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Population Dynamics

Dissertação de Mestrado

Dissertation presented to the Postgraduate Program in Mathematics of the Departamento de Matemática, PUC-Rio as partial fulfillment of the requirements for the degree of Mestre em Matemática.

Advisor : Prof. Carlos Frederico Palmeira
Co-Advisor : Prof. Paulo José Abreu Leitão de Almeida

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Abstract

Ghansah, Baaba Abassawah; Palmeira, Carlos Frederico (Advisor); Leitão de Almeida, Paulo José Abreu (Co-Adviser). **Population Dynamics**. Rio de Janeiro, 2010. 64p. MSc. Dissertation - Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Population Dynamics has traditionally been the dominant branch of mathematical biology, which has a history of more than 200 years. In this text, we present some of the fundamental ideas of Population dynamics. Using differential equations, we study the dynamical behavior of a fishery with time. First, we look at a dynamical model of the fish population with different Harvesting functions and in addition, the effect of market price on the dynamics of the fishery. In some cases multiplicity of stable equilibria is observed.

Keywords:

Differential Equations; Modeling; Population Dynamics; Fisheries.

Resumo

Ghansah, Baaba Abassawah; Palmeira, Carlos Frederico; Leitão de Almeida, Paulo José Abreu. **Dinâmicos de População**. Rio de Janeiro, 2010. 64p. Dissertação de Mestrado- Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Dinâmica populacional tem sido, tradicionalmente, o ramo dominante da biologia matemática, que tem uma história de mais de 200 anos. Neste texto, apresentamos algumas das idéias fundamentais da dinâmica de populações. Usando equações diferenciais, estudamos a evolução no tempo de uma população de peixes. Primeiro, olhamos para um modelo dinâmico com diferentes funções de pesca e, depois, o efeito do preço de mercado sobre a dinâmica da atividade pesqueira. Em alguns casos, a multiplicidade de equilíbrios estáveis é observada.

Palavras- chave

Equações Diferenciais; Modelagem; Dinâmicos de População; Pesqueiros.

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