4
Planning of the Oil Supply Chain under Uncertainty

The supply chain in the oil industry is constituted by several activities such as exploitation , oil production, transportation, refining, and distribution of refined products. As mentioned before, due to the complexity of the oil chain, planning models for this chain are divided according to the planning levels (strategic, tactical, and operational) to consider the key decisions at each level. In this chapter, the proposed tactical and operational planning models are presented and a numerical example using real data from the Brazilian oil industry was utilized to demonstrate the effectiveness of the proposed models.

The tactical model maximizes the total revenue of the supply chain, whereas the operational model is a local optimization problem to maximize the revenue of each refinery. The tactical planning allocates the production of the different products to the various refineries in each time period, while taking into account inventory holding costs and transportation costs. The operational model determines the optimal planning trajectory by computing the amount of material that is processed at each time interval within each unit at each refinery (Khor *et al*., 2008).Whereas the tactical model only makes distinction between product families, the operational model takes more detailed information into account and differs between products within a family.

Both optimization models are based on a scenario analysis approach, and are linear. Following Pongsakdi *et al*. (2006), Lakkhanawat and Bagajewicz (2008), and Al-Qahtani and Elkamel (2008), many nonlinear features were reasonably simplified at the tactical level in order to gain computation speed, which allows the decision-maker to better explore the uncertainty issues in the model. As the operational model has a smaller-scale than the tactical model, it was possible to consider a variety of intermediate products indexed by type of oil as needed for the properties calculation using linear equations.

The associated prices, costs, and demands are assumed to be externally imposed in both planning models. The models consider a fixed market, i.e. the models ensure the total fulfillment of the market demand. It is assumed that the physical settings in the supply chain have already been established, the configuration of the chain is given, and the number of facilities at each stage is known. It is also assumed a discrete planning horizon divided into a finite number of periods. The models have been formulated as two-stage stochastic programs with fixed recourse (Dantzig, 1955). Uncertainties are discretely represented by *SC* possible realization scenarios (finite sample space) and modeled as a scenario tree. A scenario is a path from the root to a leaf of the tree. The probability that the *sc-th* scenario will occur is represented by p_{sc} ($p_{sc} \geq 0, \sum_{sc=1}^{SC} p_{sc} = 1$, $sc \in SC$). Based on these assumptions, the stochastic models this thesis proposes can be represented as follows:

$$\underset{x}{Max}\left\{ z(x) = c^T x + \sum_{sc \in SC} p_{sc} q_{sc}^T y_{sc} \right\} \text{ subject to } Ax \leq b, \qquad \textbf{(4.1)}$$
$$Wy_{sc} \leq h_{sc} - Tx, \; x \geq 0, \; y_{sc} \geq 0 \quad sc \in SC$$

First-stage decisions are assumed to be made before the realization of random variables (here-and-now decisions), represented by a vector x, while second-stage decisions, denoted by y_{sc}, are made under complete information about the realization of sc, becoming scenario-dependent variables.

The objective function in Equation (4.1) contains a deterministic term $c^T x$, which models the oil purchase decisions in both planning levels. At the tactical level, the decision is concerned with oil supply by long-term contracts, whereas at the operational level the oil purchase decision is represented by the purchase of additional oil in the spot market. The second term of Equation (4.1) contains the expected value of the second-stage objective $\sum_{sc \in SC} p_{sc} q_{sc}^T y_{sc}$ which models the stochastic operational profit due to the first-stage decision. A set of deterministic inequalities ($Ax \leq b$) is used to model decisions related to oil purchase. Stochastic constraints ($Wy_{sc} \leq h_{sc} - Tx$) are used to represent refinery operation and to model all operative relations between the inputs (or different petroleum types) and the outputs (or final products) and the necessary network flows through the installed transportation network. At the tactical level, uncertainty is introduced through the product prices, oil prices, and market demand for final products. At the operational level, oil supply and capacity of the process units depending on the equipment maintenance constitute the uncertainty sources.

In order to properly evaluate the added-value of including uncertainty in the problem parameters, the models can be evaluated using the Expected Value of Perfect Information (EVPI) (Birge and Louveaux, 1997) and the Value of the Stochastic Solution (VSS) (Birge and Louveaux, 1997). The EVPI measures the loss of profit due to the presence of uncertainty which is also the measure of the maximum amount the decision maker is willing to pay in order to get accurate information on the future. As stated by the constraint (4.2), the EVPI results show the expected profit difference between the solution obtained by the agent able to make the perfect prediction (wait-and-see - WS) and the one obtained by the agent that solved the problem under uncertainty (recourse problem - RP).

$$EVPI = \left|WS - RP\right| \tag{4.2}$$

A solution based on perfect information would yield optimal first stage decisions for each realization of the random parameters (Madansky, 1960). So, assuming that the uncertainty is represented by a finite number of scenarios and that ζ is a random variable set of scenarios, the problem associated with each scenario of ξ can be defined as:

$$\begin{aligned} &\underset{x}{Max}\, z(x,\xi) = c^T x + \max\left\{q^T y \mid Wy \le h - Tx,\, y \ge 0\right\} \\ &X = \left\{x : Ax \le b,\, x \ge 0\right\} \end{aligned} \tag{4.3}$$

It is assumed that for all ξ there is at least one feasible solution $x \in R^n$. Let $x^*(\xi)$ an optimal solution to the problem (4.3) and $z\left(x^*(\xi),\xi\right)$ the optimal objective function value for a scenario ξ. The wait-and-see solution corresponds to the optimal value when the future realization of ξ is known, i.e., the decision maker can wait and see the future before deciding. The expected value of the wait-and-see solution is:

$$WS = E_\xi\left[\max_{x \in X} z(x,\xi)\right] = E_\xi\left[z\left(x^*(\xi),\xi\right)\right] \tag{4.4}$$

The recourse problem (RP) solution is also known as here-and-now decision, because the solution the first stage is decided without knowing the future realization of ξ, i.e., at the decision epoch the future scenario is known only probabilistically. So, the RP corresponds to the two-stage problem defined by the model (4.1) and can be written as:

$$RP = \max_{x \in X} E_{\xi}\left[z\left(x, \xi \right) \right] \quad \textbf{(4.5)}$$

The VSS, on the other hand, is defined by the difference between the average solution of the expected value problem (replacing the random events by their means - EEV) and the stochastic solution (RP) - constraint (4.6) (Birge, 1982). The VSS can be interpreted either as the benefit expected by the agent that has taken uncertainty into account or as the loss expected by the agent that opted for deterministic modeling using the average stochastic parameters ($E\left[\xi\right] = \overline{\xi}$).

$$VSS = \left| EEV - RP \right| \quad \textbf{(4.6)}$$

In order to quantify the VSS, first it is necessary to calculate the expected value solution (EV) which is defined by the solution of the problem to the expected scenario (expected value of ξ). Let $\overline{\xi} = E\left[\xi\right]$ and $x^*\left(\overline{\xi}\right)$ the optimal solution to EV, so:

$$EV = \max_{x} z\left(x, \overline{\xi} \right) \quad \textbf{(4.7)}$$

Then by fixing the first stage variables from the EV problem, the expectation of EV (EEV) can be obtained by allowing the optimization problem to choose the second stage variables with respect to different realizations:

$$EEV = E_{\xi}\left[z\left(x^*\left(\overline{\xi}\right), \xi \right) \right] \quad \textbf{(4.8)}$$

The complete formulations of the stochastic models proposed in this thesis are presented in the next sections, followed by an industrial scale example.

4.1. Tactical model

This section presents the stochastic formulation for tactical planning of oil refineries. This formulation is adapted from the model proposed by Ribas *et al.* (2010) by excluding all elements related to investment decisions which must only be considered in a strategic planning model. The proposed linear programming model aims to maximize the expected profit of the oil chain and considers the following factors that affect the domestic supply:

- Configuration of refining park;
- Refinery operations and transportation costs;
- Import of oil;
- Import and export of refined products;
- Requirements for refined products defined by regulatory organizations;
- Production of crude oil;
- Domestic consumption of refined products;
- Prices of oil and refined products in the domestic and international market.

The model decisions on oil refining determines the oil blending to each refinery, the production level at each process unit, and the operational mode[1] for each unit at each period to meet the demand and respect the quality standards on the refined products. With respect to the logistic network, the model must define the minimum cost flow combination for the refinery supply and the refined products distribution. The mathematical model is presented in the section 4.1.2. Definitions of sets, variables, and parameters of the model are provided in the section 4.1.1.

[1] An operational mode is characterized by a set of operation patterns to prioritize the production of a specific product set in order to meet a market demand, i.e., the process unit yields vary according to the operational mode.

4.1.1
Nomenclature

Table 5. Sets and variables of the tactical model

Sets		Variables	
Set of nodes ***(i1, i2)***	$\boldsymbol{I}$	**First stage variables**	
Set of process units ***(u, u')***	$\boldsymbol{U}$	Oil purchase	$qocf^{n}_{r,o}$
Set of operational modes ***(c)***	$\boldsymbol{C}$	**Second stage variables**	
Set of products ***(pi, po)***	$\boldsymbol{PR}$	Blending	$b^{n,sc^t}_{r,pi,po}$
Set of oils ***(o)***	$\boldsymbol{O}$	Distillation unit load	$qdu^{n,sc^t}_{r,u,c,o}$
Set of transport modes ***(m)***	$\boldsymbol{M}$	Other process unit load	$qpu^{n,sc^t}_{r,u,c,pi}$
Set of transport arcs ***(at)***	$\boldsymbol{AT}$	Oil import	$oimp^{n,sc^t}_{in,o}$
Time periods $\{\boldsymbol{n} \mid \boldsymbol{n} = 1,\ldots,\boldsymbol{NT}\}$	$\boldsymbol{N}$	Product export	$pexp^{n,sc^t}_{in,po}$
Set of tactical scenarios $(\boldsymbol{sc^t})$	$\boldsymbol{SC^t}$	Product import	$pimp^{n,sc^t}_{in,po}$
Refinery ***(r)***	$\boldsymbol{R \subset I}$	Transported flow - entering the refinery	$ir^{n,sc^t}_{r,po}$
Natural gas producers ***(ng)***	$\boldsymbol{NG \subset I}$	Oil flow	$ot^{n,sc^t}_{at,o}$
International nodes ***(in)***	$\boldsymbol{IN \subset I}$	Transported flow - leaving the refinery	$or^{n,sc^t}_{r,po}$
Terminals ***(tr)***	$\boldsymbol{TR \subset I}$	Product flow	$pt^{n,sc^t}_{at,po}$
Bases ***(b)***	$\boldsymbol{B \subset I}$	Stock level of oil o at refinery r	$vot^{n,sc^t}_{r,o}$
Oil Field ***(of)***	$\boldsymbol{OF \subset I}$		
Transportation arcs available for transportation of po from $i1$ to $i2$ by the mode m	$\boldsymbol{ATA \subset AT}$		

Table 6. Parameters of the tactical model

Parameters			
Operational cost	$OC^{n}_{r,u}$	Oil field production	$FP^{n}_{i1,o}$
Transportation capacity	CT_{at}	Own consumption	$CP_{r,u,po}$
Transportation cost	TC_{at}	Minimum proportion	$PRPL^{n}_{r,u,pi,c}$
Distillation unit yield	$YDU_{r,u,c,o,po}$	Maximum proportion	$PRPU^{n}_{r,u,pi,c}$
Process unit yield	$YPU_{r,u,c,pi,po}$	Oil price - internal distribution	$OPBR^{n}_{r,o}$
Minimum capacity	$UCL_{r,u}$	**Stochastic parameters**	
Maximum capacity	$UCU_{r,u}$	Probability of scenario sc^t	P^{sc^t}
Sulfur quantity - entry product	SIO^{n}_{pi}	Domestic product demand	$PD^{n,sc^t}_{b,po}$
Maximum sulphur	$SPOU^{n}_{po}$	Product price - domestic market	$PPBR^{n,sc^t}_{b,po}$
Viscosity blending index	BI^{n}_{p}	Product export price	$PPE^{n,sc^t}_{in,po}$
Minimum viscosity	$VPOL^{n}_{po}$	Product import price	$PPI^{n,sc^t}_{in,po}$
		Oil import price	$OPI^{n,sc^t}_{in,o}$

4.1.2 Model formulation

Maximize

$$TM = \left\{ \begin{aligned} & -\sum_{r\in R}\sum_{o\in O}\sum_{n\in N}\left(OPBR^{n}_{r,o}\, qocf^{n}_{r,o}\right) \\ & +\sum_{sc^t\in SC^t} P^{sc^t}\left\{\sum_{b\in B}\sum_{po\in P}\sum_{n\in N}\left(PPBR^{n,sc^t}_{b,po}\, PD^{n,sc^t}_{b,po}\right)\right. \\ & \qquad +\sum_{n\in N}\sum_{in\in I}\sum_{po\in P}\left(PPE^{n,sc^t}_{in,po}\, pexp^{n,sc^t}_{in,po} - PPI^{n,sc^t}_{in,po}\, pimp^{n,sc^t}_{in,po}\right) \\ & \qquad -\sum_{n\in N}\sum_{in\in I}\sum_{o\in O}\left(OPI^{n,sc^t}_{in,o}\, oimp^{n,sc^t}_{in,o}\right) \\ & \qquad -\sum_{n\in N}\sum_{r\in R}\sum_{u\in U}\left(OC^{n}_{r,u}\sum_{o\in O}\sum_{c\in C} qdu^{n,sc^t}_{r,u,o,c}\right) - \sum_{n\in N}\sum_{r\in R}\sum_{u\in U}\left(OC^{n}_{r,u}\sum_{pi\in P}\sum_{c\in C} qpu^{n,sc^t}_{r,u,pi,c}\right. \\ & \qquad \left. -\sum_{at\in AT}\sum_{n\in N}\sum_{po\in P} pt^{n,sc^t}_{at,po}\, TC_{at} - \sum_{at\in AT}\sum_{n\in N}\sum_{o\in O} ot^{n,sc^t}_{at,o}\, TC_{at}\right\} \end{aligned}\right. \qquad \textbf{(4.9)}$$

Refining balance

$$qocf^{n}_{r,o} + vot^{n-1,sc^t}_{r,o} = \sum_{u\in U}\sum_{c\in C}\left(qdu^{n,sc^t}_{r,u,c,o}\right) + vot^{n,sc^t}_{r,o} \quad \forall r\in R, \forall o\in O, \forall n\in N, \forall sc^t\in SC^t \qquad \textbf{(4.10)}$$

$$\begin{aligned} & \sum_{u\in U}\sum_{c\in C}\sum_{o\in O} qdu^{n,sc^t}_{r,u,c,o}\, YDU_{r,u,c,o,po} + \sum_{u\in U}\sum_{c\in C}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,c,pi}\, YPU_{r,u,c,pi,po} + \sum_{pi\in P} b^{n,sc^t}_{r,pi,po} \\ & + ir^{n,sc^t}_{r,po} = \sum_{pi\in P} b^{n,sc^t}_{r,po,pi} + \sum_{u\in U}\sum_{c\in C} qpu^{n,sc^t}_{r,u,c,po} + \sum_{u\in U}\sum_{c\in C}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,c,pi}\, CP_{r,u,po} + or^{n,sc^t}_{r,po} \\ & \qquad \forall r\in R, \forall po\in PR, \forall n\in N, \forall sc^t\in SC^t \end{aligned} \qquad \textbf{(4.11)}$$

Refining operation constraints

$$PRPL^{n}_{r,u,pi,c}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,pi,c} \le qpu^{n,sc^t}_{r,u,pi,c} \le PRPU^{n}_{r,u,pi,c}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,pi,c}$$

$$\forall r\in R, \forall u\in U, \forall pi\in PR, \forall c\in C, \forall n\in N, \forall sc^t\in SC^t \qquad \textbf{(4.12)}$$

$$UCL_{r,u} \le \sum_{o\in O}\sum_{c\in C} qdu^{n,sc^t}_{r,u,o,c} + \sum_{pi\in P}\sum_{c\in C} qpu^{n,sc^t}_{r,u,pi,c} \le UCU_{r,u}$$

$$\forall r\in R, \forall u\in U, \forall n\in N, \forall sc^t\in SC^t \qquad \textbf{(4.13)}$$

Environmental legislation requirements

$$\sum_{u\in U}\sum_{pi\in P}\sum_{c\in C} qpu^{n,sc^t}_{r,u,c,pi}\, SIO^{n}_{pi}\, YPU_{r,u,c,pi,po} \le \left(\sum_{u\in U}\sum_{c\in C}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,c,pi}\, YPU_{r,u,c,pi,po}\right) SPOU^{n}_{po}$$

$$\forall r\in R, \forall po\in PR, \forall n\in N, \forall sc^t\in SC^t \qquad \textbf{(4.14)}$$

$$\begin{aligned} & \sum_{pi\in P}\left(b^{n,sc^t}_{r,pi,po}\, BI^{n}_{pi}\right) + \left(\sum_{u\in U}\sum_{c\in C}\sum_{o\in O} qdu^{n,sc^t}_{r,u,c,o}\, YDU_{r,u,c,o,po} + \sum_{u\in U}\sum_{c\in C}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,c,pi}\, YPU_{r,u,c,pi,po}\right) BI^{n}_{po} \\ & \le VPOL^{n}_{po}\left(\sum_{pi\in P} b^{n,sc^t}_{r,pi,po} + \sum_{u\in U}\sum_{c\in C}\sum_{o\in O} qdu^{n,sc^t}_{r,u,c,o}\, YDU_{r,u,c,o,po} + \sum_{u\in U}\sum_{c\in C}\sum_{pi\in P} qpu^{n,sc^t}_{r,u,c,pi}\, YPU_{r,u,c,pi,po}\right) \end{aligned} \qquad \textbf{(4.15)}$$

$$\forall r\in R, \forall po\in PR, \forall n\in N, \forall sc^t\in SC^t$$

Logistic balance

$$\sum_{(at,i2,m)\in ATA} pt_{at,po}^{n,sc^t} + or_{i1,po}^{n,sc^t} = PD_{i1,po}^{n,sc^t} + \sum_{(at,i2,m)\in ATA} pt_{at,po}^{n,sc^t} + ir_{i1,po}^{n,sc^t}$$

$$\forall i1 \in R \cup TR \cup OF, \forall po \in PR, \forall n \in N, \forall sc^t \in SC \quad \textbf{(4.16)}$$

$$\sum_{(at,i2,m)\in ATA} ot_{at,o}^{n,sc^t} + FP_{i1,o}^{n} = qocf_{i1,o}^{n} + \sum_{(at,i2,m)\in ATA} ot_{at,o}^{n,sc^t} \quad \textbf{(4.17)}$$

$$\forall i1 \in R \cup B \cup TR \cup NG \cup OF, \forall o \in O, \forall n \in N, \forall sc^t \in SC^t$$

Logistic capacity constraints

$$\sum_{po\in P} pt_{at,po}^{n,sc^t} + \sum_{o\in O} ot_{at,o}^{n,sc^t} \leq CT_{at} \qquad \forall at \in AT, \forall n \in N, \forall sc^t \in SC^t \quad \textbf{(4.18)}$$

$$qocf_{r,o}^{n}, b_{r,pi,po}^{n,sc^t}, dfr_{r,u,c,o}^{n,sc^t}, pfr_{r,u,c,po}^{n,sc^t}, oimp_{in,o}^{n,sc^t}, pexp_{in,po}^{n,sc^t}, pimp_{in,po}^{n,sc^t}, ir_{r,po}^{n,sc^t}, ot_{at,o}^{n,sc^t}, or_{r,po}^{n,sc^t}, pt_{at,po}^{n,sc^t} \in \Re^{+} \quad \textbf{(4.19)}$$

The objective function (4.9) maximizes the expected tactical margin. This margin includes the revenue from the product sales and the product exports, minus the raw material costs, the oil and product imports, the refining operation costs, and the transportation costs. The oil purchase ($qocf_{r,o}^{n}$) represents the first stage decisions. The second-stage decisions are the amount of product and oil transported ($pt_{at,po}^{n,sc^t}$ and $ot_{at,o}^{n,sc^t}$), the amount of oil imported ($oimp_{in,o}^{n,sc^t}$), and the amount of imported product and exported product ($pimp_{in,po}^{n,sc^t}$ and $pexp_{in,po}^{n,sc^t}$).

Equation (4.10) represents the oil balance, whereas equation (4.11) constitutes the product balance. For both of them the sum of the entry flows must be equal to the sum of the output flows. Equation (4.12) establishes the proportion between the entry flows (*pi*) and the total process unit (*u*) loading. The maximum and minimum capacities of the process unit *u* in period *n* are limited by equation (4.13).

Equations (4.14) and (4.15) limit the sulfur content (SIO_{pi}^{n}) and the viscosity (BI_{pi}^{n}) of the final products. Final product properties must be within a range established by environmental regulations. Property calculations yield a set of nonlinear constraints (Moro and Pinto, 2004) where the nonlinear terms arise from the multiplication between the products' properties and their volumes. These terms can be linearized by estimating the properties of intermediate products. At the tactical level it is possible to estimate the sulfur content $SIO_{pi,n}$ and the viscosity $BI_{pi,n}$ of the intermediate products with sufficient accuracy, making the constraint that controls the final products' properties linear. The tactical model controls only these two properties because they are the ones that affect most tactical decisions such as oil purchase and oil blending.

Logistic balance constraints (equations 4.16 and 4.17) determine that the sum of the input flows must be equal to the sum of the output flows for each node (*i*), product (*po*) or oil (*o*), period of time (*n*) and scenario (sc^t). ***ATA*** represents the set of transportation arcs (*at*) for a product (*po*) from an origin node (*i1*) to a destination node (*i2*) by a transportation mode (*m*). Equation (4.18) limits the maximum volume transported by the transportation arc (*at*) in the period *n*. Finally, constraints (4.19) define the non-negativity of the variables.

4.2. Operational model (for each refinery)

This section presents the mathematical formulation of the operational model for each refinery. This model is based on the multiperiod stochastic model of Neiro and Pinto (2005) that represented a refinery as a set of units connected by streams. The time periods are linked by inventory variables. The authors considered oil and product prices, as well as demand levels as uncertainty factors.

The main contributions of the operational model of this thesis are the presentation of a detailed formulation unlike the general formulation of Neiro and Pinto (2005), the modeling of uncertainty as oil supply and capacity of the process units, and the addition of a set of operational modes into the model. Besides the priorization of the production of a specific product set in the process units, as previously mentioned, the model also uses the operational modes to identify the streams stored in the tank units and the different consumer markets of product demand in the delivery units[2] of each refinery. The main variable of the model is then the flow rate of stream *s* between two units (*u,u'*) that operates in the given operational mode (*c,c'*) at each time period *t* under scenario sc^o ($q^{t,sc^o}_{u',c',s,u,c}$). These variables are only defined for a set of viable flows (***F***) between two pairs (*u*,*c*) and

[2] Each refinery has a delivery unit to represent the market demand that the refinery needs to meet.

(*u'*,*c'*) of a refinery, where the viable flows are determined by the refinery flowchart.

Other contribution is the inclusion of a decision variable to represent an option for additional oil purchase ($ca^{t}_{u,c,s}$). The oil supply for each refinery is defined by the long-term contracts and does not add cost to the operational model. However, as the oil delivery is subject to uncertainties (delays or changes in the oil specification), the operational model may decide by the purchase of additional oil in the spot market which implies in raw material costs.

The model aims to maximize the expected profit of the refinery subject to the following types of constraints:

- Demand constraints;
- Oil availability constraints;
- Material balances;
- Inventory requirements;
- Production capacity;
- Feed flow rate in process units;
- Properties of final products;
- Choose the operation modes in process units; and
- Capacity in the storage tanks.

Definitions are given in the section 4.2.1. and the model is presented in the section 4.2.2

4.2.1 Nomenclature

Table 7. Sets and variables of the operational model

Sets			
Set of constraints	K	Pipelines	$UD \subset U$
Set of refineries *(r)*	R	Tank units (storage and blending)	$UT \subset U$
Set of process units *(u, u')*	U	Storage units	$UA \subset UT$
Set of operational modes *(c, c')*	C	Blending units	$UM \subset UT$
Set of streams *(s, s')*	S	Tank units (storage and blending) by recipe	$UTR \subset U$
Set of properties *(p, p')*	P	Processing units (separation and conversion)	$UP \subset U$
Volumetric properties p of unit u	$PV_{u,c} \subset P$	Conversion units	$UPC \subset UP$
Mass properties p of unit u	$PM_{u,c} \subset P$	Separation units	$UPS \subset UP$
Density properties p of unit u	$PD_c \subset P$	**Variables**	
Time periods $\{n \mid n = 1,\ldots, T\}$	T	**First stage variables**	
Set of scenarios (sc^o)	SC^o	Quantity purchased of additional raw material s	$ca^t_{u,c,s}$
Operational modes c performed in unit u	$C_u \subset C$	**Second stage variables**	
Inlet streams s of unit u	$SI_{u,c} \subset S$	Flow rate of stream s between (u,c) and (u',c')	$q^{t,sc^o}_{u',c',s,u,c}$
Outlet streams s of unit u	$SO_{u,c} \subset S$	Inventory level of u	$vo^{t,sc^o}_{u,c}$
Tanks of raw material	$UC \subset UA$	Feed flow rate of unit u	$qi^{t,sc^o}_{u,c}$
Delivery units for final products	$UE \subset U$	Feed flow rate of stream s *at* unit u	$qis^{t,sc^o}_{u,c,s}$
Viable flows (u,c,s,u',c'), where the pair (u,c)	$F \subset S$	Outlet flow rate of stream s at unit u	$qo^{t,sc^o}_{u,c,s}$

Table 8. Parameters of the operational model

Parameters			
Product price	$PFP^t_{u,s}$	Initial stock of unit u	$VOLI^t_{u,c}$
Maximum of additional raw material	$QOCA^t_{u,c,s}$	Minimum storing capacity of unit u	$VOLL^t_{u,c}$
Demand	$DEM^t_{u,c,s}$	Maximum storing capacity of unit u	$VOLU^t_{u,c}$
Cost of additional raw material	$CFPA^t_{u,s}$	Minimum feed flow rate of u at mode c	$QIL^t_{u,c}$
Inventory cost	$CINV^t_{u,s}$	Maximum feed flow rate of u at mode c	$QIU^t_{u,c}$
Separation unit yield	$YUPS_{u,c,s,s'}$	Minimum feed flow rate of stream s	$QISL^t_{u,c,s}$
Conversion unit yield	$YUPC_{u,c,s}$	Maximum feed flow rate of stream s	$QISU^t_{u,c,s}$
Blending recipe of stream s	$RUT_{u,c,s}$	**Stochastic parameters**	
Property value of the initial stock	$POI^t_{u,c,p}$	Probability of scenario sc^o	P^{sc^o}
Lower bound of outlet property p of unit u	$POL^t_{u,c,p}$	Minimum feed flow rate of unit u	QL^{t,sc^o}_u
Upper bound of outlet property p of unit u	$POU^t_{u,c,p}$	Maximum feed flow rate of unit u	QU^{t,sc^o}_u
Estimated value of property p of the outlet stream s of unit ua - allows the model	$POE^t_{u,c,s,p}$	Quantity of oil from long-term contracts at tank unit u	$QOCF^{t,sc^o}_{u,c,s}$

4.2.2
Model formulation

Maximize

$$OM = \left(\begin{array}{l} -\sum_{c \in C_u} \sum_{u \in UC} \sum_{s \in SO_{u,c}} \sum_{t \in T} CFPA_{u,s}^{t} ca_{u,c,s}^{t} \\ + \sum_{sc^o \in SC^o} P^{sc^o} \left(\sum_{c \in C_u} \sum_{u \in UE} \sum_{s \in SI_{u,c}} \sum_{t \in T} PFP_{u,s}^{t} qis_{u,c,s}^{t,sc^o} - \sum_{u \in UA} \sum_{c \in C_u} \sum_{s \in SO_{u,c}} \sum_{t \in T} CINV_{u,s}^{t} vo_{u,c}^{t,sc^o} \right) \end{array} \right) \quad \textbf{(4.20)}$$

Process constraints and material balances

$$qi_{u,c}^{t,sc^o} = \sum_{(u',c',s) \in F} q_{u',c',s,u,c}^{t,sc^o} \qquad \forall u \in UP \bigcup UT \bigcup UD, \forall c \in C_u, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.21)}$$

$$qis_{u,c,s}^{t,sc^o} = \sum_{(u',c') \in F} q_{u',c',s,u,c}^{t,sc^o} \qquad \forall u \in UPS \bigcup UT \bigcup UD \bigcup UE, \forall c \in C_u, \forall s \in SI_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.22)}$$

$$qo_{u,c,s}^{t,sc^o} = \sum_{(u',c') \in F} q_{u,c,s,u',c'}^{t,sc^o} \qquad \forall u \in UP \bigcup UT \bigcup UD \bigcup UC, \forall c \in C_u, \forall s \in SO_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.23)}$$

$$vo_{u,c}^{t,sc^o} = vo_{u,c}^{t-1,sc^o} + qi_{u,c}^{t,sc^o} - \sum_{s \in SO_{u,c}} qo_{u,c,s}^{t,sc^o} \qquad \forall u \in UA, \forall c \in C_u, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.24)}$$

$$\sum_{s \in SO_{u,c}} qo_{u,c,s}^{t,sc^o} = qi_{u,c}^{t,sc^o} \qquad \forall u \in UD \bigcup UM, \forall c \in C_u, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.25)}$$

$$qo_{u,c,s}^{t,sc^o} = \sum_{s' \in SI_{u,c}} qis_{u,c,s'}^{t,sc^o} YUPS_{u,c,s,s'} \qquad \forall u \in UPS, \forall c \in C_u, \forall s \in SO_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.26)}$$

$$qo_{u,c,s}^{t,sc^o} = qi_{u,c}^{t,sc^o} YUPC_{u,c,s} \qquad \forall u \in UPC, \forall c \in C_u, \forall s \in SO_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.27)}$$

$$\sum_{(u',c') \in F} q_{u',c',s,u,c}^{t,sc^o} = RUT_{u,c,s} qi_{u,c}^{t,sc^o} \qquad \forall u \in UTR, \forall c \in C_u, \forall s \in SI_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.28)}$$

Demand constraints

$$qis_{u,c,s}^{t,sc^o} = DEM_{u,c,s}^{t} \qquad \forall u \in UE, \forall c \in C_u, \forall s \in SI_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.29)}$$

Process unit capacities

$$QIL_{u,c}^{t} \leq qi_{u,c}^{t,sc^o} \leq QIU_{u,c}^{t} \qquad \forall u \in UP \bigcup UT \bigcup UD, \forall c \in C_u, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.30)}$$

$$QISL_{u,c,s}^{t} \leq qis_{u,c,s}^{t,sc^o} \leq QISU_{u,c,s}^{t} \qquad \forall u \in UPS \bigcup UT \bigcup UD, \forall c \in C_u, \forall s \in SI_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.31)}$$

$$QL_{u}^{t,sc^o} \leq \sum_{c \in C_u} qi_{u,c}^{t,sc^o} \leq QU_{u}^{t,sc^o} \qquad \forall u \in UP \bigcup UT \bigcup UD, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.32)}$$

Plant supply constraints

$$0 \leq qo_{u,c,s}^{t,sc^o} \leq QOCF_{u,c,s}^{t,sc^o} + ca_{u,c,s}^{t} \qquad \forall u \in UC, \forall c \in C_u, \forall s \in SO_{u,c}, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.33)}$$

$$ca_{u,c,s}^{t} \leq QOCA_{u,c,s}^{t} \qquad \forall u \in UC, \forall c \in C_u, \forall s \in SO_{u,c}, \forall t \in T \quad \textbf{(4.34)}$$

Stock constraints

$$VOLL_{u,c}^{t} \leq vo_{u,c}^{t,sc^o} \leq VOLU_{u,c}^{t} \qquad \forall u \in UA, \forall c \in C_u, \forall t \in T, \forall sc^o \in SC^o \quad \textbf{(4.35)}$$

Property constraints

$$\left(vo_{u,c}^{t-1,sc^o}+qi_{u,c}^{t,sc^o}\right)POL_{u,c,p}^{t}\leq vo_{u,c}^{t-1,sc^o}POL_{u,c,p}^{t-1}+\sum_{(u',c',s)\in F}\left(q_{u',c',s,u,c}^{t,sc^o}\right)POE_{u,c,p}^{t}$$
$$\forall u\in UA,\forall c\in C_u,\forall p\in PV_{u,c},\forall t\in T,\forall sc^o\in SC^o \quad \textbf{(4.36)}$$

$$\left(vo_{u,c}^{t-1,sc^o}+qi_{u,c}^{t,sc^o}\right)POU_{u,c,p}^{t}\geq vo_{u,c}^{t-1,sc^o}POU_{u,c,p}^{t-1}+\sum_{(u',c',s)\in F}\left(q_{u',c',s,u,c}^{t,sc^o}\right)POE_{u,c,p}^{t}$$
$$\forall u\in UA,\forall c\in C_u,\forall p\in PV_{u,c},\forall t\in T,\forall sc^o\in SC^o \quad \textbf{(4.37)}$$

$$\left(vo_{u,c}^{t-1,sc^o}+qi_{u,c}^{t,sc^o}\right)POL_{u,c,p}^{t}POL_{u,c,p'}^{t}\leq vo_{u,c}^{t-1,sc^o}POL_{u,c,p}^{t-1}POL_{u,c,p'}^{t-1}$$
$$+\sum_{(u',c',s)\in F}\left(q_{u',c',s,u,c}^{t,sc^o}\right)POE_{u,c,p}^{t}POE_{u,c,p'}^{t}$$
$$\forall u\in UA,\forall c\in C_u,\forall p\in PM_{u,c},\forall p'\in PD_c,\forall t\in T,\forall sc^o\in SC^o \quad \textbf{(4.38)}$$

$$\left(vo_{u,c}^{t-1,sc^o}+qi_{u,c}^{t,sc^o}\right)POU_{u,c,p}^{t}POU_{u,c,p'}^{t}\geq vo_{u,c}^{t-1,sc^o}POU_{u,c,p}^{t-1}POU_{u,c,p'}^{t-1}$$
$$+\sum_{(u',c',s)\in F}\left(q_{u',c',s,u,c}^{t,sc^o}\right)POE_{u,c,p}^{t}POE_{u,c,p'}^{t}$$
$$\forall u\in UA,\forall c\in C_u,\forall p\in PM_{u,c},\forall p'\in PD_c,\forall t\in T,\forall sc^o\in SC^o \quad \textbf{(4.39)}$$

$$ca_{u,c,s}^{t},q_{u',c',s,u,c}^{t,sc^o},qis_{u,c,s}^{t,sc^o},qi_{u,c}^{t,sc^o},qo_{u,c,s}^{t,sc^o},vo_{u,c}^{t,sc^o}\in\Re^{+} \quad \textbf{(4.40)}$$

The objective function (4.20) maximizes the expected operating margin. This margin includes the revenue from the products sales minus the raw materials costs and the inventory costs. The oil supply for each refinery is defined by long-term contracts and is represented by the stochastic parameter ($QOCF_{u,c,s}^{t,sc^o}$). The oil purchase in the spot market ($ca_{u,c,s}^{t}$) constitutes the first stage decisions. The second-stage decisions are related to the refinery operations such as flows between units ($q_{u',c',s,u,c}^{t,sc^o}$) and inventory level ($vo_{u,c}^{t,sc^o}$).

Equation (4.21) describes the mass balance at the inlet stream of the unit *u* ($qi_{u,c}^{t,sc^o}$). Equation (4.22) represents the mass balance at the inlet stream *s* of the unit *u* ($qis_{u,c,s}^{t,sc^o}$). Equation (4.23) describes the mass balance at the outlet stream *s* of the unit *u* ($qo_{u,c,s}^{t,sc^o}$). The stock balance in the storage unit UA is represented by equation (4.24), where $vo_{u,c}^{t-1,sc^o}=VOLI_{u,c}$ when *t=1*. Equation (4.25) corresponds to the mass balance for the blending units UM and pipelines UD because in both units there is no stock, so the sum of the inlet streams must be equal to the sum of the outlet streams.

Equation (4.26) describes the process in the separation unit UPS, where the outlet flow rate of stream *s* ($qo_{u,c,s}^{t,sc^o}$) is a function of the feed flow rate of stream *s'* (oils) and its yield. Equation (4.27) describes the process in the conversion unit UPC, where the outlet flow rate of stream *s* ($qo_{u,c,s}^{t,sc^o}$) is a function of the feed flow rate and the conversion yield. Equation (4.28) determines the blending recipe of the feed flow rate of stream *s* for the tank unit by recipe UTR as a function of the feed flow rate of the tank unit UTR ($qi_{u,c}^{t,sc^o}$) and the blending proportion of the inlet streams ($RUT_{u,c,s}$). This constraint requires that the volume of each inlet stream must be a predefined fraction of the total feed flow rate.

Equation (4.29) limits the inlet flow rate for the final products in the delivery units UE. Equation (4.30) restricts the feed flow rate of each unit *u* for each operational mode *c*. Equation (4.31) limits the inlet flow rate of stream *s* for the unit *u* and for each operational mode *c*. Equation (4.32) controls the feed flow rate of the unit *u*.

Equation (4.33) limits the outlet flow rate for raw material tanks UC. Fixed (long-term contracts) and additional (spot market) raw materials are available. The refinery may decide to keep some fixed raw material in the storage units and purchases the additional raw material necessary for its operation through the first stage variable $ca_{u,c,s}^{t}$. As the purchase of additional raw material adds costs to the model, the refinery decides by the purchase of additional oil only if the amount of fixed oil received is not enough to meet the market demand or if the type of oil received is not good to meet the quality standards on the refined products. Equation (4.34) limits the additional raw materials available for purchase. Equation (4.35) represents the inventory level for product tanks at each time period *t* under scenario sc^o.

Equations (4.36) to (4.39) refer to properties of the streams in the storage units UA considering the stock at the time interval before *t-1* and the inlet flow rate at the time interval *t* (where $vo_{u,c}^{t-1,sc^o} = VOLI_{u,c}$ and $POL_{u,c,p}^{t-1} = POU_{u,c,p}^{t-1} = POI_{u,c,p}$, when *t=1*). Like in the tactical model, property calculations were linearized by estimating the value of property *p* of the outlet stream *s* ($POE_{u,c,p}^{t}$). Whereas the first two constraints control the volumetric properties (such as viscosity), the last two constraints control mass properties (such as sulfur) in the outlet streams of the storage unit UA. The mass properties are converted to volume through the multiplication of the mass property by the density of the streams. Finally, equations (4.40) define the non-negativity of the variables.

4.3. Numerical example

An industrial scale study using real data from the Brazilian industry was used to evaluate the performance of the proposed models in optimizing large-scale problems. At the tactical level, the refining system includes 3 refineries (named R1, R2, and R3) and represents a general system that can be found in many industrial sites around the world. The refineries are coordinated centrally, the feedstock oil supply is shared, and the refineries collaborate to meet a given market demand. The refineries are supplied by 7 groups of national oils produced in 2 exploitation fields, and 1 group of foreign oils. The refineries process up to 50 intermediate products to produce 10 final products associated to the local market demand. The logistic network includes 2 domestic and 4 international terminals, 2 distribution bases, and 73 transportation arcs relative to road, water, rail, and pipeline modes. The time horizon in the tactical level covers 6 monthly periods.

The information in the tactical model is more aggregated than in the operational model. Whereas the former considers families of oils and products, the latter differs between oils and products within a family. The classification of the available oils is shown in Table 9. The types of oil of each family were aggregated according to their characteristics (API index, yields, and properties) and their production region, following the methodology defined by EPE (2007). The group E represents the imported oils. The other groups are formed by national oils.

Table 9. Classification of the available oils

Oil family	Types of oil	Classification
A	A1, A2, A3	Heavy
B	B1	Heavy
C	C1, C2,C3	Medium
D	D1	Medium
E	E1, E2, E3, E4, E5, E6	Light
F	F1	Medium
G	G1	Medium
H	H1, H2, H3, H4, H5	Heavy

The modeling of the three refineries (R1, R2, and R3) is detailed at the operational level. R1 is a small and low complex refinery which focuses on the production of lubricants, asphalt, and fuel oils. This refinery is supplied by three types of national oils (named here as A1, A2, and A3). As presented in Table 10, R1 processes up to 53 intermediate products with 8 properties that need to be controlled to specify the 17 final products. R2 can also be considered a low complexity refinery that aims at the production of solvents and fuels and processes 4 types of oils grouped in three families. Finally, R3 is a medium complexity refinery and processes oils from 7 families with the focus on the production of naphtha, but also has significant production of jet fuel, diesel, and gasoline. The main characteristics of R2 and R3 are also shown in Table 10. The time horizon analyzed at the operational level covers the 2 first monthly periods of the tactical model (where t1 = 30 days and t2 = 31 days).

Table 10. Main characteristics of the three refineries studied

	R1	R2	R3
Types of oil processed	A1, A2, A3	C1, C2 D1 E1	A2 B1 C1, C3 E2, E3, E4, E5, E6 F1 G1 H1, H2, H3, H4, H5
Capacity (m^3/day)	1,100	6,550	39,400
#Process units	3	9	10
#Tanks	30	27	51
#Operational modes	48	52	141
#Controlled properties	8	68	56
#Viable flows	129	117	336
#Intermediary flows	53	57	190
#Final products	17	18	17

4.3.1 Scenario generation

The method used to create the scenarios of the stochastic models was based on data collection and direct contraposition of primary (data obtained from the oil Brazilian oil industry) and secondary research (historical economic data available online). Developing methodologies for scenario generation is beyond the scope of this thesis, and the interested reader can refer to the work by Kouwenberg (2001), for example. As it is essential to test the proposed models, the scenario generation with the associated probabilities was arbitrated in consistency with the real problem studied and validated with experts of the oil industry.

Table 11 shows the probability of each possible realization of the stochastic uncertainty. The demand for refined products, oil prices, and product prices are mid-term uncertainties which are considered in the tactical planning, whereas oil supply and process capacity unit address the short-term uncertainties in the operational planning.

Table 11. Probabilities of the stochastic parameters

Model	Stochastic Parameter	Realizations	Probability
Tactical	Demand	High	25%
		Base	***50%***
		Low	25%
	Price	High	25%
		Base	***50%***
		Low	25%
Operational	Maintenance	3 days	25%
		0 days	***50%***
		5 days	25%
	Oil Supply	***Normal***	***70%***
		Delays/ changes	30%

Each stochastic parameter at the tactical level (price and demand) has three possible realizations (high, medium, and low). Assuming that the random variables are independent, these parameters were combined to create the scenarios presented in Figure 6. It is assumed complete dependence between the parameters of each scenario, i.e., high demand of one product implies in high demand of the other products. Similar pattern is presented to oil and product prices.

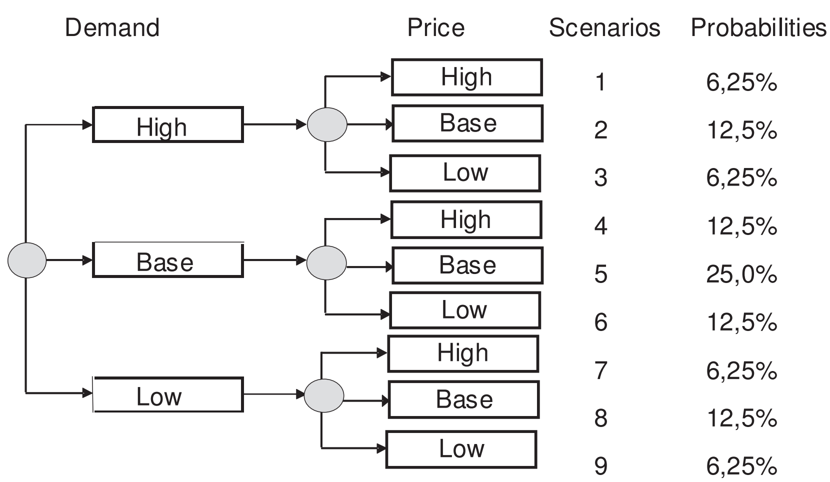

Figure 6. Tactical scenario tree

At the operational level, the available capacity in the process units has three possible realizations that consider 3, 0, or 5 days of unplanned shutdown for maintenance, affecting the total unit capacity available. As the planned stops are already considered in planning for the decision maker, they do not represent a type of uncertainty for the problem. Oil supply, the other operational uncertainty, has two possible realizations: normal supply or delays/changes in the oil received. In the first period of planning, the uncertainty is represented by a change in the oil received and, consequently, the oil specifications and yields. In the second period, the uncertainty in the oil supply is represented by a delay in the amount of oil received reducing the total available oil to 1/3. Combining the realization of these stochastic parameters (capacity and oil supply) resulted in six scenarios for each refinery as can be seen in Figure 7.

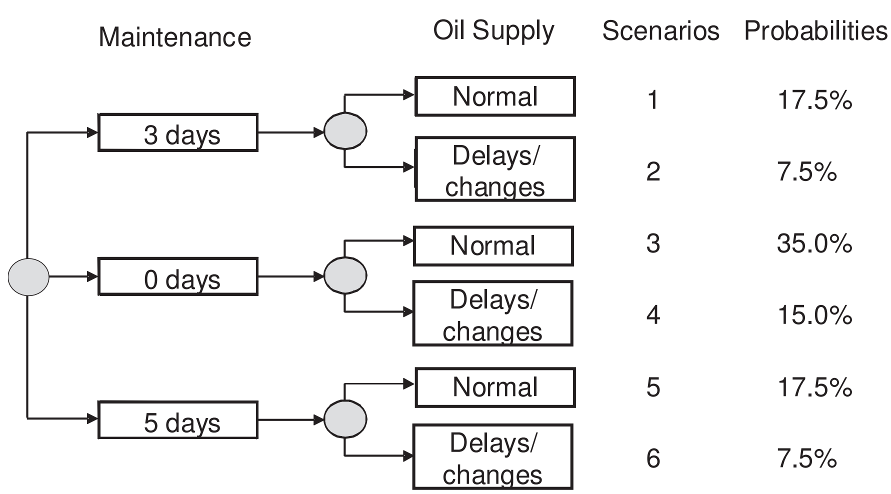

Figure 7. Operational scenario tree

The base case of each planning level (scenario 5 for the tactical and scenario 3 for the operational level) used data from the current planning system of Brazilian refineries. This system addresses only a deterministic case which was used to generate the base case. The other scenarios were constructed based on the expertise of employees of the industry under study.

4.3.2
Computational results and discussion

The models were implemented using the Advanced Integrated Multidimensional Modeling Software - AIMMS (Bisschop and Roelofs, 2007) and solved using the CPLEX 12.1 solver. A PC using an Intel Core 2 Quad processor at 3.1GHz with 8Gb RAM was used for all computations described in this thesis. Table 12 summarizes the model statistics:

Table 12. Model statistics

Planning levels	**Tactical**	**Operational**			
		R1	**R2**	**R3**	**Total**
#Variables	96,899	5,066	6,050	14,520	25,634
#Constraints	119,105	5,222	7,070	14,232	26,522
#Non zeros	218,286	14,893	20,695	44,175	79,761
Solving time (s)	0.78	0.03	0.05	0.06	0.14
E[margin] (million $)	707.9	27.2	123.9	292.9	444.0

The stochastic models (tactical and operational) maximize the expected margin at each planning level (E[margin]). In the sequel, first the solution of the tactical model is described, and then, the operational model solution is presented. The section ends with some analysis of the solutions obtained.

- **Tactical model solution**

As illustrated by Figure 8, the tactical margin for the two first periods (that are also covered by the operational model) reaches $256 million which corresponds to 57.61% of the total operational margin showed in Table 12. Besides modeling the refineries in less detail than at the operational level, the tactical level considers logistics costs and import and oil purchase costs that are not considered at the lowest level. Thus, the oil supply defined by the long-term contracts is considered only in the tactical model, not at the operational model. The operational model considers only the cost of complementary (additional) oil purchase in the spot market.

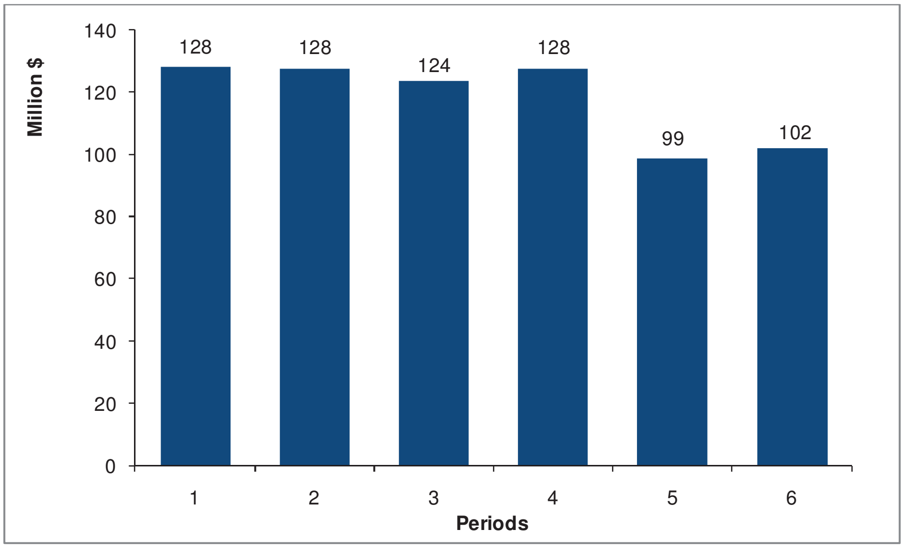

Figure 8. Tactical margin solutions by period

As shown in Figure 9, the best results for the tactical model were found in the scenarios with high prices. This finding indicates the model's sensitivity to the uncertain parameters and that the prices uncertainty had a greater impact on total profit than the demands uncertainty had.

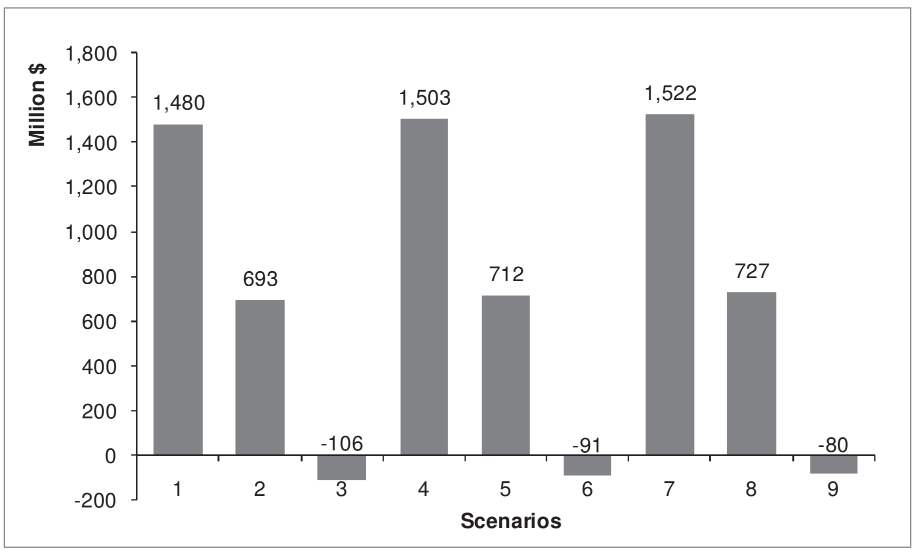

Figure 9. Tactical margin solutions by scenario

The tactical oil purchase decisions for the 6 periods of planning (n), defined by the first stage variable $qocf_{r,o}^{n}$, are presented at Figure 10. The legend represents the refinery/oil family allocated to the refinery.

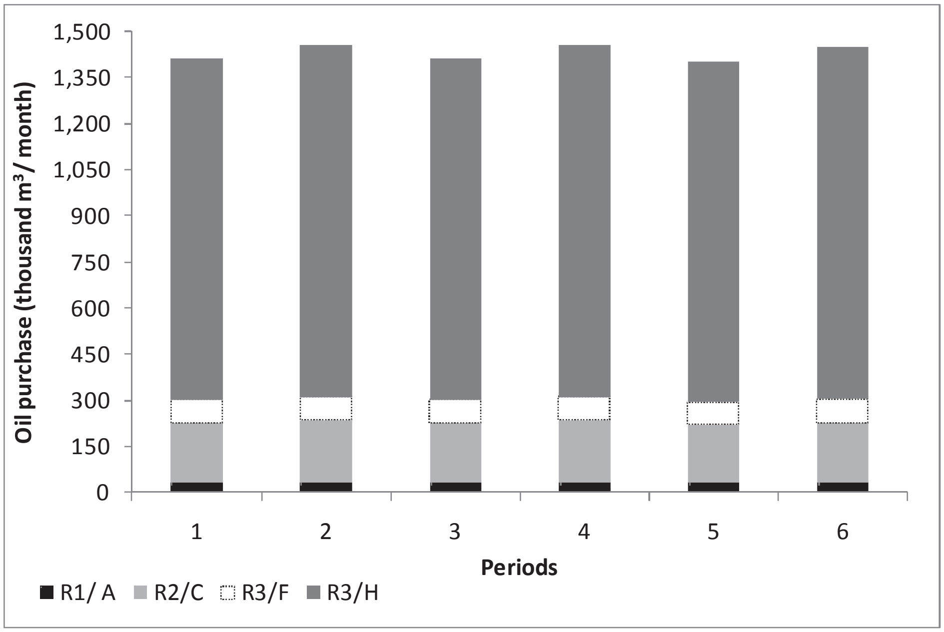

Figure 10. Tactical oil purchase decisions

Refinery R3 is responsible by 83.90% of the total oil purchases presented in Figure 10. Two oil families are allocated to R3. This refinery processes the entire amount available of the oil F (medium) and completes the maximum capacity level with heavy oil (H). In addition, 13.76% of total oil quantity is allocated to the oil family C (medium) at refinery R2. Finally, the tactical model attributes the last 2.34% of oil to the family A (heavy) for the lubricant production at R1.

- **Operational model solution**

The operational model receives the oil supply defined by the first two periods of the tactical planning through the parameter ($QOCF_{u,c,s}^{t,sc^o}$) where u identifies the tank of raw material of each refinery and c identifies that the oil received is from long-term contracts. The communication between the two models will be discussed in the chapter 6. For now, consider that the oil supplied to refinery R1 is allocated to oil type A1, oil C is allocated to type C1 in R2, and oils F and H are allocated, respectively, to types F1 and H1 in R3. These oil allocations are done for the base case (scenario 3). The other scenarios are constructed by variations from the base case.

Oils type A1, A2, and A3 can be purchased at the spot market (additional oil) to face uncertainties in the oil supply for the refinery R1. The model solutions for R1 show some additional oil purchase (A1) in period 1 (Table 13). This result is a protection for the changes in the oil specifications in period 1, when R1 receives the oil type A2 instead of A1. Oil A1 is the most appropriate oil to meet the demand and capacity constraints in R1, while avoiding large inventories of oil in the scenarios presenting a normal oil supply. The oil A2 is less expensive than A1 but it is more difficult to respect the specifications constraints due to its lower quality. Oil A3 is more expensive than A1 and A2 and the best quality oil among the three types supplied to the refinery R1. Thus, the model solution recommends only the purchasing of oil A1. Furthermore, the additional oil also helps meeting the product demand in the second period, when scenarios with reduction to 1/3 of the amount of oil received are considered.

Refinery R2 is supplied by the oil type C2 instead of C1. Despite of belonging to the same family, oil C1 has better properties than C2 (with low sulfur content) and better production yields of the products with higher added value. As a result, the model solution recommends the additional purchase of oil D1, that has also good quality and helps to meet the demand, and E1, that is a better quality oil than C1, C2, and D1, and can be used as solvent to ensure the blending specification in the scenarios in which oil quality is low (C2). Similar solution is found to refinery R3 which buys the heavy oils B1 and the ones of family H because of the delays in quantity of oil received, besides of C3 that is a better quality oil than B1 and oils type H and helps to specify the final products.

Table 13. Solution for the oil purchase at the spot market

Refinery	Type of oil	Additional oil (thousand m^3/ month)	
		t=1(30 days)	t=2 (31 days)
R1	A1	10.0	-
R2	D1	6.7	-
	E1	40.0	26.1
R3	B1	60.0	60.0
	C3	50.4	10.4
	H1	81.1	132.3
	H2	240.0	-
	H3	45.0	45.0
	H4	45.0	45.0

Figure 11, 12, and 13 present the profit per scenario at the operational level for each refinery. The solution profile follows the variation on the stops for maintenance which affects the available production capacity (3 days for the scenarios 1 and 2; 0 days for the scenarios 3 and 4; and 5 days for the scenarios 5 and 6 – as shown in Figure 7). The oil supply uncertainty is represented by the difference between two scenarios that consider the same number of stop days for maintenance (for example, $2 million for scenarios 1 and 2 of R1). As expected, the best margin is found in the scenario 3 of each refinery in which there is no stop for maintenance and normal oil supply, according to the scenario tree of Figure 7. On the other hand, the worst result is found in the scenario 6 of each refinery in which delays/ changes and the longest operation stop for maintenance (5 days) affects the refinery capacity reducing the capacity level available for its operation. In refinery R3, the scenario 5 is lower than scenario 6 due to the inventory costs incurred in keeping oil in stock because the reduction in the available process unit capacities.

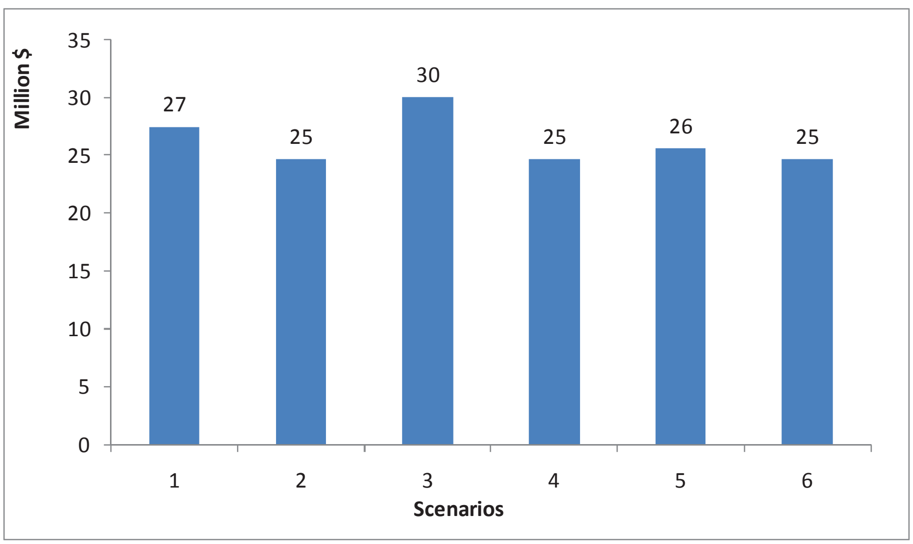

Figure 11. Operational margin solutions by scenario for R1

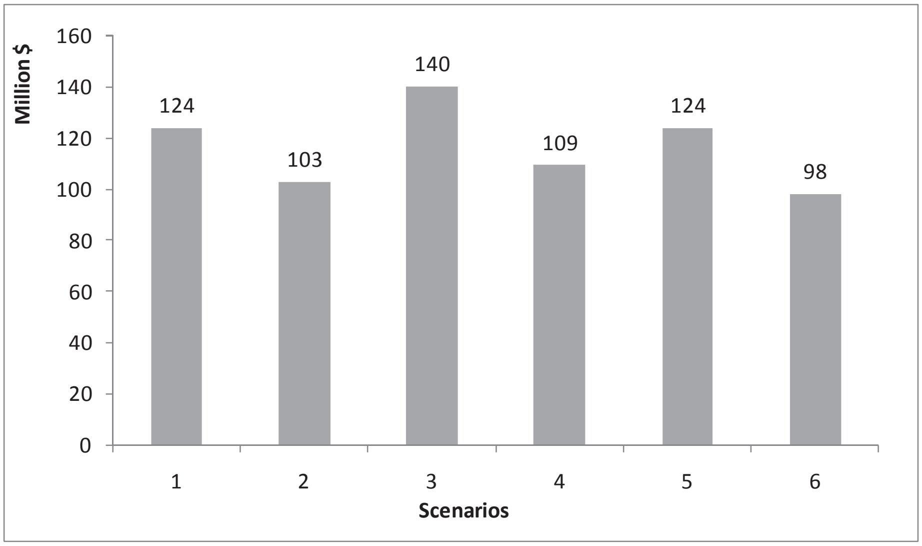

Figure 12. Operational margin solutions by scenario for R2

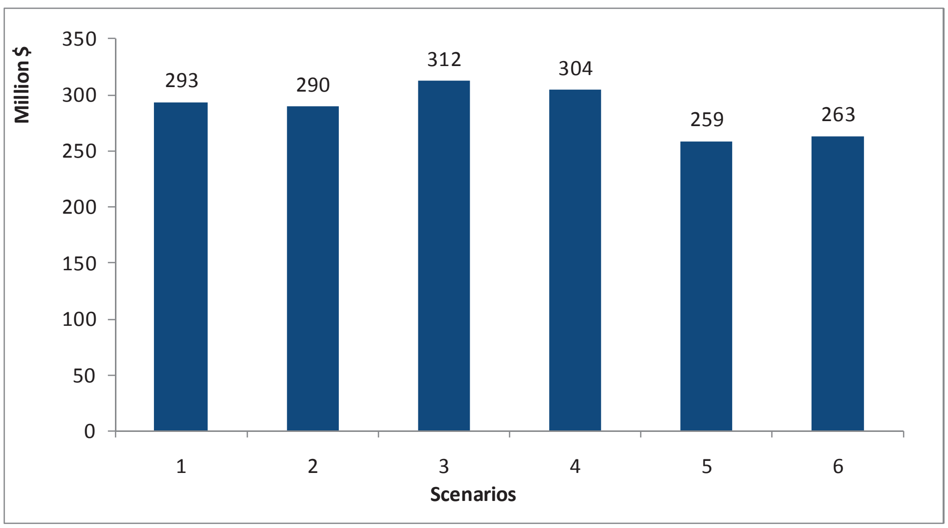

Figure 13. Operational margin solutions by scenario for R3

- **Analysis**

As the expected margin at each planning level depends on the probabilities associated to the scenarios, a sensitivity analysis was conducted to evaluate the total margin variation when the probabilities change. Table 14 presents the probabilities used in the 4 (four) cases of the analysis. Case 1 considers 5% increase on the probabilities of high prices/ demands, 3 days of maintenance, and normal oil supply, whereas case 2 considers 5% increase on the probabilities of low prices/ demands, 5 days of maintenance, and delayed/changed oil. Cases 3 and 4 consider, respectively, 10% increase and decrease in the base probability of each stochastic parameter.

Table 14. Cases of the sensitivity analysis of the stochastic parameters probabilities

Stochastic Parameter	Realizations	Original probability	Case 1	Case 2	Case 3	Case 4
Demand	High	25%	***30%***	20%	25%	25%
	Base	50%	50%	50%	***60%***	***40%***
	Low	25%	20%	***30%***	15%	35%
Price	High	25%	***30%***	20%	25%	25%
	Base	50%	50%	50%	***60%***	***40%***
	Low	25%	20%	***30%***	15%	35%
Maintenance	3 days	25%	***30%***	20%	25%	25%
	0 days	50%	50%	50%	***60%***	***40%***
	5 days	25%	20%	***30%***	15%	35%
Oil Supply	Normal	70%	***75%***	65%	***80%***	***60%***
	Delays/ changes	30%	25%	***35%***	20%	40%

As presented in Table 15, the tactical model is more sensitive to the changes in the probabilities than the operational model is. The operational margin reached up to $454.6 million that is 2.33% higher than the original solution. The tactical model, on the other hand, reached solutions 11.13% lower/bigger than the original one which means that more attention should be paid to the price and demand uncertainties at the tactical level. The use of methodologies for scenario generation may lead to great benefits in this regard.

Table 15. Sensitivity analysis solutions

Cases	Tactical		Operational	
	E[margin] (million $)	**Δ%**	**E[margin]** (million $)	**Δ%**
Case 1	785.9	+11.01	447.5	+0.79
Case 2	629.9	-11.02	440.6	-0.77
Case 3	786.7	+11.12	454.6	+2.38
Case 4	629.1	-11.13	434.3	-2.20

In this numerical study, the EVPI reaches a maximum of 1.55% of the wait-and-see solution for the tactical case and 7.28% for the operational case, as can be seen in Figures 14 and 15. The EVPI result shows the difference between the solution of the problem in which the oil purchase decisions are postponed until that the uncertainty is unfolded (wait-and-see) and the solution of the stochastic problem (recourse problem). The lower the EVPI, the better the stochastic models accommodate uncertainties. In addition, the operational solution using the average of the random variables is infeasible due to the minimum capacity of distillation unit constraint which would lead to an operational VSS of 100%. This finding indicates that incorporating uncertainty into the problem could avoid infeasibilities at the operational level. So, these results highlight the benefit of incorporating uncertainty in the different model parameters of the oil chain.

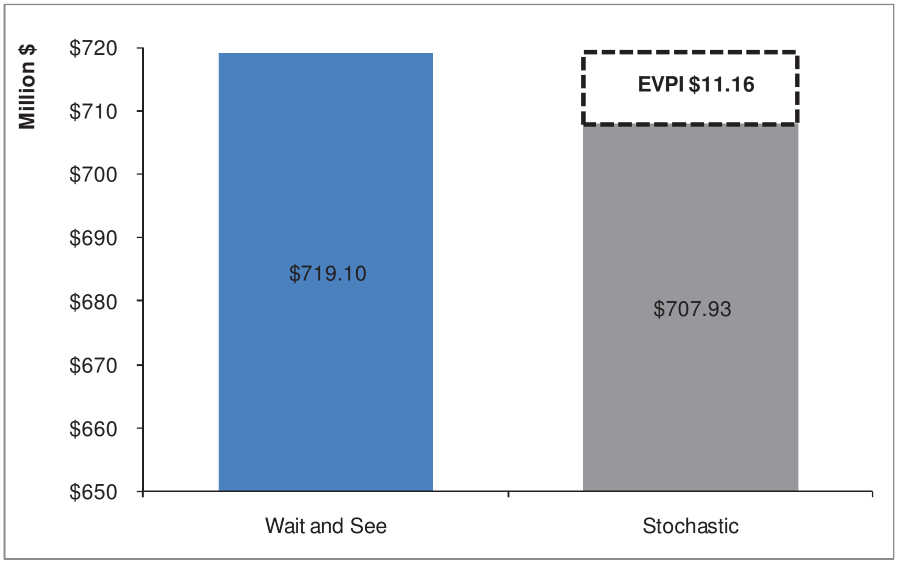

Figure 14. EVPI of the tactical model

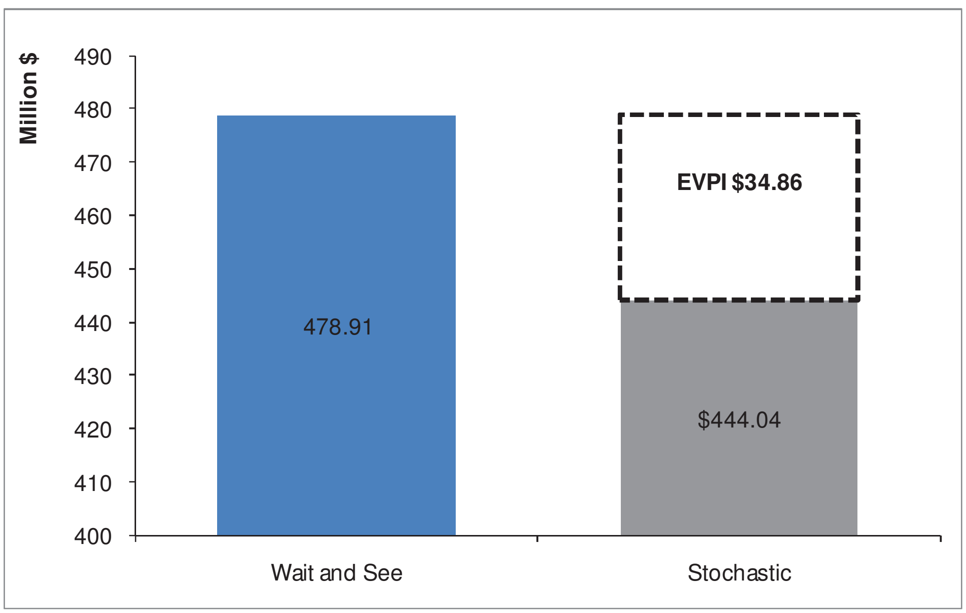

Figure 15. EVPI of the operational model

4.4. Chapter conclusions

The previously presented results indicate that stochastic formulations may lead to a more suitable solutions to the real problem. The benefits of incorporating uncertainty in the problem could be measure by the EVPI and VSS. However, as stated by Candler and Townsley (1978), large errors can be introduced by ignoring the hierarchical planning structure (i.e., solving the two models separately). For example, the operational model may need to buy a large amount of additional oil incurring in high costs. This additional purchase means that the oil allocation determined by the tactical solution was changed with implications to the oil supply and logistical constraints at the tactical level. In addition, problems of information integrity may also exist as the operating model may not receive the properly information allocated by the tactical model. Thus, it is necessary to consider the holistic issue of the problem.

The need for the integration of the tactical and the operational models is discussed in the next chapter.