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## Introduction

Unconstrained Binary Quadratic Programming (UBQP) is widely studied. It is a powerful modeling tool and its associate problem is  $\mathcal{NP}$ -hard.

It can represent problems from a wide range of sources, including but not limited to: Scheduling [alidaee1994], Combinatorial Optimization [alidaee2005], Molecular Conformation [phillips1994] and Physics [liers2003]. Some classic  $\mathcal{NP}$ -hard Combinatorial Optimization problems, like max-cut and max-clique, can be trivially modeled as UBQP.

Because of its importance, a wide number of approaches were created to try to solve UBQP. Amongst them there are exact approaches [rendl2010, billionnet2007], approximation algorithms [goemans1995] and heuristics [glover2002, merz2004, merz2002, lodi1999].

In this work two new methods are introduced, which can be used to build a powerful exact algorithm, as well as an heuristic. Also, the fundamental idea behind them can be used in an even wider family of problems.

This exact algorithm derived from the new method is highly parallelizable, which is a desired feature nowadays, when the *cloud computing* is a reality. For reasonably large instances of UBQP, the new method can parallelize to hundreds, or even thousands, of cores easily, with a near-linear speedup. So, the time of computation is (almost) inversely proportional to the budget available for the project.

The remaining of this work is organized as follows: In chapter 2 the UBQP problem is presented, and its relation with max cut is proved. In chapter 3 related works are exhibited, including the state of the art. In chapter 4 we present the Column Generation Improvement for Heuristics. In chapter 5 the new approach is outlined. In chapter 6 the new approach is used in a branch

and bound scheme. In chapter 7 there are some numerical results, showing the performance of this new method. In chapter 8 a brief conclusion is made.