Referências Bibliográficas

[1] BURNSIDE, W.. **Theory of Groups of Finite Order**. Dover Publications, New York, 2nd edition, 1955.

[2] COLBOURN, C. J.; MAGLIVERAS, S. S. ; STINSON, D. R.. **Steiner Triple Systems of Order 19 with Nontrivial Automorphism Group**. Mathematics of Computation, 59:283–295, 1992.

[3] CONWAY, J. H.; CURTIS, R. T.; NORTON, S. P.; PARKER, R. A. ; WILSON, R. A.. **Atlas of finite groups**. Oxford University Press, New York, 1985.

[4] CONWAY, J. H.; SLOANE, N. J. A.. **Sphere packings, lattices and groups**. Springer-Verlag, New York, 1999.

[5] GRIESS JR, R. L.. **Twelve sporadic groups**. Springer-Verlag, Berlin, 1998.

[6] KASKI, P.; ÖSTERGÅRD, P. R. J.. **The Steiner Triple Systems of Order 19**. Mathematics of Computation, 73:2075–2092, 2004.

[7] MATHIEU, E.. **Memoire sur l'étude des fonctions de plusier quantités, sur la manière de les former et sur les substitutions qui les laissent invariables**. J. Math Pures Appl., 6:241–323, 1873.

[8] MATHIEU, E.. **Sur la fontion cinq fois transitive de 24 quantités**. J. Math Pures Appl., 18:25–46, 1873.

[9] WILSON, R. A.. **The Finite Simple Groups**. Springer-Verlag, Berlin, 2009.

A
Uma Conta Trabalhosa

Neste apêndice veremos como verificar se uma permutação de 24 pontos pertence à M_{24}. Para tanto, basta verificar se ela preserva o código de Golay. Como exemplo, faremos essa verificação para provar que a permutação α definida em 5.1.4 é um elemento de M_{24}.

Afirmação. A permutação α definida em (5.1.4) preserva o Código de Golay.

Demonstração. Considere a seguinte base do Código de Golay formada por octads:

$$
\begin{aligned}
\beta = \{&[0,0,0,1,0,1,0,0,0,1,0,0,0,0,0,1,0,1,0,0,1,1,1,0],\\
&[1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0],\\
&[0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1],\\
&[0,0,0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,1,1,0,0,1,0,1],\\
&[0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0],\\
&[1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0],\\
&[1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,0,1,0,1,1,0],\\
&[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1],\\
&[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0],\\
&[1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0],\\
&[0,1,1,1,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0],\\
&[1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]\}
\end{aligned}
$$

Usando o programa Maple, verifica-se facilmente que β é de fato linearmente independente (crie uma matriz A cujas linhas são os vetores de β e calcule o posto de A com o comando *rank(A)*).

A ação de α em cada octad da base β é ainda uma octad. De fato,

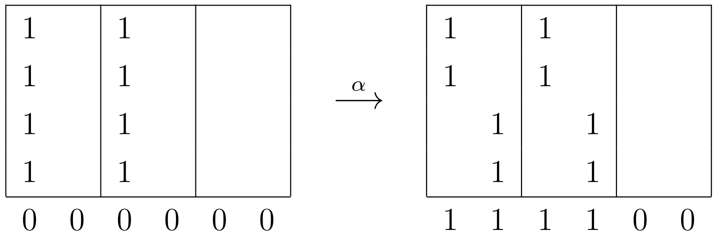
1) 0001 0100 0100 0001 0100 1110

					1
	1	1		1	1
					1
1			1		
$\overline{w}$	1	1	$\overline{w}$	1	$\overline{w}$

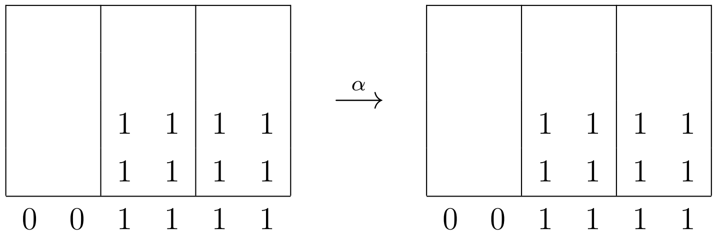
$\xrightarrow{\alpha}$

				1	1
	1	1		1	
	1	1			1
0	w	w	0	1	$\overline{w}$

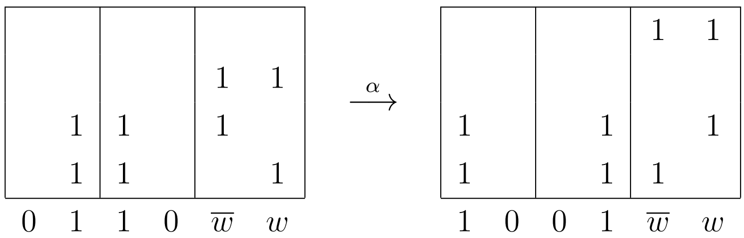
2) 1111 0000 1111 0000 0000 0000



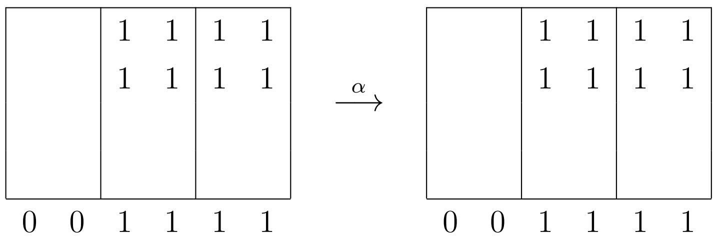
3) 0000 0000 0011 0011 0011 0011



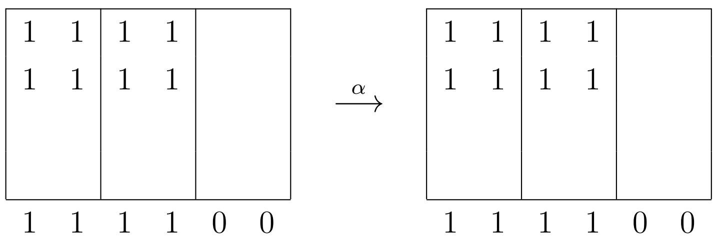
4) 0000 0011 0011 0000 0110 0101



5) 0000 0000 1100 1100 1100 1100



6) 1100 1100 1100 1100 0000 0000



7) 1100 0000 1100 0000 0101 0110

1		1			
1		1		1	1
					1
				1	
1	0	1	0	w	$\overline{w}$

$\xrightarrow{\alpha}$

1		1		1	1
1		1			
				1	
					1
1	0	1	0	w	$\overline{w}$

8) 1111 0000 0000 0000 0000 1111

1					1
1					1
1					1
1					1
0	0	0	0	0	0

$\xrightarrow{\alpha}$

1				1	
1				1	
	1				1
	1				1
1	1	0	0	1	1

9) 1111 0000 0000 0000 1111 0000

1				1	
1				1	
1				1	
1				1	
0	0	0	0	0	0

$\xrightarrow{\alpha}$

1					1
1					1
	1			1	
	1			1	
1	1	0	0	1	1

10) 1111 0000 0000 1111 0000 0000

1			1		
1			1		
1			1		
1			1		
0	0	0	0	0	0

$\xrightarrow{\alpha}$

1			1		
1			1		
	1	1			
	1	1			
1	1	1	1	0	0

11) 0111 1000 1000 1000 1000 1000

	1	1	1	1	1
1					
1					
1					
0	0	0	0	0	0

$\xrightarrow{\alpha}$

	1	1	1		
1				1	1
	1				
	1				
1	1	0	0	1	1

12) 1111 1111 0000 0000 0000 0000

$$\begin{array}{|cc|cc|cc|} \hline 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array}$$

Donde α preserva o Código de Golay, isto é, $\alpha \in M_{24}$.

□