1

Introduction

Many of the principles underlying the behavior of the natural world are statements and relations involving rates at which things happen. When they are expressed in mathematical terms the relations are equations and the rates are derivatives. Equations containing derivatives are differential equations**.** The development of differential equations is linked to the general development of mathematics. The subject of differential equations originated in the study of calculus.

The fundamental ideas of the Calculus: the processes of differentiation and integration and the fundamental theorem which relates them were independently developed by Sir Isaac Newton and Gottfried Wilhelm Leibniz in the seventeenth century. In Newton's case, a primary motivation for the development was to provide a tool for solving problems involving motion and other physical phenomena. In the first uses of differentiation and integration to solve problems, the term differential equations had not yet been formalized. Rather, the investigators used derivatives and differentials without distinction and derived particular equations for particular problems, which they attempted to solve by any method that occurred to them.

About the beginning of the eighteenth century, several categories of problems dominated scientific investigations and led to the consideration of differential equations and methods for their solution. This tool since then has being used to describe, understand and predict behavior of many physical processes or system through the construction of mathematical models of the system in diverse areas of study including physics, economics, epidemiology, fluid dynamics, pharmacology, social sciences and ecology , just to mention a few.

Ecology is one of the interesting areas that have been studied using mathematical models. Ecological science is the branch of biology that studies how organisms interact among themselves and also with the environment, and it is primarily researched at three key levels of organization; populations, communities, and ecosystems (Ricklefs, 2001). A population is a group of organisms of the same species that occupy a particular area. It is the basic unit of ecology (Berryman, 2002). The study of the short and long term changes in the size and age composition of populations, and the biological and environmental processes influencing these changes is Population dynamics.

Population dynamics is currently an interdisciplinary subject that aims at the mathematical representation, treatment and modeling of the biological processes, using a variety of applied mathematical techniques and tools. Population interactions on many spatial and temporal scales usually create many interesting problems that are sometimes too complex, numerous, small, or slow not to be approached by mathematical models. Mathematical models often make it possible to identify mechanisms behind such complex biological processes. Many researchers date the modern era of population dynamics to 1798 and the publication by Malthus "An Essay on the principle of Population" (Malthus, 1798). This principle of population dynamics is widely regarded as the exponential law of Malthus, as modeled by the Malthusian growth model. Malthus believed that human population grows exponentially while food supply grows algebraically. In 1838, Pierre Verhulst proposed his logistic model of population growth, where population size is limited by a carrying capacity. This was a refinement and adjustment of the Malthusian demographic model. Over the years, researchers have developed numerous theories that have helped understand population dynamics better.

In this study, we aim to give an accessible introduction to some of the fundamental ideas of Population dynamics in general and its application to Fisheries in particular. We study the evolution of the fish population with time. We do this by constructing dynamical systems from our assumptions about the fishery system. Using main components of the system such as the size of population (current and previous), birth rate, death rate and harvesting rate, we observe the behavior of the system and attempt to relate the variables by means of one or more differential equations. We study the existence of a solution, the stability of equilibria, and investigate bifurcations of the system. The notion of maximum sustainable yield (MSY), minimum viable population, and Harvesting (in the form of functional response) are discussed. We implement graphical representation and numerical approach to study the qualitative behavior rather than obtaining the explicit solutions for the models which do not allow for an analytical solution due to their complexity. The results of the analysis are discussed incorporating realistic and relevant ecological issues such as species extinction (collapse in fisheries), sustainable yield, and management of stock levels to derive optimal harvesting policies.

Understanding the dynamical behavior of populations is essential for the conservation and management of the species (Clark, 2006). Thus, this kind of study will provide the desired analysis that can generate reflections on the fish population dynamics, which can be used as a tool in deciding on the maintenance of their populations and the livelihoods of fishers as well as the sustainability of both commercial catches and the aquatic ecosystem from which they are extracted.

We start in Chapter 2 by indicating the factors that contribute to population change, establish the variations of models, some population models and their limitations. We refer the reader to Hirsch and Smale (1974), Boyce, (2001), Strogatz (1994), and Coddington and Levinson (1955) for background material in differential equations and in particular to local stability of equilibrium points and its bifurcations. We recommend Edelstein-Keshet (1988), Murray (1981), May (1976.Models of single populations) and Weiss (2009) for further reading on Population dynamics. In Chapter 3, we present the dynamics of a fish population in a fishing zone. Using 4 different harvesting functions, we investigate how these functions affect the natural growth process of the fish population. We particularly look for the existence of sustainable fishery; in the sense that, the fish population can be exploited in a durable way but without any risk of extinction for the resource. Although we shall concentrate on fishery, much of the discussion can be translated to apply to other plant and animal population that are harvested or preyed. In Chapter 4, the evolution of the economic activity of the fishery is analysed. Here two equations are studied. The first looks at the evolution of the exploited population and the second equation is the profit equation. The total profit of the fishery is given by the difference between the total cost of production, which is the cost for the total effort used and the total revenue which is the total quantity of fish purchased by consumers from the fishery.

In Chapter 5, we discuss the results of the study; make conclusions and recommendations to the fishery.