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5
Small Sample Properties

In this section we conduct Monte Carlo experiments to assess size and power of the test in small samples. We will compare the performance of the proposed test (henceforth refereed as Taylor) with the other two available tests for nonlinearity in Error Correction Models, Hansen and Seo (2002) [10] and Seo (2007) [22], which henceforth will be referred respectively as HS and Seo.[1] These are supLM tests, designed against a specific alternative hypothesis. In HS, the alternative is a threshold model, whereas in Seo it is a logistic or an exponential smooth transition model.

The models used in each simulation will be the same used in Hansen and Seo (2002) [10] and Seo (2007) [22], so as to render the results directly comparable. The sample size is always 250 and the number of repetitions is fixed in 1000. The error is independent multivariate normal with unit variance. First, to asses the empirical size, the Data Generating Process (DGP) will be

$$\Delta \boldsymbol{y}_t = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} (y_{1t-1} - \beta_2 y_{2t-1}) + \boldsymbol{\Gamma} \Delta \boldsymbol{y}_{t-1} + \boldsymbol{\epsilon}_t,$$

where $\boldsymbol{\Gamma}$ may assume three values:

$$\boldsymbol{\Gamma}_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \boldsymbol{\Gamma}_1 = \begin{pmatrix} -0.2 & 0 \\ -0.1 & -0.2 \end{pmatrix}, \text{ or } \boldsymbol{\Gamma}_2 = \begin{pmatrix} -0.2 & -0.1 \\ -0.1 & -0.2 \end{pmatrix}.$$

The parameters $\alpha_1 = -1$ and $\beta_2 = -1$ are fixed and α_2 varies among 0, -0.5 and 0.5. The results of the simulation for the Taylor test are in Table 5.1, as well as the results from Hansen and Seo (2002) [10] and Seo (2007) [22]. The results are somewhat homogenous, showing good size in almost every case. The largest deviation being 1.4% for Seo L, followed by 0.9% for Seo E, 0.8% for HS and 0.8% for Taylor. The mean deviation is 0.38% for HS, 0.84% for Seo L, 0.24% for Seo E and 0.4% for Taylor.

[1] Seo L will refer to the test against a logistic function and Seo E will refer to the test against the exponential function.

Table 5.1: EMPIRICAL SIZE.

This table presents for the featured tests the empirical size for a linear ECM Data Generating Process calculated by a Monte Carlo experiment with 1000 repetitions for a 250 observations sample size.

α_2	0	-0.5	0.5	0	0
$\mathbf{\Gamma}$	$\mathbf{\Gamma}_0$	$\mathbf{\Gamma}_0$	$\mathbf{\Gamma}_0$	$\mathbf{\Gamma}_1$	$\mathbf{\Gamma}_2$
HS	0.054	0.056	0.047	0.054	0.052
Seo L	0.059	0.058	0.036	0.051	0.060
Seo E	0.046	0.051	0.047	0.049	0.047
Taylor	0.054	0.053	0.055	0.042	0.050

To compare the power, the first Data Generating Process will be

$$\Delta \boldsymbol{y}_t = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix} (y_{1t-1} - \beta_2 y_{2t-1}) + \begin{pmatrix} -\delta_1 \\ 0 \end{pmatrix} (y_{1t-1} - \beta_2 y_{2t-1}) \, f(z_{t-1}, \lambda, c) + \boldsymbol{\epsilon}_t,$$

where f is the exponential function, as in Equation (2.2) or the logistic function, as in Equation (2.3). The parameters δ_1 and λ take the values $(0.4, 0.8)$ and $(0.75, 3, 9)$ respectively, while α_1 is fixed in -0.2 and β_2 is fixed in 1. The simplicity of the model under the alternative is due to the computational requirements of the two supLM tests. In Hansen and Seo (2002) [10] we read "To keep the calculations manageable, we generate the data from the simple process," while Seo (2007) [22] uses "25 grid points (...) to reduce the computational costs."

It is expected that higher δ_1 values will yield higher powers, since the nonlinearity will be more pronounced. Also, for the logistic function, higher λ values should yield higher powers, since, as can be seen in Figure 2.2, a smaller λ makes the function more similar to a linear one. For the exponential function, higher powers should be associated with smaller λ values. As shown in Figure 2.2, high lambda values turn the nonlinear region very small.

In Table 5.2 we can see that, even though this model is the alternative hypothesis under which the Seo test was developed, the Taylor test has a better power for all but one of the parameters combinations for the logistic case. Even the HS test is better than the Seo test in this case. In the exponential case we have the inverse situation: the Seo test is better than the other two tests in all but one of the parameters combinations. However, the power is very small in most of the cases.

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Table 5.2: Empirical Power.

This table presents for the featured tests the empirical power for a Smooth Transition ECM Data Generating Process calculated by a Monte Carlo experiment with 1000 repetitions for a 250 observations sample size.

δ_1	0.4	0.4	0.4	0.8	0.8	0.8
λ	0.75	3	9	0.75	3	9
DGP with Logistic Function						
HS	0.514	0.626	0.610	0.927	0.904	0.897
Seo L	0.212	0.596	0.554	0.520	0.966	0.953
Taylor	0.625	0.648	0.620	0.984	0.970	0.943
DGP with Exponential Function						
HS	0.066	0.064	0.058	0.096	0.069	0.043
Seo E	0.285	0.077	0.057	0.869	0.124	0.057
Taylor	0.119	0.065	0.043	0.379	0.064	0.058

Table 5.3: Empirical Power.

This table presents for the featured tests the empirical power for a Threshold ECM Data Generating Process calculated by a Monte Carlo experiment with 1000 repetitions for a 250 observations sample size.

δ_1	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
ω	0.5	0.5	0.5	0.5	0.25	0.25	0.25	0.25
HS	0.156	0.450	0.878	0.997	0.187	0.527	0.844	0.933
Taylor	0.151	0.503	0.891	0.994	0.179	0.592	0.888	0.958

The second Data Generating Process will be

$$\Delta \boldsymbol{y}_t = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix} (y_{1t-1} - \beta_2 y_{2t-1}) + \begin{pmatrix} \delta_1 \\ 0 \end{pmatrix} (y_{1t-1} - \beta_2 y_{2t-1}) \, \mathit{1} (y_{1t-1} - \beta_2 y_{2t-1} \leq \lambda) + \boldsymbol{\epsilon}_t,$$

where $\mathit{1}(\cdot)$ is the indicator function. Parameters α_1 and β_2 will be held fixed at -1 and 1 respectively, while δ will take values $(0.2, 0.4, 0.6, 0.8)$. The parameter λ is set so that $\omega = P(y_{1t-1} - \beta_2 y_{2t-1} \leq \lambda)$ equals 0.5 or 0.25. This model is the alternative for which the HS test was developed. The empirical power is in Table 5.3.

It is worth remembering that we cannot establish the consistency of the Taylor test against the threshold alternative. Despite this, the power in small samples is the same or better than the HS test in all parameters used in this Monte Carlo.

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