business cycle using retail credit transactions. In particular, we are interested in the first part of the argument that suggests that the Basel II Accord might amplify fluctuations in business cycles. The reasoning of this argument is that economic recessions increase credit default and the losses in portfolios of lender institutions, which require a recomposition of capital requirements, causing an increase in the cost of capital and a reduction in credit supply that further intensifies the economic downturn. However, we do not study in this paper the second part of this argument, i.e., if this increase in credit default, in the losses of portfolios and the consequent recomposition of capital requirements cause a shrinking in credit supply. As already emphasized, the difficulty of this task is to separate credit supply from credit demand and this can be the object of another study.

We explore the evidence coming from time series data as well as the evidence provided by data on the individual level. The results suggest that there is a significant relationship between credit defaults and business cycles, but this relationship is less strong than previously pointed out by other studies. We also find that in general women default less than men, and the older the borrower, the lower is the probability of delinquency.

First, our time series evidence suggests that after a positive shock in the unemployment rate, identified by a Vector Autoregressive (VAR) model, credit default in retail transactions increase, but this increase is modest and short lasting. Second, the estimations based on the microdata also provide evidence that the impact of an increase in the unemployment rate (both aggregate and sectoral) or in the GDP growth rate is small. While an additional percentage point in sectoral unemployment rate and in aggregate unemployment rate seems to produces an increase in the probability of default in Consumer Credit transactions in, respectively, 1 and 3 percentage points, the same increase in the GDP growth rate increases this probability in less that one percentage point.

Third, we find that estimated default correlations among risk rating in retail transactions are low and very dispersed. Finally, our Value at Risk experiments based on the portfolio of two financial institutions showed that the losses in recessions are around 14% higher in the Consumer Credit and only 4% higher in Vehicle Financing modality, when compared to the losses during booming periods. These values are much lower than those found in recent papers.

Most of studies in the literature reporting large impacts of recessions on the probability and correlations of default, and on the potential losses in portfolios of lender institutions concentrates on corporate data. Even though additional studies about this issue are obviously needed, in particular studies focusing on the effects of the Basel II Accord on the credit supply during recessions, our results suggests that, in retail sector in Brazil, the effects of the first part of the mechanisms that are argued to generate procyclical forces are modest. We suggest that these results can be possibly explained by the fact that, in the retail sector, loans are given to a large number of individuals, which may help to diversify the influence of default events. It is worth mentioning that our results definitely do not indicate that procyclical effects do not exist. After all, retail sector represents only part of the credit market. Second, we do not study the second part of the argument, which has to do with credit supply. And our intuition says that all these features regarding the balance sheets of the banks, combined with concerns during recessions about future developments of the economy in the short and medium run, can have stronger effects through credit supply. Therefore, more research must be done, specially on the mechanisms driving credit supply.

3.8 Appendix A

Figure 3.3: Credit Default Based on Microdata, Semiannually

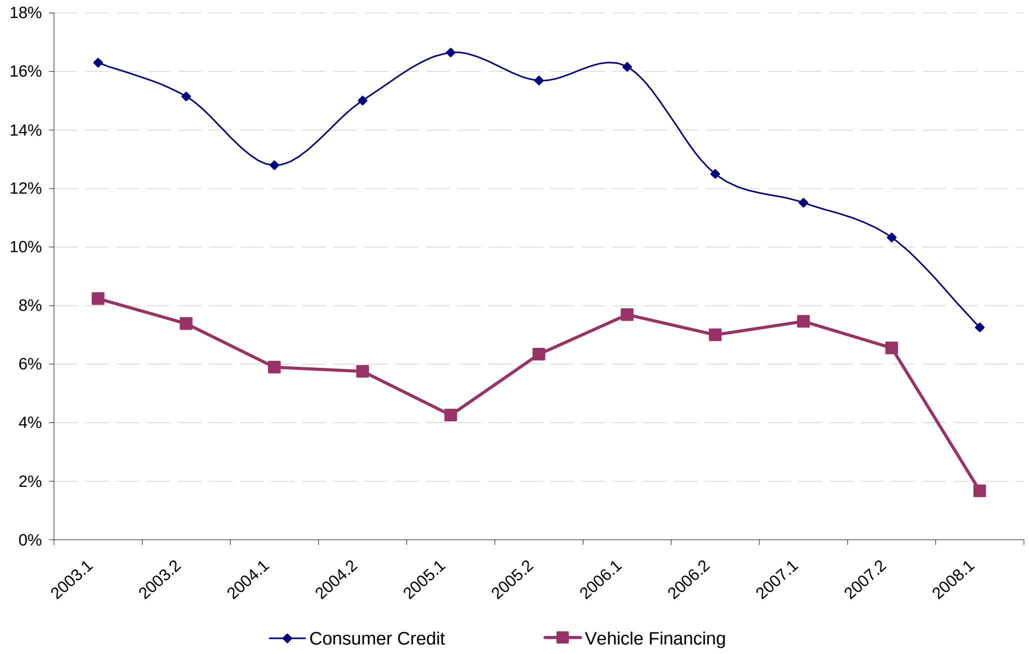


Figure 3.4: Descriptive Statistics – Consumer Credit

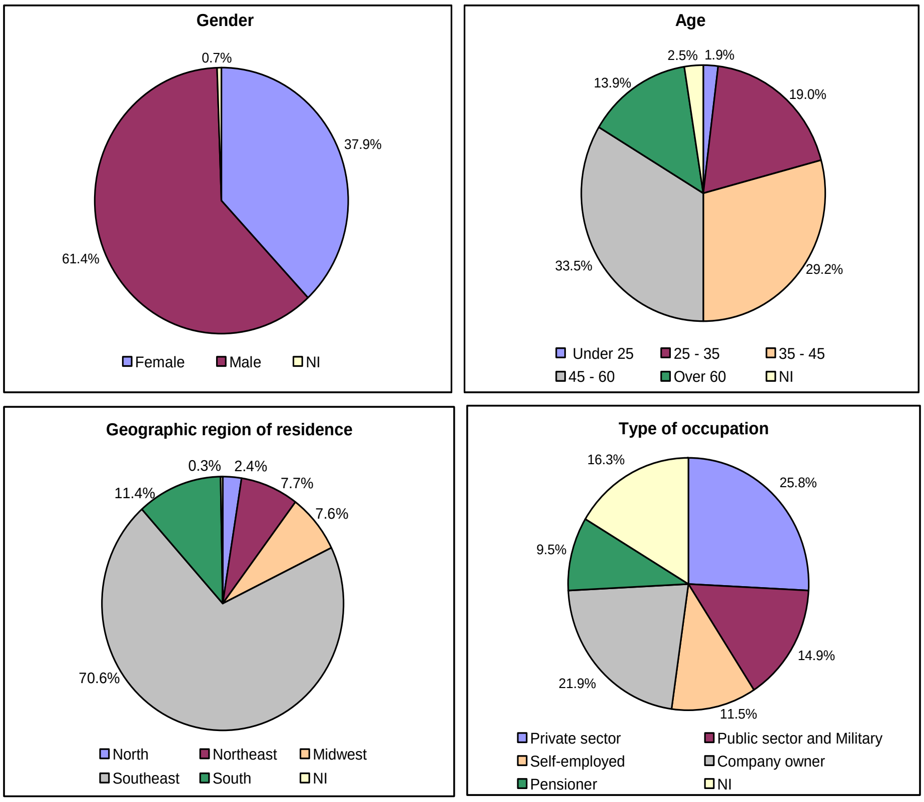


Figure 3.5: Descriptive Statistics – Vehicle Financing

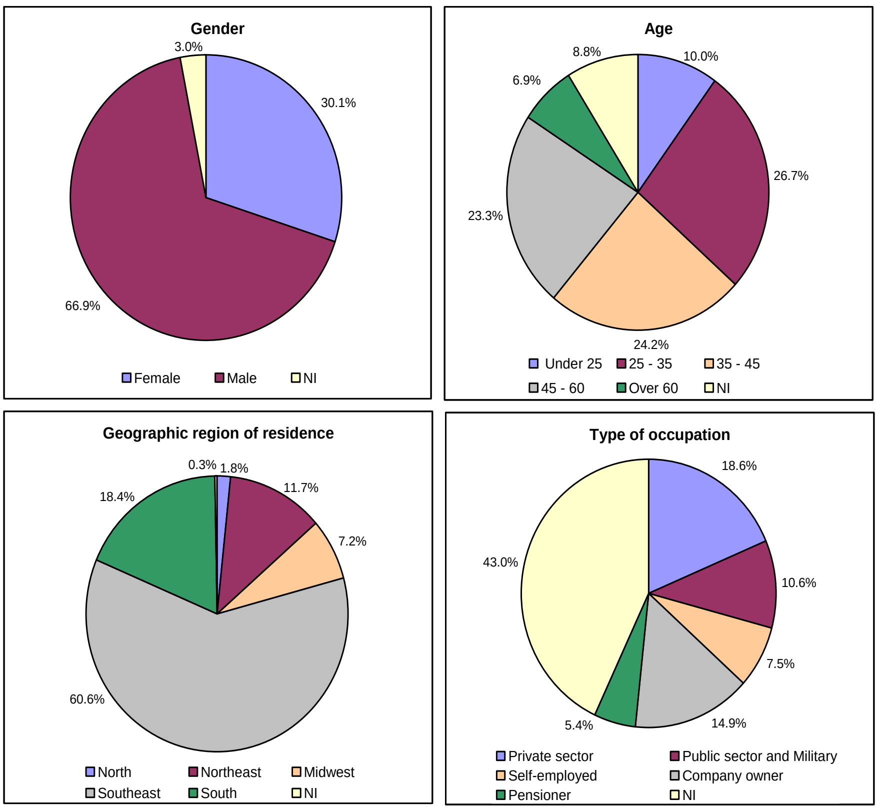


Table 3.7: Joint Transition Probabilities, Total Period

Consumer Credit

Initial Rating	Final Rating: AA, AA	AA, A	AA, B	AA, C	AA, Def	A, Def	B, AA	B, A	B, B	B, C	B, Def	C, AA	C, A	C, B	C, C	C, Def	Def, AA	Def, A	Def, B	Def, C	Def, Def
AA, AA	18.24%	11.36%	0.83%	0.95%	0.97%	2.48%	0.83%	1.53%	0.07%	0.08%	0.10%	0.95%	1.48%	0.08%	0.17%	0.11%	0.97%	2.48%	0.10%	0.11%	0.16%
AA, A	1.36%	43.87%	7.20%	1.46%	4.44%	3.69%	0.06%	1.39%	0.35%	0.06%	0.21%	0.06%	1.33%	0.34%	0.07%	0.21%	0.06%	1.96%	0.48%	0.09%	0.29%
AA, B	0.04%	3.90%	27.97%	1.97%	12.43%	14.27%	0.00%	0.23%	1.33%	0.11%	0.72%	0.00%	0.22%	1.25%	0.10%	0.70%	0.00%	0.34%	1.87%	0.15%	1.05%
AA, C	0.08%	1.47%	4.50%	26.34%	21.56%	16.97%	0.00%	0.10%	0.22%	1.01%	1.01%	0.00%	0.09%	0.21%	1.31%	1.24%	0.01%	0.16%	0.31%	1.07%	1.28%
AA, Def	0.01%	0.28%	1.59%	0.42%	46.95%	40.45%	0.00%	0.01%	0.08%	0.02%	2.22%	0.00%	0.01%	0.08%	0.02%	2.30%	0.00%	0.02%	0.12%	0.03%	3.19%
A, AA	1.36%	0.51%	0.06%	0.06%	0.06%	1.96%	7.20%	6.10%	0.35%	0.34%	0.48%	1.46%	1.06%	0.06%	0.07%	0.09%	4.44%	3.69%	0.21%	0.21%	0.29%
A, A	0.02%	0.16%	0.05%	0.01%	0.03%	4.67%	0.05%	6.11%	0.80%	0.16%	0.54%	0.01%	1.42%	0.16%	0.04%	0.12%	0.03%	4.67%	0.54%	0.12%	0.38%
A, B	0.00%	0.06%	0.39%	0.03%	0.20%	17.64%	0.00%	0.77%	5.70%	0.43%	2.35%	0.00%	0.16%	1.24%	0.09%	0.48%	0.00%	0.52%	4.06%	0.29%	1.58%
A, C	0.00%	0.02%	0.06%	0.41%	0.33%	32.20%	0.01%	0.34%	1.01%	3.71%	3.79%	0.00%	0.07%	0.21%	0.85%	0.81%	0.01%	0.23%	0.70%	2.62%	2.61%
A, Def	0.00%	0.00%	0.02%	0.00%	0.41%	79.09%	0.00%	0.04%	0.25%	0.07%	8.91%	0.00%	0.01%	0.05%	0.01%	1.91%	0.00%	0.03%	0.17%	0.05%	6.32%
B, AA	0.04%	0.03%	0.00%	0.00%	0.00%	0.34%	27.97%	24.66%	1.33%	1.25%	1.87%	1.97%	2.06%	0.11%	0.10%	0.15%	12.43%	14.27%	0.72%	0.70%	1.05%
B, A	0.00%	0.02%	0.00%	0.00%	0.00%	0.52%	0.39%	54.90%	5.70%	1.24%	4.06%	0.03%	3.56%	0.43%	0.09%	0.29%	0.20%	17.64%	2.35%	0.48%	1.58%
B, B	0.00%	0.00%	0.02%	0.00%	0.01%	2.09%	0.02%	4.75%	40.48%	2.72%	15.06%	0.00%	0.35%	2.72%	0.20%	1.08%	0.01%	2.09%	15.06%	1.08%	6.47%
B, C	0.00%	0.00%	0.00%	0.02%	0.02%	3.31%	0.07%	2.31%	7.05%	28.86%	26.09%	0.01%	0.17%	0.49%	1.85%	1.83%	0.04%	0.98%	2.75%	9.69%	10.15%
B, Def	0.00%	0.00%	0.00%	0.00%	0.03%	7.67%	0.00%	0.25%	1.63%	0.47%	60.65%	0.00%	0.02%	0.12%	0.03%	4.33%	0.00%	0.11%	0.70%	0.20%	23.46%
C, AA	0.08%	0.08%	0.00%	0.00%	0.01%	0.16%	4.50%	3.92%	0.22%	0.21%	0.31%	26.34%	14.68%	1.01%	1.31%	1.07%	21.56%	16.97%	1.01%	1.24%	1.28%
C, A	0.00%	0.06%	0.01%	0.00%	0.01%	0.23%	0.06%	8.79%	1.01%	0.21%	0.70%	0.41%	38.47%	3.71%	0.85%	2.62%	0.33%	32.20%	3.79%	0.81%	2.61%
C, B	0.00%	0.01%	0.07%	0.01%	0.04%	0.98%	0.00%	0.89%	7.05%	0.49%	2.75%	0.02%	3.08%	28.86%	1.85%	9.69%	0.02%	3.31%	26.09%	1.83%	10.15%
C, C	0.00%	0.00%	0.01%	0.03%	0.04%	1.11%	0.01%	0.29%	0.87%	3.39%	3.24%	0.03%	1.11%	3.39%	28.13%	18.72%	0.04%	1.11%	3.24%	18.72%	15.03%
C, Def	0.00%	0.00%	0.00%	0.00%	0.12%	3.64%	0.00%	0.05%	0.30%	0.09%	10.91%	0.00%	0.19%	1.03%	0.29%	41.07%	0.00%	0.19%	1.10%	0.31%	40.55%
Def, AA	0.01%	0.01%	0.00%	0.00%	0.00%	0.02%	1.59%	1.52%	0.08%	0.08%	0.12%	0.42%	0.45%	0.02%	0.02%	0.03%	46.95%	40.45%	2.22%	2.30%	3.19%
Def, A	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.02%	1.82%	0.25%	0.05%	0.17%	0.00%	0.55%	0.07%	0.01%	0.05%	0.41%	79.09%	8.91%	1.91%	6.32%
Def, B	0.00%	0.00%	0.00%	0.00%	0.00%	0.11%	0.00%	0.23%	1.63%	0.12%	0.70%	0.00%	0.06%	0.47%	0.03%	0.20%	0.03%	7.67%	60.65%	4.33%	23.46%
Def, C	0.00%	0.00%	0.00%	0.00%	0.00%	0.19%	0.00%	0.10%	0.30%	1.03%	1.10%	0.00%	0.03%	0.09%	0.29%	0.31%	0.12%	3.64%	10.91%	41.07%	40.55%
Def, Def	0.00%	0.00%	0.00%	0.00%	0.01%	0.39%	0.00%	0.01%	0.07%	0.02%	2.43%	0.00%	0.00%	0.02%	0.01%	0.70%	0.01%	0.39%	2.43%	0.70%	92.79%

Table 3.8: Joint Transition Probabilities, Total Period

Vehicle Financing

Final Rating

Initial Rating		AA, AA	AA, A	AA, B	AA, C	AA, Def	A, Def	B, AA	B, A	B, B	B, C	B, Def	C, AA	C, A	C, B	C, C	C, Def	Def, AA	Def, A	Def, B	Def, C	Def, Def
	AA, AA	81.81%	1.32%	2.31%	2.19%	2.54%	0.07%	2.31%	0.06%	0.11%	0.11%	0.12%	2.19%	0.06%	0.11%	0.10%	0.12%	2.54%	0.07%	0.12%	0.12%	0.14%
	AA, A	0.12%	76.24%	5.16%	3.91%	3.88%	0.09%	0.00%	2.44%	0.20%	0.15%	0.15%	0.00%	2.30%	0.18%	0.14%	0.14%	0.00%	2.67%	0.21%	0.17%	0.16%
	AA, B	9.75%	8.09%	48.11%	9.77%	14.18%	0.36%	0.47%	0.30%	0.99%	0.41%	0.60%	0.45%	0.29%	0.93%	0.39%	0.57%	0.52%	0.33%	1.09%	0.45%	0.66%
	AA, C	9.29%	5.75%	4.35%	42.11%	28.64%	0.71%	0.45%	0.21%	0.18%	0.68%	1.18%	0.42%	0.20%	0.17%	0.65%	1.12%	0.49%	0.23%	0.20%	0.75%	1.29%
	AA, Def	2.88%	1.08%	0.86%	1.93%	83.48%	1.46%	0.14%	0.04%	0.04%	0.08%	2.37%	0.13%	0.04%	0.03%	0.08%	2.24%	0.15%	0.05%	0.04%	0.09%	2.61%
	A, AA	0.12%	0.01%	0.00%	0.00%	0.00%	2.67%	5.16%	0.12%	0.20%	0.18%	0.21%	3.91%	0.09%	0.15%	0.14%	0.17%	3.88%	0.09%	0.15%	0.14%	0.16%
	A, A	8.83%	3.27%	0.71%	0.65%	0.59%	3.02%	0.71%	4.29%	0.35%	0.23%	0.25%	0.65%	2.76%	0.23%	0.17%	0.17%	0.59%	3.02%	0.25%	0.17%	0.19%
	A, B	1.95%	0.77%	2.57%	1.14%	1.39%	13.01%	0.51%	1.17%	2.51%	0.64%	1.04%	0.41%	0.78%	1.66%	0.46%	0.71%	0.39%	0.79%	1.80%	0.47%	0.76%
	A, C	1.58%	0.54%	0.47%	2.05%	3.33%	28.63%	0.60%	0.75%	0.40%	1.76%	2.30%	0.48%	0.50%	0.28%	1.28%	1.65%	0.46%	0.53%	0.29%	1.30%	1.71%
	A, Def	0.37%	0.09%	0.05%	0.14%	4.30%	71.97%	0.16%	0.23%	0.12%	0.16%	5.19%	0.13%	0.16%	0.08%	0.11%	3.56%	0.12%	0.15%	0.08%	0.11%	3.80%
	B, AA	9.75%	0.28%	0.47%	0.45%	0.52%	0.33%	48.11%	0.60%	0.99%	0.93%	1.09%	9.77%	0.24%	0.41%	0.39%	0.45%	14.18%	0.36%	0.60%	0.57%	0.66%
	B, A	1.95%	5.10%	0.51%	0.41%	0.39%	0.79%	2.57%	35.63%	2.51%	1.66%	1.80%	1.14%	7.85%	0.64%	0.46%	0.47%	1.39%	13.01%	1.04%	0.71%	0.76%
	B, B	1.59%	0.93%	3.03%	1.25%	1.78%	2.69%	3.03%	6.90%	25.86%	4.30%	6.89%	1.25%	1.76%	4.30%	1.28%	1.95%	1.78%	2.69%	6.89%	1.95%	3.15%
	B, C	1.75%	0.79%	0.68%	2.58%	4.40%	5.10%	3.44%	4.49%	2.51%	23.99%	14.74%	1.41%	1.12%	0.72%	3.27%	4.36%	2.03%	1.78%	1.12%	4.77%	6.73%
	B, Def	0.49%	0.14%	0.12%	0.28%	7.94%	12.54%	0.98%	1.19%	0.68%	0.99%	45.50%	0.40%	0.32%	0.18%	0.30%	9.32%	0.57%	0.47%	0.28%	0.45%	14.66%
	C, AA	9.29%	0.27%	0.45%	0.42%	0.49%	0.23%	4.35%	0.12%	0.18%	0.17%	0.20%	42.11%	0.40%	0.68%	0.65%	0.75%	28.64%	0.71%	1.18%	1.12%	1.29%
	C, A	1.58%	6.53%	0.60%	0.48%	0.46%	0.53%	0.47%	5.14%	0.40%	0.28%	0.29%	2.05%	27.19%	1.76%	1.28%	1.30%	3.33%	28.63%	2.30%	1.65%	1.71%
	C, B	1.75%	1.06%	3.44%	1.41%	2.03%	1.78%	0.68%	0.99%	2.51%	0.72%	1.12%	2.58%	3.82%	23.99%	3.27%	4.77%	4.40%	5.10%	14.74%	4.36%	6.73%
	C, C	1.77%	0.81%	0.70%	2.65%	4.50%	3.21%	0.70%	0.58%	0.38%	1.63%	2.21%	2.65%	2.64%	1.63%	24.20%	10.50%	4.50%	3.21%	2.21%	10.50%	13.67%
	C, Def	0.52%	0.16%	0.13%	0.30%	8.62%	8.12%	0.21%	0.16%	0.10%	0.16%	5.03%	0.79%	0.57%	0.36%	0.67%	38.75%	1.33%	0.81%	0.52%	0.95%	30.62%
	Def, AA	2.88%	0.08%	0.14%	0.13%	0.15%	0.05%	0.86%	0.02%	0.04%	0.03%	0.04%	1.93%	0.05%	0.08%	0.08%	0.09%	83.48%	1.46%	2.37%	2.24%	2.61%
	Def, A	0.37%	1.80%	0.16%	0.13%	0.12%	0.15%	0.05%	1.63%	0.12%	0.08%	0.08%	0.14%	2.12%	0.16%	0.11%	0.11%	4.30%	71.97%	5.19%	3.56%	3.80%
	Def, B	0.49%	0.30%	0.98%	0.40%	0.57%	0.47%	0.12%	0.39%	0.68%	0.18%	0.28%	0.28%	0.49%	0.99%	0.30%	0.45%	7.94%	12.54%	45.50%	9.32%	14.66%
	Def, C	0.52%	0.24%	0.21%	0.79%	1.33%	0.81%	0.13%	0.19%	0.10%	0.36%	0.52%	0.30%	0.26%	0.16%	0.67%	0.95%	8.62%	8.12%	5.03%	38.75%	30.62%
	Def, Def	0.15%	0.04%	0.04%	0.09%	2.51%	2.01%	0.04%	0.07%	0.03%	0.04%	1.25%	0.09%	0.09%	0.04%	0.07%	2.06%	2.51%	2.01%	1.25%	2.06%	83.14%