4 Klemperer model adapted

The theoretical model of this work is widely based on the Klemperer(1987 b). The information contained in the model of Klemperer (1987 b) that is maintained will be: the number of periods, the number of firms that appear in the industry, the differentiation of tastes among consumers and the changes of tastes that may or may not occur with the consumers between the periods. The consumers that have independent tastes between the periods will have frequency μ, the share of consumers who will come out and that will emerge from the economy will have the frequency of v, and, finally, it will have $(1 - \mu - v)$ of consumers that will not alter the tastes. The economy will be represented through the line segment $[0, t]$ and the consumers will have uniform distribution throughout this segment with density of 1. The existence of transport cost represents the differentiation of tastes by the consumers. Each firm will be placed in one of the extremes of this line segment. The A firm in the 0 point and the B firm in t.

To align the model with the portability number law effect analysis in the telephone market, it's done a little addition to the Klemperer (1987 b) model. As in it proper paper the switching cost was given and immutable, the modification done will fit in the following way: It will be an probability p ϵ $[0,1]$ of occurring an cut in switching costs. This reduction will occur in the beginning of the second period if the event happens. The recognizing of the occurrence of the switching cost cut will be the way of distinguish the effect of occurring in the first or in the second period. That is, doing comparative statics taking into account the probability, we see the effect of reduction while sticking in the first period, once this information only exists in this period. Therefore, changes in the switching cost occurred in the first period are gathered through the probability. In the model, this cut will be represented by $h \in [0, s]$.

It's relevant to highlight that the periods represents distinction to the firms and consumers. The distinction is represented by the same discount rate to the two firms in the value of λ.

Furthermore, it should be noted that each firm own marginal cost of c and the reserve price by the consumers is such that all who participate in the economy buy one of the products. In other words, $r \geq c + s + (t/2) + [t/(\mu + v)]$. Prices charged by the firm A will be p_1^A and p_2^A, respective to the two periods. q_1^A *and* q_2^A will be the amounts. Ultimately, π_1^A and π_2^A will represent the profits of the firm A in both periods. Analogously will be the nomenclature given to the prices, the amounts and the profit function of the firm B.

The model will be resolved primarily with the resolution of the second period. The resolution of the first period will use two hypotheses about how the consumers build expectative. The subgames perfect equilibrium is objectified.

4.1. Second period

The second period will compute the optimal allocation of firms after the consumers choose what firm they should consume in the first period.

The necessary condition so that consumers choose one firm and not the other will take into account not only the difference of price, but also the distance between the consumer and the two firms. For instance, a new consumer allocated in x will buy from firm A if, and only if, $p_2^A + x < p_2^B + (t - x)$. Therefore, the firm A will sell for a total of $v\left(\frac{t+p_2^B-p_2^A}{2}\right)$ of new consumers. If $r > s + \left(\frac{t+p_2^B-p_2^A}{2}\right)$, ensures that all consumers of this type will consume if the counterpart does not consume. Another condition that is supposed to the simplification of the model is to assure that both firms will sell to this type of consumer. It is supposed that $|p_2^B - p_2^A| \leq t$.

In relation to the mass $\mu\sigma^A$. In other words, the consumers those have independent preferences if compared to the previous period. The firm A will sell to the mass $\mu\left[\sigma^A\left(\frac{t+p_2^B-p_2^A+(s)}{2}\right) + \sigma^B\left(\frac{t+p_2^B-p_2^A-(s)}{2}\right)\right]$, if the reduction of the switching cost does not occur and, in an analogous manner, $\mu\left[\sigma^A\left(\frac{t+p_2^B-p_2^A+(s-h)}{2}\right)+\sigma^B\left(\frac{t+p_2^B-p_2^A-(s-h)}{2}\right)\right]$, ,if it occurs. Similar conditions as above are required so that every consumer in the economy participates consuming and that both firms sell to at least some consumer in this specific mass. This would be, respectively, $r>\left(\frac{s+t+p_2^B-p_2^A}{2}\right)$ and $|p_2^B-p_2^A+s|\leq t$.

The last consumers to be analyzed are related to those which possess the same preference in relation to the last period. Forcing the firm A, the mass related to those consumers is $(1-\mu-v)\sigma^A$. Ensuring that $r>\sigma^A t+p_2^A$ and $p_2^A+\sigma^A t+s\leq p_2^B+\sigma^B t$,[3] is sufficient to this occurs.

Therefore, when the reduction at the switching cost occurs, the amount sold by the firm A is:

$$q_2^A(p_2^A,p_2^B)=v\left(\frac{t+p_2^B-p_2^A}{2}\right)+\mu\left[\sigma^A\left(\frac{t+p_2^B-p_2^A+(s-h)}{2}\right)+\right.$$

$$\sigma Bt+p2B-p2A-(s-h)2+\ (1-\mu-v)\ \sigma At$$

$$=\frac{1}{2}\{(\sigma^A-\sigma^B)\big((1-\mu-v)t+\mu(s-h)\big)+t+(\mu+v)(p_2^B-p_2^A)\}$$

(1)

Following by:

$$\frac{\partial q_2^A(p_2^A,p_2^B)}{\partial p_2^A}=[-(\mu+v)]$$

$$\frac{\partial \pi_2^A(p_2^A,p_2^B)}{\partial p_2^A}=q_2^A+[p_2^A-c].\frac{\partial q_2^A(p_2^A,p_2^B)}{\partial p_2^A}$$

Therefore,

$$p_2^A=c+\frac{2q_2^A}{[\mu+v]}$$

[3] If happens to be a reduction at transaction costs, the required condition would be: $p_2^A+\sigma^A t+s-h\leq p_2^B+\sigma^B t$.

$$p_2^A = c + \frac{1}{[\mu+v]}\{(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu(s-h)\big) + t + (\mu+v)(p_2^B - p_2^A)\}$$

Analogously the firm A, the equation to the price of the firm B is:

$$p_2^B = c + \frac{1}{[\mu+v]}\{(\sigma^B - \sigma^A)\big((1-\mu-v)t + \mu(s-h)\big) + t + (\mu+v)(p_2^A - p_2^B)\}$$

Solving the system:

$$p_2^A = c + \frac{1}{[\mu+v]}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu(s-h)\big) + t\right\}$$

(2)

$$q_2^A = \frac{1}{2}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu(s-h)\big) + t\right\}$$

(3)

$$\pi_2^A = \frac{1}{2[\mu+v]}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu(s-h)\big) + t\right\}^2$$

(4)

Remaking the same math to when the cut in the switching cost does not occur, we have:

$$p_2^A = c + \frac{1}{[\mu+v]}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu s\big) + t\right\}$$

(5)

$$q_2^A = \frac{1}{2}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu s\big) + t\right\}$$

(6)

$$\pi_2^A = \frac{1}{2[\mu+v]}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu s\big) + t\right\}^2$$

(7)

It is noticed by observing the equation 2 that, to the firms with greater market share, the occurrence of the number portability represented by h affects negatively the price. That is, doing an weighing for market share, it is possible to say that such policy, if occurred in this period, raises the market competition.

Some cases can be highlighted so that is possible to see how the model is affected by different groups in which the consumers can belong. This analysis can be divided in three parts. The first would observe the symmetrical equilibrium where the consumers haves unaltered preferences, after observing the extreme opposite and finally focuses the equilibrium to the general case.[4]

When consumers possess unaltered preferences $(\mu + \nu = 0)$:

The quantity equation of the firm A becomes:

$$q_2^A = \frac{1}{2}\left\{\frac{1}{3}(\sigma^A - \sigma^B)(t) + t\right\}$$

Therefore,

$$q_2^A = \left\{\frac{1}{3}\sigma^A t\right\}.$$

May or may not have occurred the cut in the switching cost does not affect this result. This happens, because, the assumptions made to guarantee the purchase of all the consumers and those consume from the same firm when those preferences does not change produce this results. The firms have acted as monopolists so that their prices are restricted to the above condition.

Solving the first order condition it is reached the following standoff,

$$\frac{\partial \pi_2^A(p_2^A, p_2^B)}{\partial p_2^A} = \sigma^A t > 0$$

The only way of solving this standoff would be if the firms act in collusion.

When the consumers have independent preferences $(\mu + \nu = 1)$:

At the opposite end, case in which the consumers possesses independent preferences. Calculating the symmetric equilibrium to this case and considering a cut in the switching cost, it is observed that:

[4] The analysis made on the second period is aligned with the original Klemperer (1987) model

$$\frac{\partial \pi_2^A(p_2^A, p_2^B)}{\partial p_2^A} = 0 \iff \frac{1}{2}\left\{\frac{1}{3}(\sigma^A - \sigma^B)(s-h) + t + (p_2^B - p_2^A)\right\} - \frac{1}{2}[p_2^A - c] = 0$$

Considering the symmetric equilibrium:

$$p_2^A = t + c$$

That is, the market is as competitive as the market without switching cost. Although the switching cost reduces the number of marginal consumers, the firstly made hypothesis assure that such reduction will not happen. Should this hypothesis not be so restricted, reducing the switching cost would raise the number of consumers, but would not change the price charged by the firms in this case.

General case:

Using the equation (2) and the analogous to the firm B, we have that:

$$p_2^A = p_2^B = c + \left[{}^{t}/_{(\mu + \nu)}\right]$$

The near thing about this case is to observe that $\frac{\partial \pi_2^A(p_2^A, p_2^B)}{\partial \sigma^A} \cdot \frac{\partial^2 \pi_2^A(p_2^A, p_2^B)}{\partial \sigma^A \partial s} > 0$, worth remembering that the same goes for another firm. That is, the achieved market share in the first time haves an positive value to other firms in the second period. This derivative is increasing in the switching cost. In sum, the announce of an possible withdrawal of the switching cost could result in the diminish of the competition in markets that are in the period of obtaining market share.

4.2. second period

The analysis focus of this section will be the first period and will stick to the firm A. The analysis related to the firm B is analogous. The equation below represents the objective function that the firm A will aim at the first period.

$$\pi^A(p_1^A, p_1^B) = \pi_1^A(p_1^A, p_1^B) + \lambda \pi_2^A(p_1^A, p_1^B)$$

(8)

The reduction of the switching cost in the value of h occurs with the p probability. The equation that represents the total payoff expected by the firm is:

$$\pi^A(p_1^A, p_1^B) = [p_1^A - c]\sigma^A(p_1^A, p_1^B)t + \frac{\lambda p}{2[\mu+v]}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu(s-h)\big) + t\right\}^2 + \frac{\lambda(1-p)}{2[\mu+v]}\left\{\frac{1}{3}(\sigma^A - \sigma^B)\big((1-\mu-v)t + \mu s\big) + t\right\}^2$$

(9)

The market share of the firms will depend at how the consumers expectative are formed. Following the Klemperer (1987 b) model, It will be analyzed the results that has occurred from the creation of one type of expectations, the myopic consumers, which do not consider the second period.

The case: myopic consumers

The market share to this expectation type is simple. Just consider the same model of only one period. The result of this consideration would be:

$$\sigma^A(p_1^A, p_1^B) = \left(\frac{t+p_1^B-p_1^A}{2t}\right)$$

(10)

Given that $\sigma^A - \sigma^B = 2\sigma^A - 1$, follows that:

$$\pi^A(p_1^A, p_1^B) = [p_1^A - c]\left(\frac{t+p_1^B-p_1^A}{2}\right) + \frac{\lambda p}{2[\mu+v]}\left\{t + \frac{1}{3}\left(\frac{p_1^B-p_1^A}{t}\right)\big((1-\mu-v)t + \mu(s-h)\big)\right\}^2 + \frac{\lambda(1-p)}{2[\mu+v]}\left\{t + \frac{1}{3}\left(\frac{p_1^B-p_1^A}{t}\right)\big((1-\mu-v)t + \mu s\big)\right\}^2$$

(11)

consequently,

$$\frac{\partial\pi_1^A(p_2^A, p_2^B)}{\partial p_1^A} = \left(\frac{t+p_1^B-2p_1^A+c}{2}\right) - \frac{\lambda p}{2[\mu+v]}\frac{1}{3t}\big((1-\mu-v)t + \mu(s-h)\big)\Big\{t +$$

13p1B−p1At1−μ−vt+μ(s−h)-

$$\frac{\lambda(1-p)}{2[\mu+v]}\frac{1}{3t}\big((1-\mu-v)t + \mu(s)\big)\left\{t + \frac{1}{3}\left(\frac{p_1^B-p_1^A}{t}\right)\big((1-\mu-v)t + \mu(s)\big)\right\}$$

(12)

Analogously we arrive at the result to the firm B. Considering just the symmetric equilibrium, the equation of the resulting price is:

$$p_1^A = t + c - \frac{2\lambda}{3[\mu+v]}\{((1-\mu-v)t + \mu(s-ph))\}$$

(13)

Follows that:

$$\pi^A(p_1^A, p_1^B) = t^2\left[1 + \lambda + \frac{\lambda}{3(\mu+v)}((1-\mu-v) - \frac{2\mu(s-ph)}{t}\right]$$

(14)

Making the estimation to the case in that the number portability occurs in this period when it would be expected with $p_0 < 1$ probability, we get the following result:

$pA1p$=1−$pA1p$=$p0$=$2\lambda3\mu$+{μh(1−$p0$)}

(15)

In other words, it is noticed observing the equation (15) that the occurrence of the number portability in the first period, when the consumers possesses myopic expectations, is of price increase. Therefore, there is a distinction of effect depending on the industry maturity when this model is observed.

By the above equations some results are noticed. By the equation (13) it is noticed that the switching cost reduces the price charged in the first period, even when consumers have myopic expectation. This happens due to the competition between the firms for a greater market share. The reduction of switching cost, to this case, raises the price charged by the firm A and, in analogously manner, the same occurs with the price charged by the competitor firm.

For the case under review, the reduction in the switching cost, is characterized by the occurrence of the number portability, in the period of the market maturation, entails the reduction of competition between the firms. This characterizing positive price shocks as raise of the dispute. The competition intensifying, possibly the objective of creators of laws like that, would occur only in the posterior period. As we could see in the difference between the possible prices offered in such period considering the two possible states, of the occurrence or not of such reduction. As seen, the reduction of the cost, save some conditions, would result in a reduction of prices in mature market.

Therefore, the model above allows distinguishing the effect of the reduction in switching cost between both periods in question.