5

Discussion and Conclusion

Mathematical and computational approaches to population questions provide some of the most powerful tools in learning about nature; such approaches guide empirical work and provide a framework for analysis. Population dynamics has already generated many new examples of dynamical systems for mathematicians to study and many believe it will be one of the main driving forces of new mathematics during this century.

In this work, we have presented a fishery model with the logistic growth function. We then considered four harvesting functions; Constant harvesting, Holling´s Type I , Holling's Type II and Holling's Type III harvesting functions. For each functional response, we varied the harvesting function and our results have shown that taking into account variation of the harvesting function induces non-trivial effect. The analysis shows that according to parameter values and the type of Harvesting function used, one, two, three or four positive equilibria can coexist. These equilibria can change their stabilities or appear and disappear as a result of changing parameter values. Also we observed that changing market prices can creates two cases: (i) a stable no fishing equilibrium (ii) a stable persistent fishing equilibrium.

Indeed the existence of multiple equilibria occurs in the fishery using Hollings' Type II with $a<1$ and Holling´s Type III with k sufficiently large. What this means is that the harvesting functions saturate to a population size significantly below the carrying capacity of the fish population and in addition, the exploited fish population reproduces rapidly and has a large carrying capacity.

The case where two strictly stable positive equilibria can coexist with an unstable equilibrium in the middle for the same fishery is interesting. We noticed that the unstable equilibrium in the domain of attraction acts as a breakpoint or a threshold. This demonstrates that the initial stock (past history) of the fishery plays a very important role in its evolution and in its actual state. Therefore where a population will be in the immediate future depends on whether the initial condition is smaller or greater than the breakpoint value.

These two coexisting stable equilibria correspond to two choices for the fishery. One choice is for the fishery to support huge economic activities but maintain the stock at a low level with risk of extinction. The other choice is for the fishery to maintain the stock at a large level, far from extinction but support only small economic activities.

The results caution against harvesting at the maximum sustainable yield (MSY) when operating with Constant Harvesting, Type II and Type III harvesting function since the population in the long-term either settles to zero or a very low population.

The results suggests that for sufficently small initial population, Type I,II and III harvesting function are better options than constant harvesting because these functions decrease as the fish population decreases. This allows the natural growth rate of the population to increase above the harvesting rate.

It also suggests that sustainable fishery; where harvesting is done without the risk of extinction of the population, can be achieved when Holling´s Type III functional response is used. Here, given any initial population, that population will never decrease to zero or go extinct.

The results can be used for the estimation of certain parameters like catchabilty, fishing effort, fish price from stocking and harvesting data. This can prevent the risk of extinction since better management decision can be taken.

Governments can use taxation to control the fishing effort avoiding extinction of species.

A practical problem will be to know how much population to harvest at each time, t. For constant harvesting, this may not be a problem. However, for Holling´s Type I, II and III, this can be found as follows; Using an approximation method (Euler or Runge-Kutta Method), the net growth equation is solved for an approximated value *n(t).* This value is put into the harvesting function and this gives the catch at each time, t.

Modelling population dynamics as part of fishery management is frequently recommended but the associated complexity and uncertainty will always limit the extent to which the effects of fishing can be predicted (Auger, 2009). Finally, it will be very interesting to apply these models to empirical data to analyze the performance of each model.