

## 6 Integrated Planning under Uncertainty of the Oil Chain

In this chapter the tactical and the operational models presented in Chapter 4 are integrated using the *hierarchical* and the *iterative* approaches described in Chapter 5. An attempt of implementation of the third approach (*full-space*) using bilevel programming was conducted. The Karush-Kuhn-Tucker (KKT) optimality conditions of the operational (slave) model were added to the tactical (master) model, resulting in a single level optimization problem – see Appendix A for the KKT conditions. However, due to the nonlinearities introduced through the complementarity constraints of the KKT conditions<sup>3</sup>, the models could not be solved under this approach. As future research, methods to deal with this kind of additional complexity in large scale problems can be studied.

In this chapter, first, the communication between the two planning levels is discussed. The two integrated modeling approaches of the oil supply chain are then explained in section 6.2. Finally, a numerical study was conducted to compare the results of the *hierarchical* and *iterative* methods.

### 6.1. Integration of the tactical and the operational models

When the communication between the tactical and the operational models described in the Chapter 4 is established, the parameter of the operational model that defines the quantity of oil  $s$  received from long-term contracts ( $QOCF_{u,c,s}^{t,sc^o}$ ) becomes a variable (named as  $qocfOP_{u,c,s}^{t,sc^o}$ ) which allows the operational model to choose the best oil allocation to process at each refinery. This choice is made under the constraint that the total amount allocated to all types of oils  $s$  from a tactical oil family  $o$  must be equal to the quantity supplied to that family  $o$ .

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<sup>3</sup> For additional information about the formulation of the KKT conditions, the interested reader can refer to Karush (1939) and Kuhn and Tucker (1951).

Each refinery has tanks of raw material  $uc$  where the oils from long-term contracts (tactical allocation) are stored. Due to the operational uncertainties in the oil supply (delays/ changes), however, the quantity or quality of the oils received in the raw material tanks at the operational level ( $qocfOP_{u,c,s}^{t,sc^o}$ ) may be different the ones allocated by the tactical model. Thus, additionally, an auxiliary variable (named as  $aux\_qocfOP_{u,c,s}^t$ ) is defined to receive the value of the tactical variable  $qocf_{r,o}^n$  (the oil supplied by long-term contracts). Then, the variable  $qocfOP_{u,c,s}^{t,sc^o}$  (amount received in the raw material tanks at the operational level) for each scenario is defined by variations of this auxiliary variable according to the uncertainty in each scenario.

So, let  $sbo_{o,s}$  a parameter that defines if the type of oil  $s$  belongs to the oil family  $o$ . The breakdown of information from the tactical to the operational level is defined by the allocation of the oil supplied to a tactical oil family among the types of oil that belongs to that family (according to the classification presented in Table 9). Consider, for instance, the refinery R1 presented in the example of chapter 4 and the first planning period for both models ( $n=t=1$ ). So, the breakdown of information can be stated by constraint (6.1).

$$\sum_{n=1 \in N} \sum_{r=R1 \in R} qocf_{r,o}^n = \sum_{t=1 \in T} \sum_{s \in S} \sum_{u=uc_1 \in UC} \sum_{c=c_1 \in C_u} aux\_qocfOP_{u,c,s}^t sbo_{o,s} \quad \forall o \in O \quad (6.1)$$

The left-hand-side constraint defines the total amount of an oil family supplied to a refinery in a period. The right-hand-side constraint establishes the allocation for the types of oils within a family. The same calculus could be done for the other refineries (R2 and R3) and other period (2).

As defined by constraint (6.1), the auxiliary variable does not depend on the scenarios. To represent the uncertainties that affect the oil supply, additional constraints (6.2 to 6.4) define the quantity/type of oil effectively received at the raw material tanks of the refineries. Constraint (6.2) defines that the refinery receives the exact tactical allocation in the scenarios with normal supply (the odd scenarios, as defined in the Chapter 4 of this thesis). Constraint (6.3) and (6.4) define the scenarios in which uncertainties affect the oil supply (the even scenarios, as defined in the Chapter 4). Constraint (6.3) refers to the scenarios with changes

in the oil received, where the parameter  $change_{s',s}$  defines a change from an oil  $s'$  to another  $s$ . Finally, constraint (6.4) shows that the uncertainty in the quantity of oil received reduces the total available oil to 1/3. In the numerical example of Chapter 4, constraints (6.3) and (6.4) were considered for periods 1 and 2, respectively. In the present chapter, these constraints are used separately in different case studies.

$$qocfOP_{u,c,s}^{t,sc^o} = aux\_qocfOP_{u,c,s}^t \quad \forall u \in UC, \forall c \in C_u, \forall s \in SO_{u,c}, \quad (6.2)$$

$$\forall t \in T, \forall sc^o \in SC^o \mid sc^o = 1,3,5$$

$$qocfOP_{u,c,s}^{t,sc^o} = \sum_{s' \in SO_{u,c}} (aux\_qocfOP_{u,c,s'}^t \cdot change_{s',s}) \quad \forall u \in UC, \forall c \in C_u, \forall s \in SO_{u,c}, \quad (6.3)$$

$$\forall t \in T, \forall sc^o \in SC^o \mid sc^o = 2,4,6$$

$$qocfOP_{u,c,s}^{t,sc^o} = \frac{aux\_qocfOP_{u,c,s}^t}{3} \quad \forall u \in UC, \forall c \in C_u, \forall s \in SO_{u,c}, \quad (6.4)$$

$$\forall t \in T, \forall sc^o \in SC^o \mid sc^o = 2,4,6$$

## 6.2. Integrated modeling approaches of the oil chain

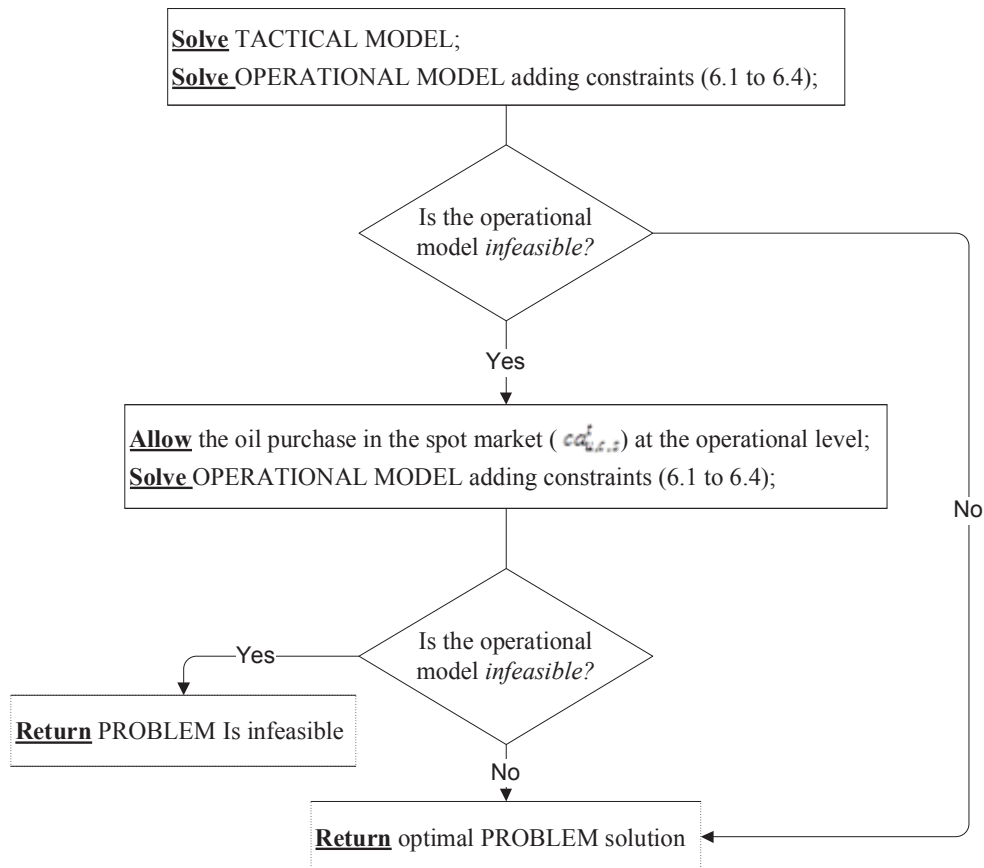
This section presents the two integration approaches proposed in this thesis. As the nature of uncertainty is different at each planning level, medium-term exogenous uncertainties (price and demand) are considered only in the tactical model and short-term endogenous uncertainties (oil supply and process capacity unit) are considered only in the operational model which means that the time of the decisions of the two models is different. First, the tactical model finds an optimal stochastic solution and, once the uncertain events have unfolded (which means that the price and demand patterns are now known and the oil purchase from long-term contracts -  $qocf_{r,o}^n$  - has already been defined), the operational model makes the necessary adjustments to face the short term uncertainties through the oil purchase at the spot market ( $ca_{u,c,s}^t$ ).

It is important to highlight that the oil purchase at the spot market is not foreseen in the tactical model, so this purchase can make the tactical solution infeasible by logistics constraints or oil supply constraints. In this regard, the solution strategies for the integrated approaches (hierarchical and iterative) evolve in the sense of trying to eliminate the oil purchase in the spot market ( $ca_{u,c,s}^t$ ) at the

operational level. In the first step of both integration approaches, the additional oil purchase ( $ca'_{u,c,s}$ ) is not allowed in the operational model. However, in the remaining steps the variable  $ca'_{u,c,s}$  may act as a slack variable to avoid the infeasibility of the operational model (if the amount or quality of the fixed oil received is not enough to meet the market demand). In this case, the cost of additional raw material is more expensive than in the case that the oil purchase is considered by the tactical planning through the variable  $qocf_{r,o}^n$ . The solution strategies are discussed in the next sections.

### 6.2.1 Hierarchical approach

In the hierarchical approach, the tactical and the operational models are solved successively and there is no feedback from the slave (operational) to the master (tactical) model. The solution procedure for the hierarchical method is summarized in the Figure 18. The proposed heuristic consists in allowing the additional oil purchase at the operational level in the case that the operational model is infeasible. This purchase works as a slack variable when the operational model infeasibility is caused by the amount/quality of the oil(s) allocated by the tactical model. Then, the operational model is solved again and the procedure terminates returning an optimal solution or an infeasible solution for the integrated problem under the hierarchical approach (where a solution of the integrated problem is composed by a solution for the tactical model and a solution for the operational model). The operational model gains feasibility with the additional oil purchase, but this purchase may lead to an infeasible tactical solution (due to logistical and commercial constraints). The iterative method presented in the next section tries to overcome this challenge by considering feedbacks from the operational to the tactical model.



**Figure 18.** Solution procedure in the hierarchical approach

### 6.2.2 Iterative approach

Unlike the hierarchical approach, the iterative approach considers a feedback loop from the slave back to the master model. The solution procedure in the iterative method is an extension of the one presented in Figure 18 and consists of trying to eliminate the oil purchase in the spot market by fixing the additional oil solution of an iteration as part of the oil allocation of the tactical model in the next iteration. The idea is to force the tactical level to buy the oils that the operation needs to meet the demand.

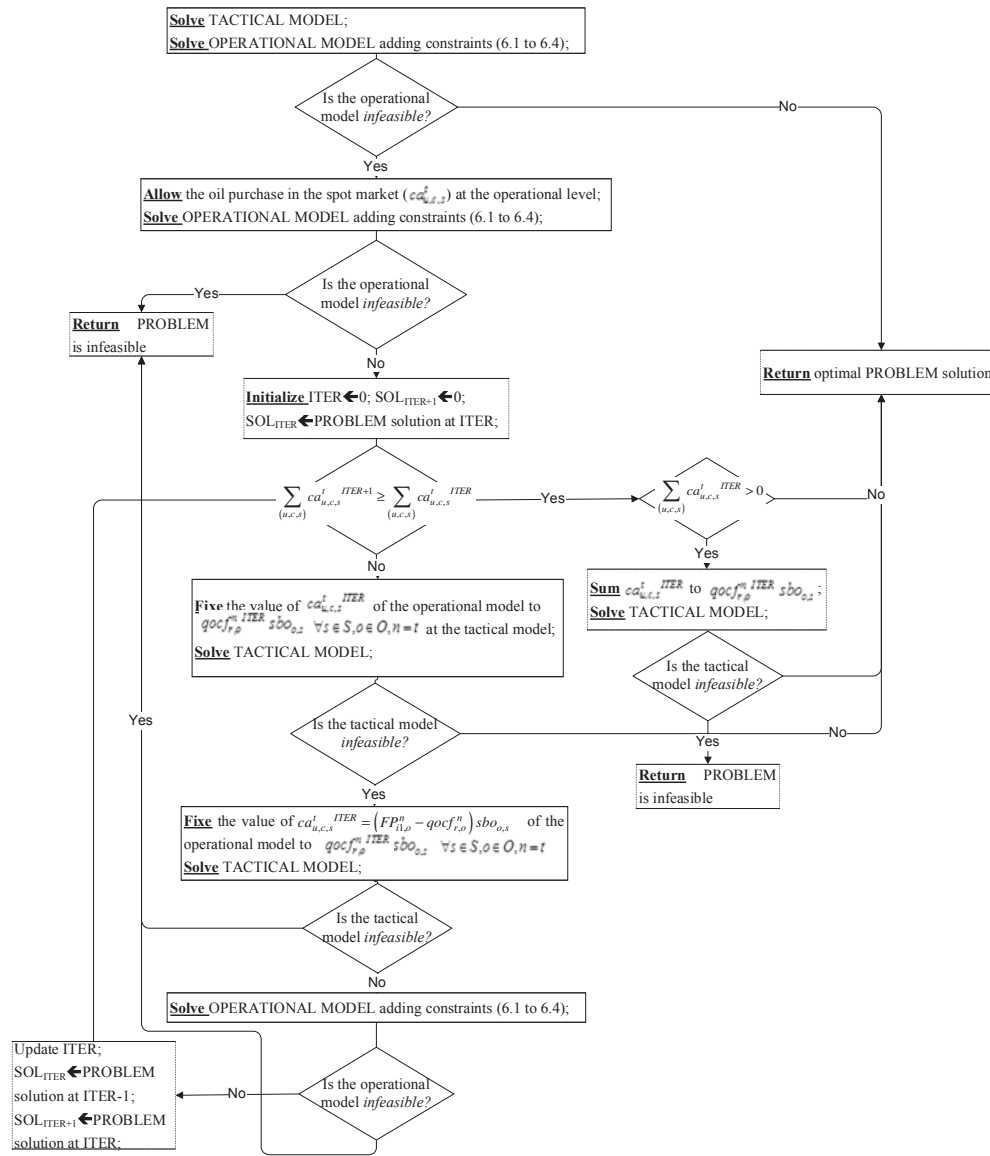
The iterative solution procedure is presented at Figure 19. After allowing the oil purchase at the spot market in case of operational infeasibility, as stated in the hierarchical procedure, the iterative procedure starts a loop to try eliminating the additional oil purchase. Let  $ITER$  an iteration of the iterative method,  $SOL^{ITER}$  the solution of an iteration, and  $SOL^{ITER-1}$  the solution of a previous iteration. The

first step of the loop is the initialization of  $ITER$ ,  $SOL^{ITER}$ , and  $SOL^{ITER-1}$ . The procedure verifies all the additional oil purchases and terminates when the total quantity of additional oil of an iteration is higher than in the iteration before ( $\sum_{(u,c,s)} ca_{u,c,s}^{t, ITER+1} \geq \sum_{(u,c,s)} ca_{u,c,s}^{t, ITER}$ ). While this condition is not reached, the heuristics fixes each additional purchase value of oil  $s$  ( $ca_{u,c,s}^{t, ITER}$ ) to the corresponding oil family  $o$  ( $qocf_{r,o}^n sbo_{o,s}$ <sup>4</sup>) at the tactical level and solves the tactical model again – i.e., the tactical solution is changed. The tactical model then evaluates the logistical constraints and the oil supply constraints associated with this additional purchase. A feasible tactical solution indicates that the tactical model could meet the need of the operational additional purchase. Otherwise, the quantity of oil that the operation needs is not available due to commercial or logistical constraints. In this case, the additional purchase of oil  $s$  is limited to the amount available for its oil family at the tactical level, that is represented by the maximum oil field production at node  $il$  (defined by tactical parameter  $FP_{il,o}^n$ ) less the amount that has already allocated to this oil family –  $ca_{u,c,s}^{t, ITER} = (FP_{il,o}^n - qocf_{r,o}^n) sbo_{o,s}$ . If the tactical model is still infeasible, so the integrated problem is infeasible. If not, the operational model must be evaluated with the new tactical solution. Finally, if the operational model is now infeasible, the loop continues. Otherwise, the loop terminates. Once the loop is finished, if the amount of additional oil is still above zero ( $\sum_{(u,c,s)} ca_{u,c,s}^{t, ITER} > 0$ ), the procedure adds this additional oil purchase to the solution obtained in the last iteration of the loop – i.e., tactical solution of the last iteration is now fixed and the additional oil quantity is added to it. The tactical model is solved again and an optimal solution is returned, if it exists.

At the end of the loop, the model may return an infeasible solution, a solution without additional oil purchase or a solution with a quantity of additional oil lower than in the original solution.

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<sup>4</sup> As explained in the section 6.1, parameter  $sbo_{o,s}$  defines the types of oil  $s$  that belongs to an oil family  $o$ .



**Figure 19.** Solution procedure in the iterative approach

To exemplify the procedure, suppose that 100 thousand  $m^3$  of the oil family A and 30 thousand  $m^3$  of the oil family B are available at the tactical level. Consider that 80 thousand  $m^3$  of oil family A were allocated to a refinery at the tactical solution. Consider also that family A is composed by oils A1 and A2 at the operational model and that family B is composed only by oil B1. The operational model broke down this 80 thousand  $m^3$  as follows: 60 thousand  $m^3$  for oil A1 and 20 thousand  $m^3$  for oil A2. However, due to the operational uncertainties (delays/changes in the oil supply), when the operational model was solved, there was an additional oil purchase of 20 thousand  $m^3$  of oil B1 at the spot market to

specify the final products (because oil B1 is a higher quality oil than A1 and A2). In the iterative approach, this additional purchase was then fixed as part of the tactical allocation which means that 20 thousand  $m^3$  of oil family B were forced to be part of the tactical solution. Then, the tactical model was solved again and an optimal solution was obtained, because the amount of oil required was available and the logistic constraints could be met.

Suppose now that only 10 thousand  $m^3$  of the oil family B are available at the tactical level, besides of the 100 thousand  $m^3$  of the oil family A. In this case, the tactical model would be infeasible when the 20 thousand  $m^3$  of additional oil family B, required by the operational model, would be fixed as part of the tactical solution. Then, the next step would be offer to the operational model the amount that is available at the tactical level, i.e., 10 thousand  $m^3$  of the oil family B. After being run again, the operational model could return an optimal solution (if the available oil B blended with oils A1 or A2 could meet the demand), or be infeasible.

Finally, suppose that the operational model broke down the 80 thousand  $m^3$  allocated by the tactical solution to the family A only for the oil A1 at the operational level (80 thousand  $m^3$  of oil A1). Additionally, the model opted by the purchase of 20 thousand  $m^3$  of oil A2 at the spot market. In this case, if this additional purchase would be fixed as part of the tactical allocation for family A, the model would return the same solution because this purchase is already part of the tactical solution ( $20 < 80$ ). To overcome this restriction, after the loop is finished, the 20 thousand  $m^3$  is added to the 80 thousand  $m^3$  of family A, and then, 100 thousand  $m^3$  of family A would be forced to be part of the tactical solution.

The previously presented integration approaches are discussed in the next section in the context of a numerical example.

### **6.3. Numerical example**

The proposed integrated modeling approaches of the oil chain are evaluated using the numerical example presented in section 4.3. First, a deterministic case is presented to point the importance of the uncertainty to the integration of the



tactical and operational planning of oil refineries. Next, stochastic cases are discussed.

As in the hierarchical approach the tactical and the operational models are solved successively, the computational time is equal to the sum of solution times when the models are solved separately ( $0.78s + 0.14s = 0.92s$ , according to Table 12). In the case of the iterative approach, after first run of the models, there is only re-optimization which lead to irrelevant increment of the computational effort.

### 6.3.1 Deterministic case

The deterministic case presented in this section considers the base case of both planning levels (scenario 5 of Figure 6 for the tactical and scenario 3 of Figure 7 for the operational level). As summarized by Table 16, in the deterministic case there is no oil purchase at the spot market which means that the integrated approaches could find a solution to the problem without the oil purchase at the spot market is allowed, as stated by the solution procedure described in the Figures 18 and 19. Actually this result was expected because, without uncertainty, the different levels of aggregation between the two models would be the only reason for the additional oil purchase. But as the tactical model considers an aggregated demand for the same products of the operational model, the tactical solution is able to allocate the oil families that the operation needs, leaving to the operational model only the choice between oils within that family. So, these findings indicate that, without uncertainty, the operational planning would follow the tactical planning. The stochastic cases are presented below and the effect of uncertainty is the integrated planning is discussed.

**Table 16.** Deterministic case: solutions of the integrated approaches

Refinery	Additional oil (thousand m <sup>3</sup> / month)			
	Hierarchical approach		Iterative approach	
	t=1	t=2	t=1	t=2
R1	-	-	-	-
R2	-	-	-	-
R3	-	-	-	-
<b>Operational margin</b> (million \$)	795.6		795.6	
<b>Tactical margin</b> (million \$)	707.9		707.9	
<b>Op. + Tactical margin</b> (million \$)	1,503.5		1,503.5	

### 6.3.2 Stochastic cases

In the stochastic cases discussed in this section, the operational model evaluates the tactical base case (scenario 5 of Figure 6) considering the operational uncertainties, which means that the tactical stochastic model was solved and that the actual realizations of the tactical random parameters (price and demand) were considered to correspond to the base case of the tactical model. Similar analysis can be done to the other scenarios.

As shown in Table 17, three case studies are considered. Uncertainties in oil supply and capacity of the process units affects the operational planning but the type of uncertainty in the oil supply varies from one case to another. In the first two cases, the oil supply uncertainty is represented by a change in the oil received. In case 1, however, the oils are exchanged among oils from the same oil family, whereas in case 2 exchanges among different families are also considered. Case 3 considers only delays in the oil supply.

**Table 17.** Cases for the evaluation of the integrated approaches

Type of uncertainty	Case 1	Case 2	Case 3
Capacity of the process units	x	x	x
Changes in the oil supply (same oil family)	x	x	
Changes in the oil supply (among oil families)		x	
Delays in the oil supply			x

The fixed tactical oil purchases (long-term contracts, defined by the first stage variable  $qocf_{r,o}^n$ ) follow the same decisions presented at Figure 10. These decisions are summarized at Table 18 for the first two planning periods which are also covered by operational planning.

**Table 18.** Tactical oil purchase decisions for the first two periods of planning

Refinery	Oil family	Fixed oil (thousand m <sup>3</sup> / month)	
		t=1 (30 days)	t=2 (31 days)
R1	A	33.0	34.1
R2	C	196.5	203.5
R3	F	74.9	77.4
	H	1,107.1	1,143.9

In the integrated approaches, the tactical oil allocation among the types of oil depends on the uncertainties considered at the operational level. The refineries choose the best oil allocation through the constraint (6.1) and deal with the uncertainties through the constraints (6.2) to (6.4). Table 19 illustrates the tactical allocation (defined by the variable  $aux\_qocfOP_{u,c,s}^t$ ) and the quantity/types of oil from long-term contracts (fixed oil) that the refinery R3 effectively receives in  $t=1$  – the first planning period (represented by  $qocfOP_{u,c,s}^{t,sc^o}$ ) for the three case studies previously presented. The results are presented to refinery R3 because it is the larger refinery and the one that is supplied with larger number of oils. The other refineries present similar pattern considering the oils received by each one.

**Table 19.** Tactical allocation and oils received by R3 in  $t=1$

Cases	Scenarios	Tactical allocation (thousand m <sup>3</sup> / month)						Oils received by the refinery (thousand m <sup>3</sup> / month)							
		Types of oil						Types of oil							
		F1	H1	H2	H3	H4	H5	A1	B1	F1	H1	H2	H3	H4	H5
1	1	75	791	316	-	-	-	-	-	75	791	316	-	-	-
	2	75	-	-	-	316	791	-	-	75	791	316	-	-	-
	3	75	1107	-	-	-	-	-	-	75	1107	-	-	-	-
	4	75	-	-	-	-	1107	-	-	75	1107	-	-	-	-
	5	75	122	985	-	-	-	-	-	75	122	985	-	-	-
	6	75	-	122	-	985	-	-	-	75	-	985	-	122	-
2	1	75	445	-	662	-	-	-	-	75	445	-	662	-	-
	2	75	-	-	-	-	1107	-	1107	-	-	75	-	-	-
	3	75	205	902	-	-	-	-	-	75	205	-	-	-	902
	4	75	-	1107	-	-	-	-	1107	-	-	75	-	-	-
	5	75	149	199	759	-	-	-	-	75	149	199	759	-	-
	6	75	-	-	479	-	628	479	628	-	-	75	-	-	-
3	1	75	748	359	-	-	-	-	-	75	748	359	-	-	-
	2	75	767	340	-	-	-	-	-	25	256	113	-	-	-
	3	75	1059	48	-	-	-	-	-	75	1059	48	-	-	-
	4	75	739	368	-	-	-	-	-	25	246	123	-	-	-
	5	75	-	1107	-	-	-	-	-	75	-	1107	-	-	-
	6	75	-	1107	-	-	-	-	-	25	-	369	-	-	-

Cases 1 and 2 in Table 19 present changes in the oil received in the scenarios 2, 4, and 6. As Case 1 considers exchanges for oils within the same oil family, the set of oils received by the refinery is similar to the one in tactical allocation. On the other hand, when exchanges for oils of different oil families are considered (as shown in Case 2), the group of oils that the refinery receives changes considerably, which can strongly impact the production profile of the

refinery. Finally, in Case 3 of Table 19, R3 faces delays in the amount of oil received reducing the available oil to 1/3 in the scenarios 2, 4, and 6 and, consequently, reducing the capacity of the refinery meets its product demand.

The solutions for the proposed cases are presented below. In the integrated approaches, each planning level maximizes its own expected margin ( $E[\text{margin}]$ ). As the additional purchase adds costs to the operational model, this purchase is done only if the quantity or quality of fixed oil received is not enough to meet the product demand.

- **Case 1 - changes in the oil supply (same oil family)**

In Case 1, the oils are exchanged by similar oils from the same oil family. As a result, there is no need to allow the oil purchase at the spot market as stated in the procedures presented at Figures 18 and 19. Table 20 summarizes the results of the hierarchical and the iterative approaches. These findings indicate that the uncertainty among oils from a same oil family is well accommodated by the integrated approaches.

**Table 20.** Case 1: solutions of the integrated approaches

Refinery	Additional oil (thousand m <sup>3</sup> / month)			
	Hierarchical approach		Iterative approach	
	t=1	t=2	t=1	t=2
R1	-	-	-	-
R2	-	-	-	-
R3	-	-	-	-
<b>Operational margin</b> (million \$)	771.4		771.4	
<b>Tactical margin</b> (million \$)	707.9		707.9	
<b>Op. + Tactical margin</b> (million \$)	1,479.3		1,479.3	

- **Case 2 - changes in the oil supply (among oil families)**

In Case 2, the oils are exchanged by oils from other families. Table 21 shows the oil purchase decisions at the spot market for R1 in the hierarchical and the iterative approaches. In the iterative approach, two iterations were needed until the additional oil purchase converge to zero.

As refinery R1 processes oils from only one oil family, the solution of Case 2 is the same of the Case 1 in which there is no additional oil purchase. In refinery R2, however, oil C1 was exchanged to the oil type C2, that is a lower quality oil

than C1, despite of belonging to the same oil family. With this exchange, in the hierarchical approach, the operational model was infeasible when only the tactical allocation was considered and the purchase at the spot market needed to be allowed. So, the exchanges from C1 to C2 led to the additional purchase of better quality oils (D1 and E1) for the product specification. The total purchase of oils D1 and E1 was then fixed as part of the tactical allocation in the iterative approach, as proposed by the procedure described at Figure 19. As a result, the additional oil purchase was eliminated by the iterative method in its first iteration. However, the iterative method needed two iterations to converge, due to the additional oil purchase of R3.

**Table 21.** Case 2: oil purchase decisions of the integrated approaches

Refinery	Type of oil	Fixed oil (thousand m <sup>3</sup> /month - weighted average of the scenarios)		Additional oil (thousand m <sup>3</sup> /month)					
		t=1	t=2	Hierarchical approach		Iterative approach			
				First iteration	Second iteration	First iteration		Second iteration	
		t=1	t=2	t=1	t=2	t=1	t=2	t=1	t=2
R1	A1	27.2	28.8	-	-	-	-	-	-
	A2	3.0	2.7	-	-	-	-	-	-
	A3	2.8	2.6	-	-	-	-	-	-
	<b>Total</b>	<b>33</b>	<b>34.1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
R2	C1	126.7	142.1	-	-	-	-	-	-
	C2	69.7	60.9	-	-	-	-	-	-
	D1	-	-	8.3	13.14	-	-	-	-
	E1	-	-	40.0	40.00	-	-	-	-
	<b>Total</b>	<b>196.4</b>	<b>203</b>	<b>48.3</b>	<b>53.14</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
R3	A2	35.9	24.1	-	-	-	-	-	-
	B1	296.2	319.1	-	-	-	-	-	-
	E2	-	-	65.0	65.0	-	-	-	-
	E3	-	-	50.0	50.0	-	-	-	-
	E4	-	-	217.1	-	-	-	-	-
	E5	-	-	32.5	3.4	-	-	-	-
	F1	52.5	54.2	-	-	-	-	-	-
	H1	175.9	-	-	-	-	-	-	-
	H2	57.4	23.2	-	-	12.1	-	-	-
	H3	248.6	242.9	-	-	-	-	-	-
	H4	-	125.9	-	-	-	-	-	-
	H5	315.6	431.9	-	-	-	-	-	-
<b>Total</b>	<b>933.5</b>	<b>1221.3</b>	<b>364.6</b>	<b>118.4</b>	<b>12.1</b>	<b>0</b>	<b>0</b>	<b>0</b>	

In R3 the oil changes in the oil supply led to the importation of light oils from family E in the hierarchical approach. After fix the importation amount to the tactical model in the first iteration of the iterative approach, the purchase of oil H2 could not be eliminated because it was already part of the tactical allocation for

the oil family H in period 1 as presented at Table 18 (12.1 is lower than 1,107.1 m<sup>3</sup>/month). So, when the additional oil purchase is fixed in the iteration of the iterative scheme (as proposed by the procedure described at Figure 19), the model returns the same solution of the hierarchical method. Despite the additional purchase in R3 was not completely eliminated in the first iteration, a reduction of 97.5% of the hierarchical solution was obtained. The residual amount of oil H2 (12.1 thousand m<sup>3</sup>/ month) was then added to the fixed quantity at the tactical model (the total oil purchase – long term + spot market – was fixed at the tactical level). The tactical model was run once again (second iteration), the logistical and commercial constraints were evaluated, and a feasible solution was obtained. So, an optimal solution to the integrated problem without any additional oil purchase was found.

In the hierarchical solution (corresponding to the iteration 0 at Table 22), a total amount of 584.5 thousand m<sup>3</sup>/ month of additional oil was purchased for the refineries operations. However, the total purchase decreased to 2.1% of the hierarchical solution in the first iteration of the iterative method, demonstrating the effectiveness of the proposed approach. These results were translated into an increase of 18.9% in the value of the objective function. Despite the tactical margin had decreased, the sum of the tactical and the operational margins increased which indicates a benefit of the iterative approach. In the second iteration, the tactical objective function has slightly changed because the additional purchase is now included in the tactical costs as oil purchase from long-term contracts. Moreover, the costs of purchasing additional oil are reduced of the operational margin, but as in this case the purchase is only of 12.1 thousand m<sup>3</sup>/ month, the operating margin has not changed.

**Table 22.** Case 2: solutions of the iterations of the iterative approach

Iteration	E[margin] (million \$)			Total additional oil (thousand m <sup>3</sup> / month)
	Operational	Tactical	Op. + Tactical	
0	497.4	707.9	1,205.3	584.5
1	766.8	666.9	1,433.7	12.1
2	766.8	666.8	1,433.6	0

- **Case 3 - delays in the oil supply**

Table 23 shows the solutions of the hierarchical and the iterative approaches for Case 3. In the hierarchical method (corresponding to the iteration 0 at Table 23), the total additional oil purchase reaches 1,690.9 thousand m<sup>3</sup>/month, considering the fixed oil allocated by the tactical planning. In the first iteration of the iterative method, the total amount of additional oil is slightly reduced (0.7%) and the operational margin increases 5.4%. In the second iteration, the margin decreases to \$162.7 and the total oil purchase increases to 1,680.2 thousand m<sup>3</sup>/month. Thus, as stated in the Figure 19, the solution obtained at the first iteration is a better solution for the iterative approach and the solution of the second iteration is disregarded. However, as a large amount of additional oil was still present in the solution of the first iteration, the residual amount of oil 1,679.3 thousand m<sup>3</sup>/ month was then added to the fixed quantity at the tactical model. The tactical model was run once again (third iteration) and a feasible solution was found. Thus, the integrated problem has an optimal solution without any additional oil purchase. When the additional oil purchase was added to the tactical level, the tactical model incurred in costs of oil supply by long-term contracts and the cost equivalent to this purchase was subtracted to the operational level, changing both the tactical and the operational margins. The sum of the tactical and the operational margins in the third iteration was 7.5% lower than the one obtained at the first iteration. Due to the costs of additional oil purchase, the operational expected margins are approximately 5 times lower than the one of Case 1.

**Table 23.** Case 3: solutions of the iterations of the iterative approach

Iteration	E[margin] (million \$)			Total additional oil (thousand m <sup>3</sup> / month)
	Operational	Tactical	Op. + Tactical	
0	156.6	707.9	864.5	1,690.9
1	165.1	700.2	865.3	1,679.3
2	162.7	701.1	863.8	1,680.2
3	164.5	635.6	800.4	0

As summarized at Table 24, refinery R1 receives the fixed oils type A1, A2, and A3 in both planning periods. In the hierarchical approach, the operational model opts to the purchase of additional oil type A1 to face the delays in the oil

received. The iterative approach does not eliminate this purchase in its first iteration, because the additional quantity is already part of the tactical allocation for family A (similarly to R3 in Case 2) – for example, the additional oil purchase in the first period (16 thousand m<sup>3</sup>/month) is lower than the 33 thousand m<sup>3</sup>/month allocated by the tactical planning for family A as presented at Table 18. In addition, one more iteration, fixing the tactical solution was necessary to that the additional purchase equals zero. The second solution of the iteration was skipped because this iteration was disregarded, as mentioned before.

**Table 24.** Case 3: oil purchase decisions of the integrated approaches at R1

Iteration	Type of oil	Fixed oil (thousand m <sup>3</sup> /month - weighted average of the scenarios)		Additional oil (thousand m <sup>3</sup> /month)	
		t=1	t=2	t=1	t=2
0	A1	20.3	22.1	16.0	16.5
	A2	2.0	1.5	-	-
	A3	4.1	3.7	-	-
	<b>Total</b>	<b>53.7</b>		<b>32.5</b>	
1	A1	20.3	23.0	16.0	16.5
	A2	3.5	1.0	-	-
	A3	2.5	3.2	-	-
	<b>Total</b>	<b>53.5</b>		<b>32.5</b>	
2	-	-	-	-	-
3	A1	36.3	39.5	-	-
	A2	3.5	1.0	-	-
	A3	2.5	3.2	-	-
	<b>Total</b>	<b>86.0</b>		<b>-</b>	

In the hierarchical approach, correspondent to the iteration 0 described in Table 25, refinery R2 is supplied by the oils type C1 and C2 from the tactical planning and buys oil E1 as additional oil. In the first iteration of the iterative method, however, oil C2 is exchanged by E1 in the fixed allocation and the operational model purchases additional oil type D1, besides of E1. The total purchase in the spot market for this refinery is reduced in 20.5% by the iterative method, considering the hierarchical solution. As done to refinery R2, the second iteration was skipped in Table 25.



**Table 25.** Case 3: oil purchase decisions of the integrated approaches at R2

Iteration	Type of oil	Fixed oil (thousand m <sup>3</sup> /month - weighted average of the scenarios)		Additional oil (thousand m <sup>3</sup> /month)	
		t=1	t=2	t=1	t=2
0	C1	139.4	151.0	-	-
	C2	17.8	11.4	-	-
	D1	-	-	-	-
	E1	-	-	39.7	16.3
	<b>Total</b>	<b>319.6</b>		<b>56.0</b>	
1	C1	132.9	154.7	-	-
	C2	-	-	-	-
	D1	-	-	-	6.5
	E1	31.8	13.3	21.4	16.6
	<b>Total</b>	<b>332.7</b>		<b>44.5</b>	
2	-	-	-	-	-
3	C1	132.9	154.7	-	-
	C2	-	-	-	-
	D1	-	6.5	-	-
	E1	53.2	29.9	-	-
	<b>Total</b>	<b>377.2</b>		<b>-</b>	

In the hierarchical approach presented at Table 26, refinery R3 receives oils type F1, H1, and H2 and buys the oils B1, C3, H1, H2, H3, and H4 at the spot market. The total oil purchase was not reduced from the hierarchical to the iterative solution, but the purchase profile has changed from one solution to another when the additional purchase decision was fixed as part of the tactical solution in the first iteration (less C3 and more B1 and H2 were purchased at the spot market). Once again, the solution of the second iteration is not presented here.

**Table 26.** Case 3: oil purchase decisions of the integrated approaches at R3

Iteration	Type of oil	Fixed oil (thousand m <sup>3</sup> / month - weighted average of the scenarios)		Additional oil (thousand m <sup>3</sup> / month)	
		t=1	t=2	t=1	t=2
0	B1	-	-	41.9	60.0
	C3	-	-	49.1	9.3
	F1	59.9	61.9	-	-
	H1	557.7	820.3	500.0	500.0
	H2	327.9	94.9	240.0	21.9
	H3	-	-	45.0	45.0
	H4	-	-	45.0	45.0
	<b>Total</b>		<b>1,922.6</b>		<b>1,602.2</b>
1	B1	33.5	48.0	60.0	60.0
	C3	39.2	7.5	8.15	-
	F1	60.0	62.0	-	-
	H1	559.9	759.9	500.0	500.0
	H2	253.0	13.9	240.0	54.1
	H3	-	85.9	45.0	45.0
	H4	-	-	45.0	45.0
	<b>Total</b>		<b>1,922.6</b>		<b>1,602.2</b>
2	-	-	-	-	-
3	B1	93.5	108.0	-	-
	C3	47.4	7.5	-	-
	F1	60.0	62.0	-	-
	H1	1,059.9	1,259.9	-	-
	H2	493.0	68.0	-	-
	H3	45.0	130.9	-	-
	H4	45.0	45.0	-	-
	<b>Total</b>		<b>3,525.1</b>		<b>-</b>

The fact that the three refineries presented additional oil purchases until at the end of the first iteration of iterative method in Case 3 – before the total oil purchase be fixed and evaluated by the tactical model solution – is an indication that uncertainty in the quantity of oil received is not completely circumvented by the integration approaches. However, this type of uncertainty could be faced keeping the considered amounts of additional oil purchased in the safety stock of the refineries. Safety stocks are a type of protection for delivery delays and changes in the oil specification but lead to high storage costs and act against improvements in the supply chain problems.

## 6.4. Chapter conclusions

This chapter presented the two integrated modeling approaches and a numerical example to evaluate these approaches. Whereas the formulation of the two models separately (single-level formulation) can be visualized as centralized decision making systems, the integrated formulation takes into account the reaction of the second level and solves the problem of coordinating in a decentralized system by improving the objective of the highest level, while dealing with the maximization of the lower level objectives. This feature allows the integrated model to react to the second level uncertainties incorporating these uncertainties in the first level solution. In this regard, the tactical planning model of the oil supply chain was improved when the reaction (additional oil purchase) was considered. Though complete reduction/ elimination of the oil purchase at the spot market may not be optimal its reduction or elimination is an indication of the effectiveness of the proposed approaches. The refineries choose the amount allocated to the types of oil through the constraints (6.1) to (6.4) which allows the operational model to choose the best oil combination to process at each refinery.