4. Collusion Strategy

Subsection 4.1 describes one of the strategies, suggested by Skrzypacz & Hopenhayn (2004a, b), for a simplified version of the model described above. Subsection 4.2 shows us a modification of the preceding subsection strategy to adapt it to the present work's model.

4.1. Literature: The Model With A Known Winner

Skrzypacz & Hopenhayn (2004a, b) model's is different from the above for items 4 and 5: provision of the good is public, so that all participants know the identity of the winner but, on the other hand, participants cannot communicate among themselves after each auction. In this situation, Skrzypacz and Hopenhayn suggest the following collusion scheme:

Participants are divided between "included" and "excluded", the latter ones indexed from 1 to T. At each auction, the included participants can offer bids for the auctioned good, while the excluded participants are not supposed to. With probability α, the winner of the auction will become excluded of order T, the excluded participant of index 1 will become included (and therefore be able to make a bid on the next auction), and all other excluded participants will change their index from $t \in \{2, \dots, T\}$ to $t - 1$. With probability $(1 - \alpha)$, however, no changes are made to the statuses of the participants (the winner stays included, and the excluded participants maintain their old indexes).

Excluded participants are convinced to respect the collusion scheme (and not to offer bids - they cannot simply offer bids unlikely to win as no such bids exist: even a bid of 0 might win) through the threat that, if they win, the collusion is canceled, and all participants will bid according to the myopic Nash Equilibrium, which implies in a reduction of the present expected value of the payoffs for all participants (including, as Skrzypacz and Hopenhayn proves, for an excluded participant of order T with valuation v^H for the good).

For this scheme to be stationary, before the first auction T participants are randomly chosen to become excluded, each with one index $t \in \{1, \dots, T\}$.

Also, before each auction, one of the included participants is chosen to be the sure bidder, and he should make a bid even when he does not wish to (that is, when his valuation of the good plus the value of being in the collusion $\frac{T}{\alpha}$ periods in the future is lower than his expected valuation in a myopic Nash equilibrium). To force such a bid, it is sufficient to threaten the end of the collusion.

In their paper, Skrzypacz and Hopenhayn show that, for a number N of participants sufficiently high, there is δ^* such that, for all discount rates $\delta \geq \delta^*$, this strategy is sustainable and asymptotically efficient. They use this result to prove Proposition 4 in their paper:

> ***Proposition 4 (Skrzypacz and Hopenhayn)*** *In any repeated standard auction the optimal collusive scheme is asymptotically efficient for large cartels (large cartels can achieve almost first best without transfers or communication, conditioning only on the history of wins)*

For this proposition to be better understood, it is important to define an optimal collusive scheme:

> ***Definition 2 (Skrzypacz and Hopenhayn)*** *The optimal symmetric Public Perfect Equilibrium (optimal collusive scheme) is asymptotically efficient for large cartels if for any* $p < 1$ *and* $\epsilon > 0$ *we can find* N^* *large enough such that for every* $N \geq N^*$ *there exists* δ^* *so that for all* $\delta \geq \delta^*$*, there exists a Public Perfect Equilibrium that satisfies for every auction along the equilibrium path:*
>
> *1. The good is always obtained by some bidder*
>
> *2. The expected payment is at most* ϵ*.*
>
> *3. With probability at least* p *the winner has value at most* ϵ *less than the highest realized value.*

4.2. The Model With A Hidden Winner

The strategy described above can be adapted to the model without the auctioneers' announcement described in section 2. Notice that the strategy described by Skrzypacz and Hopenhayn depends on the identity of past winners, a piece of information that is not trivially obtained in this model. Therefore, to apply the suggested strategy, it comes to mind to ask the participants to inform in a message whether they won or not the auction. To make truth-telling the optimal strategy, it is sufficient to threaten the return to the myopic Nash Equilibrium in case some participant lies. With all players answering simultaneously the query of whether they won or not the last auction (and unaware of each other's responses), it is possible to check whether there was a lie by checking if $k \neq 1$ players admit victory. With this, telling the truth is part of the Nash Equilibrium in this node of the game[2].

Therefore, it is possible to ascertain:

Proposition 1: *In any repeated standard auction the optimal symmetric Communication Equilibrium (optimal collusive scheme) is asymptotically efficient for large cartels (large cartels can achieve almost first best without transfers, conditioning only on the history of communications and the last period's private information)*

The proof of the above Proposition is similar to Proposition 4 in Skrzypacz & Hopenhayn (2004a, b), with an added step to show that truth-telling is optimal. It follows in Appendix A.

Notice that the added steps' new inequalities are satisfied whenever the original inequalities are. Therefore, this scheme, in the new scenario, attains the same expected payoff as obtained in Skrzypacz & Hopenhayn (2004a, b)

[2]However, there are N Nash Equilibria, with each player i announcing victory and all other players $j \neq i$ announcing a loss; truth-telling is, however, the only correlated equilibrium