4

Effects of Price on the Dynamics of the Fishery Model

The inclusion of economic factors in fishery models is a very important topic which is aimed at assessing fishery management and its economic consequences.

We consider a system of two equations, the first one describes the net growth of the fish population i.e. the evolution of the fish stock which grows naturally and is harvested by a fishing fleet with time. The second equation describes the evolution of the fishing effort, *e(t)* which depends on the difference between the benefit and the cost of the fishery, fixing the investment of revenues for fishing effort.

$$\frac{dn}{dt} = f(n) - h(n,e) \qquad 4.1$$
$$\frac{de}{dt} = ph(n,e) - ce(t)$$

Where *f(n)* is the natural growth function of the fish stock. *h(n,e)* is the harvesting function depending on the fish stock, *n* and the fishing effort, *e*. *c* is the cost per unit of fishing effort due to fuel costs, taxes, salaries and so on. *p* is the fish price per unit of harvested fish. According to classical economics, the price change depends on the gap between a demand function (that is the quantity of fish purchased by the consumers) and the supply which is no more than the catch. We assume that the market price is not constant. We assume that the adjustment of the market price occurs rapidly with respect to fish growth, investment (new boats entering the fishery) and catch.

Using the logistic growth model and Schaefer harvesting function, the model now reads;

$$\frac{dn}{dt} = rn(1-\frac{n}{K}) - qen \qquad 4.2$$
$$\frac{de}{dt} = e(pqn - c)$$

We have a two-dimensional nonlinear system with the form

$$n' = f(n,e)$$
$$e' = g(n,e) \qquad 4.3$$

where *r,q,p, K* and *c* are non- negative parameters.

The effort, *e* increases when *g(n,e)> 0* i.e. *pqn >c* and *g'(n,e) >0* i.e *pqn >c* else it decreases. On the other hand, the stock increases when *f(n,e)>0* i.e. $e < \frac{r}{q}(1-\frac{n}{K})$ and *f'(n,e)>0* i.e $e < \frac{r}{q}(1-\frac{2n}{K})$. This is illustrated in Figure 4.1.

4.1

Equilibrium Points and Local Stability Analysis

We find the equilibrium solution of Eq.(4.3) by calculating $f(n^*,e^*)=0$ and $g(n^*,e^*)=0$ simultaneously. From $g(n^*,e^*)=0$, we get *e=0* or $n=\frac{c}{pq}$.

Considering $f(n^*,e^*)=0$, we derive the following

$$rn(1-\frac{n}{K}) = qen$$
$$r(K-n) = Kqe$$
$$e = \frac{r}{q} - \frac{r}{Kq}n \qquad 4.4$$

Thus there are three equilibrium points: (0,0) , $(K,0)$ and $(\frac{c}{pq}, \frac{r}{q}(1-\frac{c}{Kqp}))$.

The Jacobian matrix at the general point (n,e) is

$$J(n,e) = \begin{pmatrix} \frac{\partial f}{\partial n} & \frac{\partial f}{\partial e} \\ \frac{\partial g}{\partial n} & \frac{\partial g}{\partial e} \end{pmatrix} = \begin{pmatrix} r(1\text{-}\frac{2n}{K})\text{-}qe & \text{-}qn \\ pqe & pqn\text{-}c \end{pmatrix} \qquad 4.5$$

We study the local stability of each equilibrium by find the eigenvalues of the respective Jacobian matrix. We use the formular below

$$\lambda_{1,2} = \frac{1}{2}(\tau \pm \sqrt{\tau^2 - 4\Delta}) \qquad \tau=\lambda_1+\lambda_2 \qquad \Delta=\lambda_1\lambda_2 \qquad 4.6$$

1. At $E_o = (0,0)$ $\quad J(0,0) = \begin{pmatrix} r & 0 \\ 0 & \text{-}c \end{pmatrix}$

has two real eigenvalues with opposite signs ($\lambda_1 = r$ *and* $\lambda_2 = -c$). Hence, the equilibrium point (0, 0) is always a saddle point.

2. At E_k = (K,0); $J(K,0) = \begin{pmatrix} -r & -qK \\ 0 & -c+qKp \end{pmatrix}$

has two real eigen values, ($\lambda_1 = -r$ *and* $\lambda_2 = -c + qpK$).

Two cases appear:
if $K < \left(\dfrac{c}{qp}\right)$ then (K,0) is a stable node.
if $K > \left(\dfrac{c}{qp}\right)$ then (K,0) is a saddle-point.

3. At E = $(\dfrac{c}{pq}, \dfrac{r}{q}(1 - \dfrac{c}{Kqp}))$; $J(n^*, e^*) = \begin{pmatrix} \dfrac{-rc}{qpK} & -cp \\ r(p - \dfrac{c}{qK}) & 0 \end{pmatrix}$

$det\ J(n^*,e^*) = -rcp(p - \dfrac{c}{qK})$ and $trJ(n^*,e^*) = -\dfrac{rc}{qpK}$

It is straightforward to see that $trJ(n^*,e^*)<0$
Two cases appear:

If $detJ(n^*,e^*) < 0$ which is equivalent to $K < \left(\dfrac{c}{qp}\right)$, there are two eigenvalues with opposite signs. Then (n^*,e^*) is a saddle point.

If $detJ(n^*,e^*) > 0$ which is equivalent to $K > \left(\dfrac{c}{qp}\right)$, there are either two real eigenvalues with the same sign (which is the sign of their sum) or complex conjugate and since $trJ(n^*,e^*)<0$, both eigenvalues have negative real part, so (n^*,e^*) is either a stable node or a stable spiral.

It is observed that the local stability of the equilibrium points depends on the equilibrium population, $n^* = \dfrac{c}{qp}$. When $n^* > K$, the equilibria *(0 ,0)* is a saddle point, *(K,0)* is a stable node and (n^*,e^*) is a saddle point.

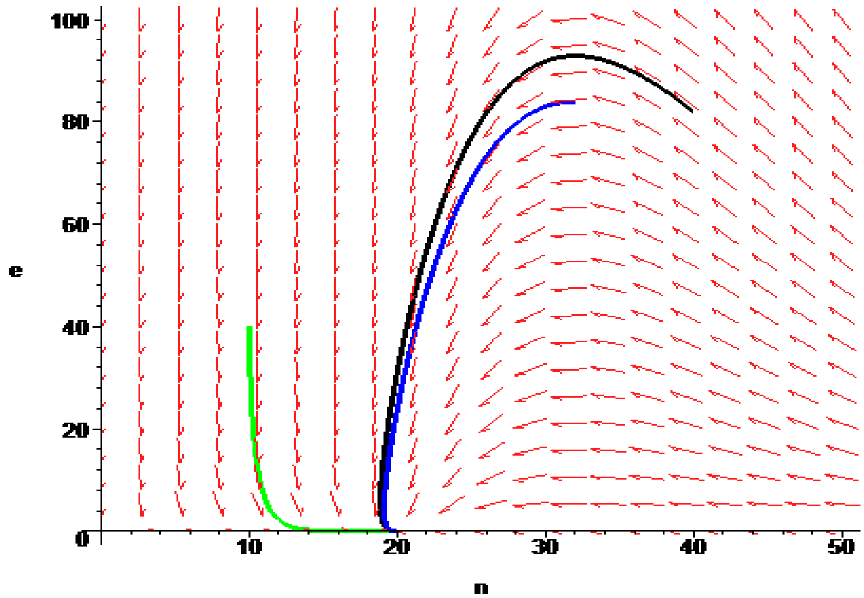

Figure 4.1 Phase portrait of the system with values *c=4, q=0.005, p=25* and *K=20.* The trajectories of *(10,40), (40,82)* and *(32,84)* are shown by the curves from left to right respectively.

When *n** < *K*, the equilibria *(0, 0)* is a saddle point, *(K, 0)* is a saddle point and *(n*,e*)* is a stable node or stable spiral.

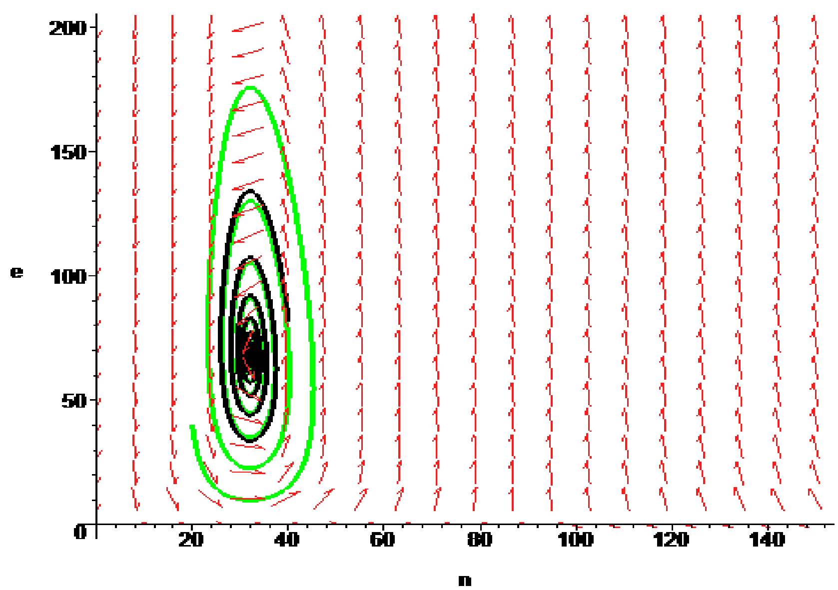

Figure 4.2 Phase portrait of the system with values *c=4, q=0.005, p=25* and *K=100.* The trajectories of *(20,40)*and *(40,82)* are shown by the green and black curves respectively.

Changing Price or Cost

We notice that that changing the price, cost and catchability changes the behavior of the system. The case of (*0, 0*) and (*K, 0*) is fishing free equilibrium. We focus on the third equilibrium (*n*, e**) which is more interesting. We look at the case where *n** <

K as shown in figure 4.3. In quadrant I, effort is increasing and population size is decreasing. In quadrant II, effort is decreasing whiles population size is increasing. In quadrant III, the population size is increasing and the effort is decreasing and in quadrant IV, both the population size and effort are increasing.

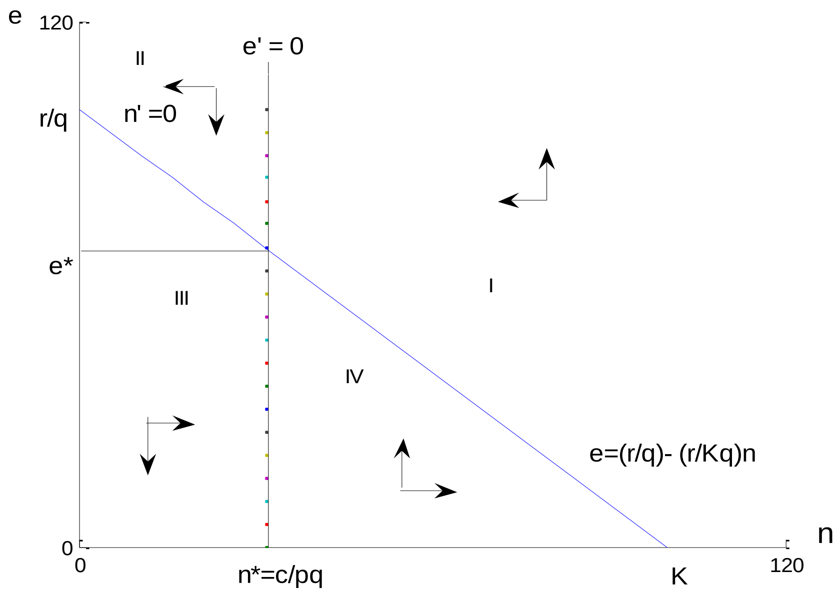

Figure 4.3 Plot of *e(t)* versus *n(t)*. The arrows show the direction of flow. Here *K* is greater than *n*=c/pq*.

Assume the system is in equilibrium, as shown in Figure 4.3. If price increases on the market, the line *(e'=0)* shifts to the left. We get a new equilibrium point (n_1^*, e_1^*) as shown in Figure 4.4. Thus the effort increases while the population size decreases. This is because when there is an increase in price of fish, the fishermen in an effort to increase their total revenue, increase the amount of fish harvested per day/time at sea. As a result, harvesting will eventually exceed the natural growth of the fish population causing the population size to decrease. If the price is increased infinitely, the equilibrium population size will tend to extinction.

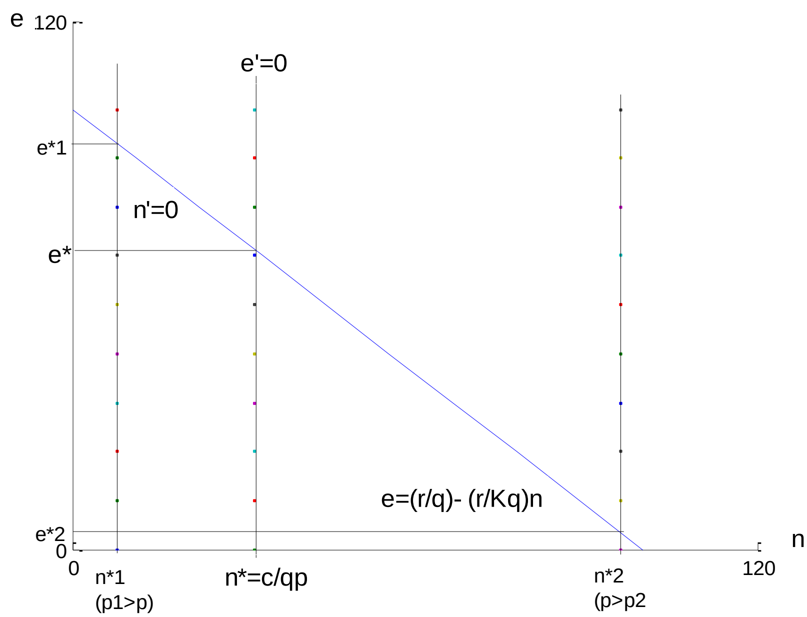

Figure 4.4

On the other hand, increasing cost, shifts the line *(e'=0)* to the right as shown in Figure 4.4. Here the effort equilibrium falls, whiles the population equilibrium increases. If cost rises infinitely, the total cost of production increases, tending the equilibrium effort to zero.

From the analysis, there is the possibility of extinction of the population if prices on the market should grow infinitely. One way to stop this could be by artificially increasing production costs by imposing taxes.