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A
Ricatti equations

In this Appendix we show how to obtain equations (2-20). The purpose here is not to give a thorough demonstration of the algebra, but rather provide the reader with a sketch of how to proceed. We will be using and adapting Appendix A in Joyce, Lildholdt and Sorensen (2009) to our notation and particular form of movement laws for the discount factors.

Starting from the fundamental asset pricing equation for real bonds as given by our equation (2-7) and taking the natural logarithms of both sides, we have

$$p_{t,\tau}^R = E_t\left(m_{t+1}^R + p_{t+1,\tau-1}^R\right) + \frac{1}{2}Var_t\left(m_{t+1}^R + p_{t+1,\tau-1}^R\right)$$

With the movement law (2-18) for the SDF, we know the real bond price is given by

$$p_{t,\tau}^R = \mathbf{A}_\tau^R + \mathbf{B}_\tau^{R'} x_t$$

with $\mathbf{B}_\tau \sim 4 \times 1$. Substituting for the next period's SDF and for the bond price, after some algebraic manipulation we obtain:

$$\begin{aligned}
\mathbf{A}_\tau^R &= -\bar{r} + \mathbf{A}_{\tau-1}^R - \lambda^{R^\top}\mathbf{B}_{\tau-1}^R + \frac{\mathbf{B}_{\tau-1}^{R^\top}\mathbf{B}_{\tau-1}^R}{2} \\
\mathbf{B}_\tau^R &= -\gamma^R + \Phi\mathbf{B}_{\tau-1}^R
\end{aligned}$$

To obtain initial conditions, we need only remember that the price of a bond maturing today is $P_{t,0}^R = 1$, so that

$$\mathbf{A}_0 = 0 \text{ and } \mathbf{B}_0 = 0$$

This restricts yields of different maturities to the no-arbitrage condition.

Finally, to obtain our usual formula for yields, we must only see that real yields in continuos time is given by

$$y_{t,\tau}^R = -\frac{p_{t,\tau}^R}{\tau} = -\frac{\mathbf{A}_\tau}{\tau} - \frac{\mathbf{B}_\tau}{\tau}x_t = A_\tau + B_\tau x_t$$

and so replace the right maturities. For instance, the one period rate is $y^R_{t,1} = -\bar{r} + \gamma^R x_t$.

Mutatis mutandi, applying the same method for nominal yields gives us their Ricatti equations:

$$\begin{aligned} \mathbf{A}^N_\tau &= -\left(\bar{r} + \bar{\theta}\right) + \mathbf{A}^N_{\tau-1} - \left(\lambda^{R^\top} + \lambda^{\theta^\top}\right)\mathbf{B}^N_{\tau-1} + \frac{\mathbf{B}^{N^\top}_{\tau-1}\mathbf{B}^N_{\tau-1}}{2} \\ \mathbf{B}^N_\tau &= -\left(\gamma^R + \gamma^\theta\right) + \Phi\mathbf{B}^N_{\tau-1} \end{aligned}$$