4
Identification

4.1 Lack of identification of model's parameters with constant q^W and q^P

For the empirical implementation of the model, suppose that I have panel data of several countries i across years t that implicitly give me the probabilities of war for each set of observables Z_{it}. I denote these probabilities by $p(W_{it}|Z_{it})$. For the rest of the paper, I interpret player 1 as being the government in period t while player 2 is the rebel movement in period t.

Suppose at first that q^W and q^P are constant in the whole sample. Consider the two alternative structures: structure $S1$ is given by $c_{2it} = c_2(Z_{it}) + \delta^c_{it}$, by $X_{it} = X(Z_{it}) - \delta^X_{it}$ and by $d_{it} = d(Z_{it}) + \delta^d_{it}$, where $\delta^c_{it}, \delta^X_{it}$ and δ^d_{it} are random unobservable terms orthogonal to Z_{it}. Structure $S2$ is given by $\tilde{c}_{2it} = c_2(Z_{it}) + h(Z_{it}) + \delta^c_{it}$, by $X_{it} = X(Z_{it}) - m\frac{h(Z_{it})}{q^W - q^P} + \delta^X_{it}$ and by $\tilde{d}_{it} = d(Z_{it}) + (1-m)\frac{h(Z_{it})}{(1-q^P)} + \delta^d_{it}$, where m is an arbitrary constant and $h(Z_{it})$ is an arbitrary function of Z_{it}.

It is useful to discuss somewhat more the functional forms adopted. First of all, the inclusion of the observable variables Z_{it} are meant to capture heterogeneity in the model's parameters. Given the fact that the data I am dealing with is a panel of countries, it is expected that these parameters are heterogeneous, and it might be interesting to see how much these parameters vary across countries.

Still, these functional forms are not meant to capture some causal model leading from the covariates Z_{it} to the model's parameters. Instead, these functional forms are meant to capture an approximation of these parameters based on their correlation with variables Z_{it}. I allow c_2, X and d to depend on the same set of covariates. The reason for that is that X might depend on c_2 and d (say, since X is the maximum attainable value of being in government, it might include the value of facing a war in the future, which is correlated with c_2 and d). In the same way, it can be argued that d might depend on c_2 and X and that c_2 might depend on X and d.

To allow for a more compact notation, denote by $v_{it} = \delta^c_{it} + (q^W - q^P)\delta^X_{it} + (1 - q^P)\delta^d_{it} \sim F(v_{it})$. For sake of simplicity, I will take $F(v_{it})$ as known, so that I do not have to discuss identification of $F(v_{it})$. It is relatively easy to check for the following proposition:

Proposition 2 *Suppose q^W and q^P are constant in the data. Then, structures S_1 and S_2 are observationally equivalent.*

Proof: Applying the functional forms in structures $S1$ and $S2$ to the inequality (2) will yield the same inequalities and, thus, the same probabilities of war. ■

In other words, even taking $F(v_{it})$ as given, without observing variation in q^W and q^P, it is not possible to credibly identify c_2, X and d, since the structures $S1$ and $S2$ are observationally equivalent. In the next subsection, I move to the case in which I suppose I observe variation in q^W and q^P.

4.2 Allowing for varying q^W and q^P across units of observation

Suppose now that I observe a pair q^W_{it}, q^P_{it} that vary across countries i and years t. More explicitly, given the interpretation that the rebel movement in t is player 2, q^W_{it} (q^P_{it}) will measure the probability that, in year $t+1$, the rebel movement has come to government through war (peace). Moreover, suppose that q^W_{it} is linearly independent from q^P_{it}. First, I will show that the model's parameters are not identified if c_2, X and d are allowed to depend on an arbitrary manner on q^W_{it} and on q^P_{it}. The data also gives the probability of war given $Z_{it}, q^W_{it}, q^P_{it}$, denoted by $p(W_{it}|Z_{it}, q^W_{it}, q^P_{it})$. Then, I show that I can identify the model's parameters if (i) c_2 does not depend on q^W_{it} and q^P_{it} and (ii) X and d may depend on q^W_{it} and on q^P_{it}, but in a separable manner.

In this way, first, take the structure $\tilde{S}_1$, given by $c_{2it} = c_2(Z_{it}) + \delta^c_{it}$, by $X_{it} = X(Z_{it}) - \delta^X_{it}$ and by $d_{it} = d(Z_{it}) + \delta^d_{it}$. Also, suppose that $\delta^c_{it}, \delta^X_{it}$ and δ^d_{it} are random unobservable terms orthogonal to $Z_{it}, q^W_{it}, q^P_{it}$. For brevity, denote by $v_{it} = \delta^c_{it} + (q^W_{it} - q^P_{it})\delta^X_{it} + (1 - q^P_{it})\delta^d_{it} \sim F(v_{it}|q^W_{it}, q^P_{it})$. Note that the existence of the error terms δ^X_{it} and δ^d_{it} - given the assumption that they are orthogonal to Z_{it}, q^W_{it} and q^P_{it} - only add heteroskedasticity to v_{it}. Again, to simplify the discussion, I will suppose that $F(v_{it}|q^W_{it}, q^P_{it})$ is known and the non-parametric identification of $F(v_{it}|q^W_{it}, q^P_{it})$ will not be discussed.

It is important to mention that I suppose c_{it}, X_{it} and d_{it} do not depend on q^W and q^P. Without that, it is not possible to identify the parameters c_{2it}, X_{it} and d_{it} without imposing restrictive functional forms. This is a strong assumption: for instance, it could be that military investments affect both c_{2it}

and q_{it}^W. Alternatively, a more democratic country (with higher q^P) may not be able to repress the population, which could bring a lower cost of war for the rebel movement. Given the assumptions necessary for identification, one should focus only on variation in q_{it}^W and q_{it}^P that, conditional on Z_{it}, is exogenous to v_{it}.

With that in hands, the following proposition can be proved:

Proposition 3 *Suppose I observe variation in q_{it}^W, q_{it}^P. Suppose also that q_{it}^W, q_{it}^P and Z_{it} are also linearly independent. Then, structure $\tilde{S}_1$ is identifiable.*

Proof: Apply the equation from structure $\tilde{S}_1$ to inequality (2). This will give, for structure $\tilde{S}_1$, that:

$$F^{-1}(p(W_{it}|Z_{it}, q_{it}^W, q_{it}^P)|q_{it}^W, q_{it}^P) = -c(Z_{it}) + (q_{it}^W - q_{it}^P)X(Z_{it}) - (1 - q_{it}^P)d(Z_{it}) \tag{1}$$

Now, note that:

$$\frac{F^{-1}(p(W_{it}|Z_{it}, q_{it}^W, q_{it}^P)|q_{it}^W, q_{it}^P) - F^{-1}(p(W_{it}|Z_{it}, q_{it}^{W\prime}, q_{it}^P)|q_{it}^{W\prime}, q_{it}^P)}{q_{it}^W - q_{it}^{W\prime}} = X(Z_{it}) \tag{2}$$

and, since the left hand side of the above equation is observed in the database, $X(Z_{it})$ is identified.

In the same way, note that:

$$\frac{F^{-1}(p(W_{it}|Z_{it}, q_{it}^W, q_{it}^P); q_{it}^W, q_{it}^P) - F^{-1}(p(W_{it}|Z_{it}, q_{it}^W, q_{it}^{P\prime}); q_{it}^W, q_{it}^{P\prime})}{q_{it}^P - q_{it}^{P\prime}} = d(Z_{it}) - X(Z_{it}) \tag{3}$$

so that once equations (2) and (3) are summed, $d(Z_{it})$ is identified. Finally, applying the terms for $X(Z_{it})$ and $d(Z_{it})$ to equation (1), the term $c(Z_{it})$ is identified.■

Intuitively, the parameter of commitment is identified as the increase in the probability of war once q^P and q^W grow together by the same amount. If there was no commitment, the probability of war would not grow in response to symmetric growth in q^W and q^P. However, for such a strategy to identify credibly the parameters of the model, it is necessary that the variation in q^W

and q^P is exogenous to costs of war, values of going to war and to commitment (at least given Z_{it}).

Moreover, it is useful to note that while the parameters X_{it} and d_{it} capture the relationship between q_{it}^W, q_{it}^P and war, the parameter c_{it} captures factors other than q_{it}^W and q_{it}^P that should determine war. In this way, a way to understand the role of the parameter c_{it} is to view it as capturing other factors influencing war besides limited commitment and the value under dispute.