1 Introduction

1.1 Motivation and contributions

Computer simulations of mechanical phenomena heavily rely on vector field generation, sampling, visualization and analysis. For their validation several techniques allow for measuring such vector fields, generating discrete data, which helps understanding physical behaviors. For example, Particle Image Velocimetry (PIV) is concerned with the quantitative investigation of fluids by imaging techniques (24) and has been used typically in mechanical engineering, in particularly modern aerodynamics and hydrodynamics research (29). The problem is that such real data or even simulation is typically corrupted by noise, which harms further simulation and puzzles the interpretation. Therefore, the processing of such vector fields typically involves a denoising step.

Classical denoising approaches rely on local coherences. They further consider that noise has a vanishing mean, and thus can be cancelled by averaging a piece of data with its neighbors. This has a smoothing effect which is widely used to generate scale-spaces of scalar fields such as images. However, as opposed to scalar physical quantities, for which one expects a globally smooth behavior in real experiments, vector fields can present rapidly changing directions. In fact those discontinuities are generally the most interesting part to analyze: they correspond to interfaces in fluid simulation, structural tissues when measuring brain water movement, faults and fractures in geophysical interpretation of soils. The problem is that when applying the classical filters theses features are generally removed.

We address this problem with our first contribution, in Chapter 3, where we propose a feature-preserving vector field filter based on random walks (21). This filter preserves the field's discontinuities while denoising. Moreover, this formulation allows for processing unstructured or irregularly sampled fields. Moreover the filter is intuitive because it has a probabilistic interpretation from the random walks formulation. There is still a persistent issue with such vector field denoising. Such random walk filters along with other classical convolution filters (37), rely on the assumption that the information is present in the measured data at a constant scale compared to the noise. In practice, an optimal filter scale can be automatically or manually chosen from a scale-space such as a hierarchical representation of the original data obtained by successive applications of such convolution filters (15, 4). However, for real vector fields with rapidly varying noise levels, using a *single* scale may keep either both noise and information or neither, leading to a delicate tradeoff.

As our second contribution, this problem is addressed in Chapter 4 where we propose a topology aware vector field denoising methodology (18) that lets the user control the topological changes caused by classical vector field filtering by the use of a suitable interface. The main part of this user interaction provides local tradeoffs between information and noise. The reconstructed field is then a smooth combination of different denoising scales.

Instead of preserving the discontinuities, as in our first contribution, we focus on controlling the topology of the vector field while denoising. In a variety of applications, in particular fluid dynamics, the field's singularities are the main features to be considered (13) and the interpretation is eased by detecting and identifying its singularities, like sinks, sources and saddles. Such topological features give a *global* information of the field which guides the user to adapt *locally* the scale filter.

1.2 Related work

In this section we briefly present some of the research literature related to our two contributions, the random walk and the topology-aware denoising techniques. We review denoising in general and the specific use of scale-spaces on vector fields. Then we describe the use of random walks and topology aware techniques in the literature.

Denoising. Among vector field filtering techniques on structured grids, several are specifically dedicated to colored image processing (23). In particular, color image filters focus on the reduction of impulse noise (34, 27, 17). In geometry processing several works have been proposed for noise reduction on surface normals (33, 19, 30, 31). Recently, Westenberg and Erlt (37) proposed a 2D vector field denoising algorithm that suppress additive noise by threshold-ing vector wavelet coefficients. Their method is restricted to work only on a structured grid of points. They compare their method to Gaussian filters, as

we will do throughout this dissertation. Close to our method, a class of filters has been introduced as generalized random walks for images (32) and meshes (30, 31).

Scale-spaces on vector fields. Scale-space techniques have become popular in computer vision for their capability to represent the multi-scale information inherently contained in real data. In particular, Bauer and Peikert (1) use scale-spaces to track vortices in 2D-time dependent computational on fluid dynamics simulations. Klein and Ertl (13) proposes a strategy to track singularities over multiples scales in order to evaluate the importance of the critical points to the analysis and interpretation of the vector field. The methodology developed in Chapter 4 employs such scale-space representations to let the user choose *locally* which scale to utilize for reconstruction.

Random walks. Random walk has many applications nowadays not only in visual computing but also in genetics, physics, medicine, chemistry, computer science, just to cite a few. The first work using random walks in computer vision is in the application of texture discrimination (36), and recently has been applied to image segmentation (8). In the field of image processing, random walk has been used to image enhancement (28) and filtering (32). The use of random walks in geometry processing was recently proposed by Sun *et al* (30, 31) for mesh denoising and also for mesh segmentation (14). Chapter 3 is inspired in their work where instead we deal with unstructured 2D vector fields.

Topology-aware techniques. Turbulent vector fields usually have structures in different scales which complicates their analysis. A possible solution to this problem is to analyze the topology of the vector field in order to automatically simplifying while keeping the most persistent features (35). Another strategy, proposed in this work, relies on the user knowledge of the vector field, letting him decide interactively which topological singularities to keep or to smooth. Such approach has already been proposed in the problem of surface reconstruction (12, 25).

1.3 Organization

This dissertation is organized as follows. Chapter 2 briefly introduces vector field topology and its application to discrete vector fields. In Chapter 3 the notion of random walks is introduced along with its interpretation. We then describe how to use these concepts to build the meshless feature-preserving filter. At the end of the chapter we present the results and implementation details. In Chapter 4 we describe the methodology for topology-aware denoising, detailing the techniques used in it and how they fit together. Again at the end, results are presented and analyzed. Finally, Chapter 5 concludes the dissertation by pointing out some limitations with both denoising techniques and suggesting improvements to be made.