5. Extensions: What Can The Auctioneer Do?

5.1. Punishing Communications

On the attempt to reduce his losses due to the formation of the ring, the auctioneer may invest resources on the search for proof of communication between the participants. It is not the objective of this paper to study how much the auctioneer chooses to invest on the pursuit of cartels and, in fact, such a decision may not come from the auctioneer, but from the police or an antitrust agency, so that the costs of such a task may not fall on the principal. The result of such a pursuit is that, if communication is discovered, then each of the members of the cartel who sent communications on the preceding period will be forced to pay a fine.

Therefore, it is assumed that communication will be discovered with probability ρ (if it was done on the current period), and in this case each participant will pay a fine of value κ.

In this case, it might be inefficient to send messages every period. Therefore, the ring might make cycles of size $\tau \geq 1$ periods, and requesting that its participants send signals only at the end of each cycle. Thus, at these moments, each participant should send a message containing all his history of wins and losses during the previous cycle. Nevertheless, the random mechanism still works at each period, so that the winner might become excluded one period after he wins, and excluded participants might become included participants during the cycle.

The following proposition states that Proposition 1 holds only when the punishment is applied indiscriminately against all participants whenever at least one of them is caught communicating.

Proposition 2: *In case of a communication seeking mechanism, it is possible to keep the collusion above, with its asymptotic efficiency properties, as long as parties are punished whenever communication is observed by anti-trust authorities. However, if only parties which were observed communicating can be punished, then the collusion scheme is not even sustainable for large enough N.*

The additional proof needed is in the Appendix B. Intuitively, there are three differences when compared to the original proof: (1) to guarantee that, even though punishment happens only after some periods, it is still enough to sustain the collusion; (2) to guarantee that truth-telling is still optimal; and (3) to guarantee that the participants from the auction will be better off by participating in the collusion (and therefore will send their communications, despite the risk of being fined). There could be a problem with efficiency, as participants will expect to pay $\rho\kappa$ at each communication, but as τ can be arbitrarily large with sufficiently high δ, the total expected cost of paying the fine becomes small relative to the total expected payoffs.

However, when the fine applies only to those against whom there is evidence of communication, it is easy to see that for N large enough, the expected value of the cartel, which is limited by v^H/N will become smaller than the expected value of the fine, that is, $\rho\kappa$, making it profitable to deviate by not sending a communication. This result actually applies to any collusion scheme that depends on sending signals between participants, as long as the valuation is limited from above.

This result is also applicable to situations in which auctions occur very frequently (that is, when there might not be a fine, but there is no time for communication between auctions). In this case, τ might exogenously set to be positive, though this may not be a problem for the cartel, as long as δ is sufficiently close to 1.

5.2. Withholding The Good

Another possible response by the auctioneer would be to secretly withhold the good in some of the auctions (that is, after the bids have been made so that no participant knows that the good was not delivered). It will also be assumed that the highest bidder does not know his bid is the highest (otherwise, it would still be a correlated equilibrium for him to announce his "victory", as mentioned in Step 6B of the proof of Proposition 2). The idea behind this model is similar to the idea developed by Harrington & Skrzypacz (2010), in which firms repeatedly compete in a marked deciding their product's prices, in which both prices and sales' volume are private information, but total demand is a random variable with known distribution. In this article, they construct a cartel scheme in which the firms truthfully announce their individual sales, and in which firms who sold higher quantities of the product make payments to firms who sold less. Truth telling is made optimal by the threat of ending the collusion with a probability that depends negatively on the sum of reported sales.

In the present work, demand for the good follows a Bernoulli distribution, in which the auctioneer will not hold back the good (and therefore will deliver the good) with probability ψ, and will retain the object with probability $(1-\psi)$. As in both the schemes described in subsection 4.2, and in the article by Harrington and Skrzypacz, this collusion scheme will inhibit false announcements with the threat of reversal to the myopic Nash Equilibrium with probability $\phi(m)$, where $m \in \{\, 0, \dots, N\}$ is the sum of the announcements of the participants (assuming the participants announce 0 if they have lost and 1 if they have won the auction. As the punishment will happen with positive probability, it becomes useful to make the myopic Nash Equilibrium phase last for a finite number of periods, P. After this punishment stage, the collusion stage returns, with all participants maintaining their previous statuses (included or excluded of order $j \in \{1, \dots, T\}$).

By setting $\phi(m) = 1\ \forall m > 1$, the participants will avoid announcing a victory when they actually lost the auction. Also, making $\phi(1) = 0$ will make it become optimal for the winner to tell the truth (considering Proposition 1). The next Proposition, whose proof can be found in Appendix B, describes a result about collusion in this model.

Proposition 3: *In case the auctioneer retains the good with positive probability, the collusion scheme described above is still sustainable, with the possibility of temporary "punishment" (myopic Nash Equilibrium) phases in case the auctioneer cancels the auction. It is sufficient, though not necessary, that* $P \geq T/\alpha$ *and* $\phi(0) \equiv \phi = 1$