2
The Model

2.1 Hypotheses

Suppose there are two players, 1 and 2 - in the empirical analysis, these will be respectively the party initially in government and the party initially in the opposition - disputing a cake of size $X > 0$. The players fight for the government, which gives the right to set up the allocation of the cake X. More specifically, the players play the following game:

1. Both players make a promise of the amount of the cake to give to the player 1 in case they become the government in the stage (4) of the game.

2. After observing the promises by both players, the player 2 decides whether to go to war ($W = 1$) or to remain in peace ($W = 0$). Each player faces a cost of war relative to peace, given by $c_i, i \in \{1, 2\}$. I suppose that $c_2 > 0$ and that $c_1 + c_2 > 0$.

3. If $W = 1$, nature selects the player 2 as the next periods' government with probability q^W. If $W = 0$, nature selects the player 2 as the next periods' government with probability $q^P < q^W$.

4. The player in government can implement the promise he made in the first stage of the game, or pay d to implement something else.

I allow the promises in the first stage of the game to be conditional on the decision of the player 2 to go to war in the second stage of the game. For this, we will denote as $V^i(W)$ the part of the cake promised in the first stage to player 1 by player i conditional on W (and, residually, player i will have promised to player 2 the amount $X - V^i(W)$). I also suppose that $V^i(W) \in [0, X]$.[1]

[1]In appendix A, I show a natural dynamic extension of this model that yields these hypothesis endogenously. More precisely, supposing that the government can make a transfer to the opposition in each period, denote by $V^i(W)$ the continuation value to player 1 associated with an equilibrium of the dynamic model. In the equilibrium of the dynamic game in the appendix, the continuation value to player 2 will be $X - V^i(W)$ in the model of the appendix, where X is the maximum social welfare attainable in equilibrium. Normalizing the agents utility functions yields continuation values between 0 and X.

The assumption that only the player 2 can go into war is not a trivial assumption. However, this model can be seen as a model in which both players go into war, and the action of one of the players is "almost given"(for instance, because player 1 has a very low cost of going to war). Mostly, this hypothesis is useful in simplifying the analysis.

Finally, note that d can be interpreted as a measure of commitment. If d is high, it means that it is very costly for a player to deviate from his promise, and thus, he has a high level of commitment. Alternatively, if d is low, the cost of not implementing a promise is low, and consequently, the players have low commitment levels. The term d might be interpreted as a reduced form of punishments arising from repeated interactions, or as checks and balances institutions that limit the scope to break previous promises. Even more, the term d may be related to the cost of changing the constitution, or the political costs of loosing one's reputation with voters or costs of "buying out"legislators to approve a change in the law.

2.2 Solution

We solve this game by backward induction. On the last stage, if the player 1 was selected as the government, he will only implement his promise if $V^1(W) \geq X - d$ (otherwise, he prefers to pay d and implement X). In the same way, if the player 2 was selected as the government, he will only implement his promise if $X - V^2(W) \geq X - d$ (otherwise, the player 2 prefers to pay d, implement $V^2(W) = 0$ and get $X - 0 - d$). Given this, we can restrict attention, with no loss of generality, to promises $V^1(W) \in [X - d, X]$ and $V^2(W) \in [0, d]$. Throughout the text, I will refer to these constraints on $V^i(W)$ as *commitment constraints*. I will also say that player 1 (2) is *bound by commitment* if player 1 (2) would be better off promising $V^1(W) < X - d$ (or, if player 2 would be better off promising $V^2(W) > d$).

Moving to the second stage of the game, the player 2 decides to go to war if:

$$-c_2 - q^W V^2(1) - (1 - q^W)V^1(1) \geq -q^P V^2(0) - (1 - q^P)V^1(0) \qquad (1)$$

Finally, moving to the first stage of the game, the players will anticipate that player 2 will decide to go to war according to (1) and choose $V^i(W)$. Formally, the problem player 1 faces when making promises is given by:

$$
\begin{aligned}
P1 = \max_{V^1(1),V^1(0)} \; & \{W[q^W V^2(1) + (1-q^W)V^1(1) - c_1] + \\
& \quad (1-W)[q^P V^2(0) + (1-q^P)V^1(0)]\} \\
s.t. \; & \begin{cases} W = 1 \leftrightarrow (1) \\ V^1(W) \in [X-d, X], \; W = \{0,1\} \end{cases}
\end{aligned}
$$

To put it in words, player 1 maximizes his expected utility subject to player 2 going into war if and only if (1) is valid and subject to his commitment constraint. The problem for player 2, which is referred to as $P2$, is analog to $P1$, with changes only to the commitment constraints, which become $V^2(W) \in [0, d]$, and to the fact that $V^i(W)$ enter the objective function multiplied by -1. Solving for both player 1 and player 2's problem yields the following proposition:

Proposition 1 *War occurs if*

$$c_2 < (q^W - q^P)X - (1-q^P)d \tag{2}$$

Proof: I start by showing the optimal $\{V^i(W)\}$, and then I show when these optimal promises imply inequality (1).

Result 1 *The optimal promises $\{V^i(W)\}$ by each player i are given by:*

- $V^2(1) = 0$
- $V^2(0) = 0$
- $V^1(1) = X$
- $(1-q^P)V^1(0) = \begin{cases} X & if \; c_2 > q^W X \\ (1-q^P)(X-d) & if \; c_2 < (q^W - q^P)X - (1-q^P)d \\ c_2 + (1-q^W)X & otherwise \end{cases}$

This can be proven in two ways: the first is to solve for the Lagrangean for $P1$ and for $P2$ supposing that $W = 1$ and that $W = 0$. After solving for these, check whether the players' objective function attains a higher value with $W = 1$ or with $W = 0$. I show an alternative, more intuitive proof.
Proof: To see why the $\{V^i(W)\}$ are as stated, start by $V^1(1)$. The higher player 1 sets $V^1(1)$, the higher is his payoff and the lower are the incentives of the player 2 to go to war (which would cost c_1 to player 1). For that, it is optimal to set $V^1(1) = X$. Also, note that the player 2 sets, on the optimum, $V^2(0) = 0$:

with this, the player 2 increases his payoff and decreases his incentives to go to war.

Regarding the choice of $V^2(1)$, note that the trade-off player 2 faces is that, by setting a lower $V^2(1)$, he increases his payoff, but raises his incentives to go to war. Hence, his trade-off is between setting $V^2(1) = 0$ or setting $V^2(1)$ in order to make inequality (1) turn to an equality. Consider the cases where the first option ($V^2(1) = 0$) is different from the second option (namely, that $V^2(1)$ is set to turn (1) into an equality). Note that, under the first option, player 2 receives the payoff from war with $V^2(1) = 0$. Under the second option, the player 2 receives the payoff from peace, which is set to be equal to the payoff from war with $V^2(1) > 0$. Thus, player 2 finds it optimal to set $V^2(1) = 0$.

Player 1 also faces the same trade-off when setting $V^1(0)$: by setting a higher $V^1(0)$, he increases his payoff from peace, but he increases the incentives for war. His trade-off is between setting $V^1(0) = X$ or setting $V^1(0)$ to turn (1) into an equality. Again, consider the cases in which the first option (setting $V^1(0) = X$) is different from the second option (setting $V^1(0)$ to turn (1) into an equality). In the first option, player 1 gets the payoff from war $-c_1 + q^W \times 0 + (1 - q^W)X$. In the second option, he gets the payoff from peace $(1 - q^P)V^1(0) + q^P \times 0 = c_2 + q^W \times 0 + (1 - q^W)X$ (where the last equality comes from the fact that (1) must be valid as an equality under this option). After some calculations, it is possible to see that player 1 would like to turn (1) into an equality whenever $c_1 + c_2 > 0$, which we supposed to be true. The fact that $c_2 > 0$ is enough to show that $V^1(0) > 0$, something required for $V^1(0)$ to be feasible. In this way, if player 1 was not bound by commitment issues, he would promise $(1 - q^P)V^1(0) = c_2 + (1 - q^W)X$ whenever that is smaller than X. When that is larger than X, player 1 can promise $V^1(0) = X$ and still avoid a war.

Still, player 1 can only make such a promise of $V^1(0)$ credibly if $V^1(0) \geq X - d$. This yields the result for $V^1(0)$. ■

Back to the proof of the proposition, the only cases in which there are wars are the cases in which turning (1) into an equality yields $V^1(0) < X - d$ (after all, these are the cases in which player 1 cannot promise to turn (1) into an equality). Replacing $(1 - q^P)V^1(0) = c_2 + (1 - q^W)X$ on the inequality $V^1(0) < X - d$ will give the equation (2). ■

Before the discussion of the equation of war, it is useful to discuss the intuition behind the optimal promised transfers. First, player 2 promises no transfers in this model because he does not have to provide incentives for player 1 to remain in peace. Second, player 1 promises to take the whole cake X for himself if player 2 goes into war, and to concede a part of the cake to player

2 in case he does not go into war.[2] These promises decrease the incentives for player 2 to go into war, and they only do so when player 1 has some (limited) capacity to commit to these promises (if player 1 had no commitment capacity, such promises would not be credible).

Even more, the proof of proposition 1 shows that player 1 would always like to set the transfer after peace, $V^1(0)$, to turn (1) into an equality. That would avoid the occurrence of a war. However, whenever inequality in (2) is valid, the level of $V^1(0)$ that turns (1) into an equality is lower than $X - d$. Consequently, when (2) is valid, player 1 is bound by his commitment constraint and cannot credibly promise to player 2 a share of the cake that would be big enough to avoid war. Consequently, a war happens in this model only when player 1 is sufficiently bound by commitment.

The equation (2) derives from applying the promised transfers to inequality (1). Player 2 goes into wars if the costs of going into wars are smaller than the benefits one can get by becoming the incumbent (given by X) times the increase in the probability of becoming an incumbent brought by wars $(q^W - q^P)$. Out of this, we subtract a term of commitment, since commitment allows player 1 to make a concession to player 2 in cases of peace, and thus, reduce what is up for grab with a war. Moreover, this term of commitment is multiplied by $(1 - q^P)$ because that is the probability of player 1 having to make concessions to player 2 (and, consequently, that is the probability that commitment issues come up).

It is also worth noting that, if $d = X$ - or, if the players can credibly promise any $V^i(W) \in [0, X]$ -, there are no wars, since, in such a case, a war would require $c_2 < 0$. I call the case in which $d = X$ *full commitment*.

Finally, it is worthy to highlight the importance of assuming that $c_2 > 0$ (or, player two faces a cost of war instead of a benefit) and that $c_1 + c_2 > 0$ (that war is inefficient). If war was not inefficient, the proof of the proposition above implies that player 1 would prefer not to provide incentives for player 2 not to go into war. In such a case, there would be war whenever $c_2 < (q^W - q^P)X$. Finally, if player 2 face $c_2 < 0$, it may not be possible for player 1 to make a transfer $V^1(0)$ that would turn player 2 indifferent between war and peace, since $V^i(W) > 0$. More explicitly, in the model above, if $c_2 < -(1 - q^W)X$, player 1 would be unable to promise $V^i(W)$ that would make player 2 indifferent between war and peace, and there would necessarily be a war.

[2] In this sense, the model allows for endogenous punishment in response to the occurrence of war.

2.3 Discussion of the model

To arrive at the result from proposition 1, I have made several simplifying hypotheses. Particularly, I assumed that only one player can go into war; that the cost of war and the probability of becoming the government are exogenous; and that information regarding costs of war is symmetric, despite the fact that there is a large literature studying wars due to asymmetric information (for a survey, see for instance Fearon [1995]). Moreover, I adopted a two period model instead of an infinite horizon model. In this section, I briefly discuss the motivation for these hypothesis.

I do not write a infinite horizon model, simplifying the structure to a two period model. The first reason for that is that while some countries have parties playing a war game infinitely, in other countries parties are short lived, so it is not necessarily reasonable to model the game as an infinitely repeated game. Even when it is reasonable to model the relationship between parties as an infinitely repeated interaction, the model proposed here can do the job: the transfers made in the last period of my model could be seen as continuation values associated with different equilibria of an infinitely repeated game. The players, by promising $V^i(W)$, would be promising to induce one equilibrium of an infinitely repeated game. This last argument is detailed in the appendix.

Moreover, the objective is to capture the extent to which players are committed to Coasian bargaining. Assuming an infinite game where players play, say, a trigger strategy, violates the purpose of the paper: this option would suppose a commitment mechanism is being used. Instead, the paper aims at evaluating if there is a commitment mechanism being used.

I do not take into account that war might happen due to asymmetric information (for instance, the parties in conflict do not know the other's cost of war), which is a traditional theory of war. However, it might be reasonable to assume that parties can learn on war the other's capacity of fighting and their cost of war. Hence, if war happened solely due to asymmetric information, the conflict would be over once the parties learned whichever information was not common knowledge and the duration of wars would be small. In my database, on average, the countries/years at war face a war duration of more than 7 years. In that way, I simplify the analysis and do not consider such a possibility of asymmetric information bringing about the results.

I also do not model the way that players endogenously choose the parameters q^W, q^P, c_2, X and d. Adding these components would just add structure to the determination of the model's parameters. Given countries are heterogeneous in the way these parameters are determined, I prefer to use a more parsimonious specification of the model, as opposed to an option imposing more structure. Additionally, I want to state, at least in theory, when d can be identified without the imposition of more structure into the process determining them.

It is true that many of these model's parameters are jointly determined with wars. For instance, the level of q^P could be determined by the level of democracy in the country, which could be determined through wars. Still, for the reasons highlighted before, I prefer to look at (assumedly) exogenous shocks to the model's parameters in order to identify commitment to Coasian bargaining.

Finally, considering that both players can go into wars could complicate considerably the empirical analysis by potentially adding multiple equilibria to the game. Simply adding asymmetric information in a global games fashion does not help, since promises signal whichever information was private. In order not to complicate the model too much, I opt for a simpler specification allowing only for one player to go into war.