3
The Model

We incorporate public employment in an overlapping generations framework with incomplete markets similar to Huggett (1996) and Imrohoroglu et al. (1999). In particular, we consider a public sector in which the government opens a given number of vacancies every period. Agents can choose to apply for these jobs or to work in the private sector. Candidates who are not hired by the public sector work in the private sector. The aim is to study the welfare implications of public employment policies.

3.1.
Demographics, Preferences and Endowments

The economy is populated with overlapping generations whose decisions follow a well-defined life-cycle structure. At any point in time there is a measure one of agents indexed by age $t \in \{1,2,\ldots,T\}$, who face an age-dependent probability π_t of surviving up to age t conditional of surviving up to age $t-1$. Once they reach age T, death is certain so $\pi_{T+1} = 0$. We assume an equal measure of agents is born at every period, so that the age distribution remains stationary. Thus, at every period, agents at age t constitute a constant fraction $\mu_t \in (0,1)$ of the population.

At t = 1, agents have identical preferences over streams of consumption $\{c_t\}_{t=1}^{T}$, given by

$$E\sum_{t=1}^{T}\beta^{t-1}\left(\prod_{i=1}^{t}\pi_i\right)u(c_t), \text{ with } u(c)=\frac{c^{1-\gamma}}{1-\gamma},\ \gamma>0$$

where β is the discount factor. We assume there is not altruism, so bequests are accidental and distributed lump sum to all agents alive.

Agents are not endowed with assets when they enter the labor market at

$t = 1$ (i.e. when they are born). However, they are endowed with one unit of labor, which is supplied inelastically until the age of $t = Tr < T$, when they are forced to retire. Moreover, each agent experiences a productivity profile that determines the value of this unit of labor over time. In particular, this productivity profile depends on: (i) the experience at the labor market, which is equal to age t in our model; (ii) a fixed level of human capital $\theta \in \{\theta_1, \theta_2, \ldots, \theta_m\}$ drawn by nature at the time the agent is born from a distribution in which each θ has mass $\mu_\theta \in (0,1)$ and $\sum_\theta \mu_\theta = 1$; and (iii) an uninsured idiosyncratic risk z that follows a finite state Markov chain with transition probabilities $\Pi(z', z) = \Pr(z_{t+1} = z' | z_t = z)$, where z, $z' \in \{z_1, z_2, \ldots, z_n\}$.[1]

Let $s \in \{g, y\}$ be the sector an agent is working in, where g stands for the public sector while y stands for the private sector.[2] We assume that the productivity profile, which may vary across sectors, is given by:

$$q_s(t, \theta, z) = \exp\left\{\gamma_1^s t + \gamma_2^s t^2 + \gamma_3^s(\theta) + \gamma_4^s(z)\right\}, \quad s \in \{g, y\}$$

Notice that γ_1^s and γ_2^s are parameters whereas $\gamma_3^s(\cdot)$ and $\gamma_4^s(\cdot)$ are functions to be specified in the next section. Importantly, these objects may depend on the sector $s \in \{g, y\}$ the agent is working in. We assume that in the private sector $\gamma_4^s(z) = z$, but, as we discuss later, it is not clear how one's idiosyncratic risk affects productivity (or wages) in the public sector.

3.2. Private Production

There is a representative firm that produces consumption goods with a Cobb-Douglas function augmented with public goods,

$$Y = G^{\xi} K_y^{\alpha} H_y^{1-\alpha}, \quad \xi, \alpha \in (0,1)$$

where K_y and H_y are aggregate capital and efficient labor units, respectively, employed at the private sector. Each period capital K_y depreciates at rate δ_y. Finally, we assume that public goods G, which are produced by the government, enhance productivity in the private sector.

[1] We rule out aggregate risk by assuming that this stochastic process is independent and identically distributed across agents.

[2] Since the public sector produces public goods G and the private sector produces consumption goods Y, we choose g and y, respectively, to denote these sectors throughout the paper.

3.3.
Markets Arrangements

There are no insurance markets for the idiosyncratic risk z. In particular, markets are incomplete in the sense that agents can only accumulate wealth trough risk-free bonds. Moreover, agents are subject to a no-borrowing constraint. We consider a closed economy with competitive markets. Hence, at every period, the interest rate r and the private wage rate w_y clear the markets for capital and efficient labor units, respectively. Finally, accidental bequests are distributed lump sum to all agents alive.

3.4.
Public Sector

We assume that the government taxes linearly labor income (τ_h), financial income (τ_a), consumption (τ_c) and bequests (τ_{beq}) in order to finance its consumption (C_g), investment in public capital (I_g), lump-sum transfers (Υ) and payroll bill ($w_g H_g$), where w_g is the public wage rate set by the government. The government can also issue public debt D, at the equilibrium interest rate r, to finance its deficit.

The government also produces public goods G with efficient labor units H_g and capital K_g, which depreciates at a rate δ_g.[3] We assume a Cobb-Douglas production function:

$$G = A_g K_g^{\eta} H_g^{1-\eta}, \quad \eta \in (0,1),$$

where A_g is the total factor productivity in the production of public goods. Since we normalize A_g to match the steady-state ratio G/Y we observe in the data, this formulation is general enough to accommodate a public sector in which only a fraction of public employment is used in productive activities.[4]

Notice that the public sector production has opposing effects on aggregate output. Since we consider a closed economy, it crowds out private production. In contrast, it also enhances productivity in the private sector.

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[3] In a stationary equilibrium, the law of motion of public capital implies that $\delta_g = I_g/K_g$. Thus, given an investment decision I_g, K_g is determined endogenously.

[4] Indeed, if ω is the fraction of efficient labor units employed to produce public goods, $G = \tilde{A}_g K_g^{\eta} (\omega H_g)^{1-\eta} = A_g K_g^{\eta} H_g{}^{1-\eta}$, where $A_g = \tilde{A}_g \omega^{1-\eta}$

Finally, the government also runs a pay-as-you-go pension system. In particular, workers of both sectors contribute with a fraction τ_{ss} of their labor income, while retired agents receive a flat benefit b. Since we calibrate the model economy to Brazil, where pension schemes are in deficit, we include the pension system in the government budget constraint, which reads

$$\tau_a r(K_y + D) + \tau_c C_y + (\tau_h + \tau_{ss})(w_y H_y + w_g H_g) + \tau_{beq} beq$$
$$= C_g + I_g + \Upsilon + rD + w_g H_g + B$$

in a stationary equilibrium. Notice that *beq* stands for accidental bequests and B stands for the aggregate level of pension benefits b.

We assume that tax instruments, public debt, pension benefits, and investment are exogenously set, in the sense that we calibrate them to capture how fiscal policy is conducted in Brazil. The government consumption is the policy variable used to balance the government's budget at the benchmark calibration. It remains to discuss how employment is chosen and wages are set in the public sector.

3.4.1.
Admission Policy

At every period, for each level of human capital $\theta \in \{\theta_1, \theta_2, \ldots, \theta_m\}$, the government is willing to employ $\lambda(\theta)$ workers. Hence, it opens the number of vacancies necessary to accomplish this goal. Agents choose either to apply for a public job or to work in the private sector. For simplicity, we assume an agent can only apply for vacancies assigned to her level of human capital. In our calibration, we proxy human capital θ by the level of schooling, which is observable by the government. In practice, depending on the complexity of the job, the government requires a minimum degree of schooling from candidates.

Depending on the model's parameters, public jobs may attract a larger number of candidates than open vacancies. If this is the case, in order to fill the vacancies, the government hires the most productive candidates.[5] This selection mechanism emulates a public exam in which performance is positively associated with productivity. Admissions to public jobs trough public exams are widely

[5] Since labor is inelastically supplied, candidates work in the private sector if they are not hired by the government.

spread across countries. In Brazil, for instance, most of the vacancies are filled with agents who perform well in a public exam designed to test the knowledge necessary to perform a specific job.

Although the age t also affects the productivity profile $q_s(t,\theta,z)$, $s \in \{g,y\}$, it is not clear how age t affects performance in a public exam. On one hand, older agents have more time to prepare themselves for the exam. On the other hand, performing well in an exam may require a specific skill that tends to depreciate over time, especially for those agents who have spent some years working in the private sector. Hence, we assume that admission to the public sector depends only on human capital θ and idiosyncratic risk z.

In a stationary equilibrium, the selection mechanism we explain above implies that, for each level of θ, there is a threshold $\underline{z}(\theta)$ such that, all open vacancies necessary to keep $\lambda(\theta)$ workers in the public sector are filled with type-θ agents who experience $z \geq \underline{z}(\theta)$. Importantly, not necessarily all type-θ agents with $z \geq \underline{z}(\theta)$ apply for a public job. Indeed, the private sector might be more attractive for some of them.

Finally, as we observe in practice, we assume public workers cannot be fired, but they may quit if the private sector becomes more attractive for them.

3.4.2. Wage Setting

Let w_y and w_g be the wage rates paid in the private and public sectors, respectively. Recall that productivity is given by:

$$q_s(t,\theta,z) = \exp\left\{\gamma_1^s t + \gamma_2^s t^2 + \gamma_3^s(\theta) + \gamma_4^s(z)\right\}, \quad s \in \{g,y\}.$$

Since we assume the private sector behaves competitively, the productivity profile $q_y(t,\theta,z)$ has a dual role. First, $q_y(t,\theta,z)$ is employed to produce consumption goods. Second, $w_y q_y(t,\theta,z)$ is the wage schedule in the private sector. Hence, by using data at the individual level on wages, experience and human capital, one can estimate γ_1^y, γ_2^y and $\gamma_3^y(\cdot)$ and, thus, calibrate the productivity profile in the private sector.

However, even in a competitive equilibrium, the government may choose to not reward productivity. In this case, $w_g q_g(t,\theta,z)$ might not be the wage schedule in the public sector. Hence, we define a wage-setting rule in the public sector denoted by $w_g \hat{q}_g(t,\theta,z)$, where

$$\hat{q}_g(t,\theta,z) = \exp\left\{\hat{\gamma}_1^g t + \hat{\gamma}_2^g t^2 + \hat{\gamma}_3^g(\theta) + \hat{\gamma}_4^g(z)\right\}.$$

In a similar fashion, we can use data on public workers to estimate $\hat{\gamma}_1^g$, $\hat{\gamma}_2^g$ and $\hat{\gamma}_3^g(\cdot)$[6] and, thus, calibrate the wage-setting rule in the public sector.

We postpone to the next section the discussion on how we set $q_y(t,\theta,z)$, $q_g(t,\theta,z)$ and $\hat{q}_g(t,\theta,z)$ to solve numerically the model.

3.5. Recursive Equilibrium

In this paper, we focus on the properties of a stationary competitive equilibrium in which the measure of agents, defined over an appropriate family of subsets of the individual state space, remains invariant over time.

3.5.1. The Agents' Problem

The agents make two types of decision during their lives. First, they choose how to allocate their disposable income between consumption and risk-free bonds. Second, they decide whether to work in the private or the public sector. Once hired by the public sector, workers cannot be fired but they may quit. Finally, as mentioned above, not all candidates have the option to work in the public sector, as their idiosyncratic productivity may not be high enough.[7]

In this context, there are five individual state variables: the age *t*, a fixed level of human capital *θ*, the idiosyncratic risk z, the previous sector *s* one works, and the amount of assets *a* accumulated. We assume that $s = y$ for those agents at the age of $t = 1$. Given our assumptions on the hiring and firing of government employees, the agent's problem prior to retirement, i.e. for $t < Tr$, is given by:

$$V_t(a,s,z;\theta) = \max_{c,a',s'}\left\{u(c) + \beta\pi_{t+1}\sum_{z'}\Pi(z',z)V_{t+1}(a',s',z';\theta)\right\},$$

[6] Many empirical studies estimate these objects for both sectors and find substantial differences across them (e.g. Braga et al. (2009)). There are two possible complementary explanations for this discrepancy. First, the productivity profile varies across sectors. Second, productivity plays a minor role when setting public wages.

[7] Recall that, for a given θ, the government only hires those type-θ agents with $z \geq \underline{z}(\theta)$.

subject to

$$(1+\tau_c)c + a' \le [1+(1-\tau_a)r]a + (1-\tau_h - \tau_{ss})w_{s'}\hat{q}_{s'}(t,\theta,z) + \Upsilon + (1-\tau_{beq})beq$$

$$c \ge 0, \quad a' \ge 0,$$

$$s' \in \begin{cases} \{y\} & if \quad z < \underline{z}(\theta) \quad and \quad s = y \\ \{g,y\} & otherwise \end{cases}$$

$$V_{Tr}(a',s',z';\theta) = \tilde{V}_{Tr}(a'), \text{ for all } s',z',\theta$$

Where $\tilde{V}_{Tr}(a')$ is the value of retiring at the age of $t = Tr$. Notice we implicitly define $\hat{q}_y(t,\theta,z) = q_y(t,\theta,z)$ for all t, θ, z, so we can write a single problem for all agents.

After retiring, i.e. for $Tr \le t < T$, the agent's problem is a cake-eating one:

$$\tilde{V}_t(a) = \max_{c,a'}\left\{u(c) + \beta\pi_{t+1}\tilde{V}_{t+1}(a')\right\},$$

subject to

$$(1+\tau_c)c + a' \le [1+(1-\tau_a)r]a + b + \Upsilon + (1-\tau_{beq})beq$$

$$c \ge 0, \quad a' \ge 0$$

$$\tilde{V}_T(a') = 0 \text{ for all } a'.$$

By solving the problems above, one obtains decision rules for consumption $c_t(a,s,z;\theta)$, savings $a'_t(a,s,z;\theta)$ and job sector $s'_t(a,s,z;\theta)$ along the life-cycle $t = 1, \ldots, T$.

3.5.2. Definition and Policy Experiment

The definition of stationary competitive equilibrium is standard, except for the role the government has in hiring workers. In particular, (i) given prices and fiscal policies, agents solve their problems; (ii) given prices and fiscal policies, the representative firm maximizes profits; (iii) accidental bequests are distributed lump-sum to all agents alive; (iv) the private wage rate w_y and the interest rate r clear the labor and capital markets, respectively; (v) the government produces public goods and chooses fiscal policy objects, which remain invariant over time, subject to a balanced budget constraint and the law of motion for public capital; (vi) for each θ the government specifies a threshold $\underline{z}(\theta)$ such that it employs $\lambda(\theta)$ workers; and (vii) for each age t and human capital θ, there is a stationary measure $\psi_{t,\theta}$ defined over an appropriate family of subsets of the individual state space.[8] A formal definition is provided in Appendix A.

We are interest in welfare properties of the stationary equilibrium. In particular, we study the welfare implications of different levels of public employment, which is given in equilibrium by:

$$L_g = \sum_{t<T_r} \mu_t \sum_{\theta} \mu_\theta \int I_{\{s'_t(a,s,z;\theta)=g\}} d\psi_{t,\theta}(a,s,z) = \sum_{\theta} \lambda(\theta),$$

where ***I*** is the indicator function.[9] The policy experiment we study is to increase or decrease $\lambda(\theta)$ proportionally for all θ.[10] In this case, public employment L_g increases or decreases, at the same time that the proportion of public workers across human capital levels remains the same.

3.5.3. Welfare Criterion

The optimal size of public employment maximizes an ex-ante utilitarian welfare criterion in a stationary equilibrium. Following Conesa et al. [2009], we consider only the welfare of newborn agents. Thus, social welfare reads

$$\sum_{\theta} \mu_\theta \int V_1(a,s,z;\theta) d\psi_{1,\theta}(a,s,z)$$

Throughout the paper, we report welfare effects in terms of consumption equivalence. In other words, the welfare effect associated with a given policy is defined by how much lifetime consumption would have to increase uniformly across newborn agents in the benchmark economy in order to equalize social welfare measures across stationary equilibriums.

By adapting the methodology from Flodén (2001) to this environment, we decompose the overall welfare effect of a change in public employment into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of insurance in the economy. See Appendix B for more details.

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[8] The individual state space is the Cartesian product of the spaces associated with the individual state variables, i.e., a, s, z.

[9] Notice that L_g is not equal to H_g, which is the aggregate level of efficient labor units employed at the public sector. In particular,

$$H_g = \sum_{t<T_r} \mu_t \sum_{\theta} \mu_\theta \int I_{\{s'_t(a,s,z;\theta)=g\}} q_g(t,\theta,z) d\psi_{t,\theta}(a,s,z)$$

[10] For each θ, $\underline{z}(\theta)$ also has to adjust so public vacancies can be filled.

We also consider a conditional welfare criterion. In particular, for each θ, we calculate the aforementioned welfare effects considering

$$\int V_1(a,s,z;\theta)d\psi_{1,\theta}(a,s,z)$$

The aim is to study how welfare effects vary across groups with different levels of human capital.