



**Bruno Lopes Vieira**

## **Extending Propositional Dynamic Logic for Petri Nets**

**Tese de Doutorado**

Thesis presented to the Programa de Pós Graduação em  
Informática of the Departamento de Informática, PUC-Rio as  
partial fulfillment of the requirements for the degree of Doutor  
em Informática

Advisor : Prof. Edward Hermann Haeusler  
Co-Advisor: Prof. Gilles Dowek

Rio de Janeiro  
March 2014



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## Abstract

Lopes, Bruno; Haeusler, Edward Hermann (advisor); Dowek, Gilles (co-advisor). **Extending Propositional Dynamic Logic for Petri Nets**. Rio de Janeiro, 2014. 101p. D.Sc. Thesis — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

Propositional Dynamic Logic (PDL) is a multi-modal logic used for specifying and reasoning on sequential programs. Petri Net is a widely used formalism to specify and to analyze concurrent programs with a very intuitive graphical representation. In this work, we propose some extensions of Propositional Dynamic Logic for reasoning about Petri Nets. We define a compositional encoding of Petri Nets from basic nets as terms. Second, we use these terms as PDL programs and provide a compositional semantics to PDL Formulas. Then we present an axiomatization and prove completeness regarding our semantics. Three versions of Dynamic Logics to reasoning with Petri Nets are presented: one of them for ordinary Marked Petri Nets and two for Marked Stochastic Petri Nets yielding to the possibility of model more complex scenarios. Some deductive systems are presented.

The main advantage of our approach is that we can reason about [Stochastic] Petri Nets using our Dynamic Logic and we do not need to translate it into other formalisms. Moreover our approach is compositional allowing for construction of complex nets using basic ones.

## Keywords

Logic; Modal Logic; Dynamic Logic; Petri Net; Stochastic Petri Nets;

## Resumo

Lopes, Bruno; Haeusler, Edward Hermann; Dowek, Gilles. **Extensões de Lógica Proposicional Dinâmica para Redes de Petri**. Rio de Janeiro, 2014. 101p. Tese de Doutorado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

Lógica Proposicional Dinâmica (PDL) é um sistema lógico multi-modal utilizada para especificar e verificar propriedades em programas sequenciais. Redes de Petri são um formalismo largamente utilizado na especificação de sistemas concorrentes e possuem uma interpretação gráfica bastante intuitiva. Neste trabalho apresentam-se extensões da Lógica Proposicional Dinâmica onde os programas são substituídos por Redes de Petri. Define-se uma codificação composicional para as Redes de Petri através de redes básicas, apresentando uma semântica composicional. Uma axiomatização é definida para a qual o sistema é provado ser correto, e completo em relação à semântica proposta. Três Lógicas Dinâmicas são apresentadas: uma para efetuar inferências sobre Redes de Petri Marcadas ordinárias e duas para inferências sobre Redes de Petri Estocásticas marcadas, possibilitando a modelagem de cenários mais complexos. Alguns sistemas dedutivos para essas lógicas são apresentados.

A principal vantagem desta abordagem concerne em possibilitar efetuar inferências sobre Redes de Petri [Estocásticas] marcadas sem a necessidade de traduzí-las a outros formalismos.

## Palavras-chave

Lógica; Lógica Modal; Lógica Dinâmica; Redes de Petri; Redes de Petri Estocásticas;

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*Logic is not a body of doctrine, but a mirror-  
image of the world. Logic is transcendental.*

**Ludwig Wittgenstein**, *Tractatus Logico-Philosophicus*.