Pontifícia Universidade Católica
do Rio de Janeiro

Daniel Haanwinckel Junqueira

A Compensating Differentials Model of Informal Labor Markets

DISSERTAÇÃO DE MESTRADO

Dissertation presented to the Programa de Pós-Graduação em Economia of the Departamento de Economia, PUC–Rio as partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Rodrigo Reis Soares

Rio de Janeiro
March 2013

Pontifícia Universidade Católica
do Rio de Janeiro

Daniel Haanwinckel Junqueira

A Compensating Differentials Model of Informal Labor Markets

Dissertation presented to the Programa de Pós-Graduação em Economia of the Departamento de Economia do Centro de Ciências Sociais da PUC–Rio as partial fulfillment of the requirements for the degree of Mestre.
Approved by the following commission:

Prof. Rodrigo Reis Soares
Advisor
Departmento de Economia — PUC–Rio

Prof. Gustavo Gonzaga
Departmento de Economia — PUC–Rio

Prof. Gabriel Ulyssea
Instituto de Pesquisa Econômica Aplicada

Prof. Monica Herz
Coordinator of the Centro de Ciências Sociais da PUC–Rio

Rio de Janeiro, March 19th, 2013

Daniel Haanwinckel Junqueira

Daniel Haanwinckel graduated in Control and Automation Engineering from Unicamp (Campinas, Brazil). He worked for two years at Espirito Santo Investment Bank before starting his graduate studies at PUC–Rio.

Bibliographic data

Haanwinckel, Daniel

A Compensating Differentials Model of Informal Labor Markets / Daniel Haanwinckel Junqueira; advisor: Rodrigo Reis Soares — 2013.

66f. : il.; 30 cm

Dissertação (Mestrado em Economia) — Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 2013.

Inclui bibliografia

1. Economia – Dissertação. 2. Informalidade. 3. Mercado de trabalho. 4. Busca. 5. Diferenciais Compensatórios. 6. Brasil. I. Soares, Rodrigo R.. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. III. Título.

CDD: 330

Acknowledgments

To my mother, Maria de Fátima, for encouraging me since I was a kid to be curious and for being a shining example of dedication, resilience and integrity.

To my advisor Rodrigo Soares for conversations that covered not only labor/development but mainly how to do good research and to be a good researcher.

To Gabriel Ulyssea and Gustavo Gonzaga for excellent comments on my research, both before and after the defense.

To the other professors in PUC–Rio Department of Economics, especially Cláudio Ferraz, Juliano Assunção and Eduardo Zilberman, for teaching me many things and for being accessible and supportive.

To my brothers Beto and Priscila for the partnership and help along many years, and to Garcia for being a model of work ethic.

To my colleagues at PUC–Rio, from whom I learned much and whose company made the graduate program easier.

To CAPES, FAPERJ and PUC–Rio for the financial support during the masters.

Finally, to Thaís, for the company, the helping hand and for giving me so much joy over the last two years.

Abstract

Haanwinckel, Daniel; Soares, Rodrigo R. (advisor). **A Compensating Differentials Model of Informal Labor Markets**. Rio de Janeiro, 2013. 66p. Dissertação de Mestrado — Departmento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This work develops a search and matching model of informality with heterogeneous workers and firms, minimum wages, and mandated benefits. In equilibrium, informal firms are smaller, less productive and employ fewer skilled workers as a result of self-selection. Informal workers are generally compensated for the lack of mandated benefits by receiving higher wages, but a simple comparison of average earnings between sectors shows a formality wage premium because of compositional effects. In addition, a binding minimum wage can break the equalizing differentials relation, so that there might be a formality wage premium among low wage workers even after controlling for individual productivity. The model is calibrated using Brazilian data and used to explain the evolution of labor market outcomes in that country from 2003 to 2012. The results suggest that rising schooling was the most important factor behind the reversal of the informality trend in Brazil, which remains a puzzle in the current literature. It is also shown that, for the calibrated model, a progressive payroll tax would lead to a decrease in both unemployment and informality without compromising tax revenues.

Keywords

Informality; Labor Market; Search; Compensating Differentials; Brazil;

Resumo

Haanwinckel, Daniel; Soares, Rodrigo R. (orientador). **Um Modelo de Diferenciais Compensatórios para Mercados de Trabalho Informais**. Rio de Janeiro, 2013. 66p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Este trabalho desenvolve um modelo de busca por emprego com trabalhadores e firmas heterogêneos, salário mínimo e benefícios trabalhistas. Em equilíbrio, as firmas informais são menores, menos produtivas e empregam menos trabalhadores qualificados devido a um efeito de seleção. Trabalhadores informais geralmente recebem salários maiores para compensar a falta de benefícios trabalhistas, mas uma simples comparação de salários médios entre setores mostra um prêmio salarial para a formalidade devido a um efeito de composiçao. Além disto, o salário mínimo pode quebrar a relação de diferenciais compensatórios, de forma que haja um prêmio de formalidade para trabalhadores pouco qualificados mesmo após controlar para produtividade individual. O modelo é calibrado usando dados do Brasil e utilizado para explicar a evolução do mercado de trabalho neste país de 2003 até 2012. Os resultados sugerem que o aumento da escolaridade média foi o fator mais importante por trás da reversão da tendência de informalidade no Brasil, que ainda é um fato não plenamente explicado na literatura acadêmica. Também mostra-se que, no modelo calibrado, impostos progressivos sobre folha salarial poderiam levar a uma redução tanto do desemprego quanto da informalidade sem comprometer as receitas do governo.

Palavras-chave

Informalidade; Mercado de trabalho; Busca; Diferenciais Compensatórios; Brasil;

Contents

List of Tables

> *I do not see how one can look at figures like these without seeing them as representing* possibilities. *Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's [between the 1960s and 70s]? If so,* what, *exactly? If not, what is it about the 'nature of India' that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.*
>
> *This is what we need a theory of economic development* for*: to provide some kind of framework for organizing facts like these, for judging which represent opportunities and which necessities.*

Robert Lucas, *On the mechanics of economic development.*

I
Introduction

Informality was a major policy concern in developing countries during the 1990's. The shadow sector was on the rise throughout the world, reaching over a quarter of GDP in most non-OECD economies.[1] Surprisingly, that trend was sharply reversed in some Latin American countries by the early 2000's, with informality rates among salaried workers in Brazil, Ecuador, Peru and Uruguay declining by one fifth or more by 2010.[2] These shifts remain largely unexplained and it is not clear whether they can be easily accounted for by current models of informality.

The Brazilian case, corresponding to a reduction in informality of 10 percentage points between 2003 and 2012, is particularly difficult to understand based on factors commonly discussed in the literature. The minimum wage increased by roughly 60% in real terms after the mid 2000's, more than twice the growth rate in GDP per capita, while changes in labor legislation and payroll taxes were minimal. At the same time, increased productivity, stricter enforcement of labor regulations, and changes in the educational composition of the workforce may have had important implications for the labor market. But some of these factors do not feature prominently in either the policy or academic debates on informality, and there is no single theoretical framework available capable of dealing with all these dimensions simultaneously, and allowing for an assessment of their respective roles in the observed increase in formalization. The understanding of the pattern of changes experienced by Brazil, besides being important on its own, has clear policy implications for many developing countries where labor informality is still prevalent and rising.

In this paper, we study the case of Brazil through the lens of a search and matching model of informality that allows for worker and firm heterogeneity, minimum wages, and mandated benefits. In the model, workers can be either skilled or unskilled. Unemployed workers simultaneously search for both formal and informal jobs in labor markets specific for each skill level. Firms are heterogeneous in a fixed capital endowment and use a technology that displays

[1] Schneider & Enste (2000).
[2] Tornarolli et al. (2012).

capital-skill complementarity. Firms decide on how many skilled and unskilled vacancies to post at each instant, and on whether to comply with labor regulations. By choosing not to comply, firms avoid paying payroll taxes and are not restricted by the minimum wage, but become subject to an informality penalty associated with firm size (representing the probability of being audited and the associated fine). The labor regulation also includes mandated benefits which, from the perspective of employees, make formal jobs more valuable than informal jobs for a given wage.

In a steady-state equilibrium, firms with lower capital endowment employ fewer workers and choose to operate informally. These firms also employ a lower fraction of skilled workers. In general, informal workers are compensated for the lack of mandated benefits by receiving higher wages, but this equalizing differentials condition can be broken by minimum wages. If the minimum wage binds for unskilled workers, then these workers strictly prefer to hold a formal job, but are willing to accept informal offers in equilibrium in order to avoid unemployment.

We calibrate the model to depict the Brazilian labor market as of October 2003 and then examine whether changes in tax rates, mandated benefits, enforcement of regulation, minimum wage and workforce composition can explain the evolution of labor market outcomes from 2003 to 2012. By assessing the contribution of each of these factors one at a time, we verify that our comparative statics are in line with many other labor market models. For instance, increases in minimum wages or payroll taxes lead to increased informality and unemployment, while increased enforcement causes formalization and unemployment to increase. An increase in the proportion of skilled workers leads to a decline in the wage gap between the skilled and unskilled in both sectors. It also causes substantial reductions in unemployment and informality. Once all factors are accounted for, the model predicts a decline in the informality rate of almost the same magnitude as that observed in the data. The predicted evolution of unemployment and wages also matches closely the data. We find that shifts in workforce composition are the most important factor behind the observed reduction in informality. Without increases in schooling levels, the informality rate in Brazil would have gone up by three percentage points, instead of decreasing by nine.

Next, we use the model to examine two policies that subsidize formal low-skill employment as a means to reduce informality. In the first policy, the subsidy is implemented in the form of smaller tax rates for low wage positions, as in a progressive payroll tax. In the second, the subsidy is instead a direct government transfer to low wage formal workers similar to a current policy

adopted in the Brazilian labor markets (*Abono Salarial*). Our results show that the first alternative can improve labor market outcomes and increase government revenues, while the second is much less cost-effective. The reason behind the sharp contrast in outcomes of these apparently similar policies lies in the binding minimum wage. While a reduction in payroll taxes induces employers to create formal jobs, there are no incentives for employers under the second policy, since they do not benefit from the government transfer to workers if wages cannot adjust downward.

Our model also explains the fact that the wage gap between formal and informal jobs is decreasing along the wage distribution, becoming null or even negative at the top. This heterogeneity in the formality wage premium suggests that the informal sector is composed of two distinct tiers.[3] For the older, more educated workers at the top tier, informality is a matter of opportunity, which is reflected in their wages being equal to or higher than they would be in the formal sector. However, for the bottom tier, informality appears to be strictly worse than formal employment, since these informal workers earn lower wages and lack valuable mandated benefits.[4] In our model, the two tiers are clearly identified by the two skill levels. The pattern of decreasing wage gaps can be replicated in the case where the minimum wage binds for the unskilled workers, but not for the skilled. The hypothesis that the minimum wage is the cause behind this pattern is in line with the discussion in Bargain & Kwenda (2011) and Botelho & Ponczek (2011). To our knowledge, the only other model that explains this pattern among salaried workers is Araújo & Ponczek (2011), which uses a setup where there is asymmetric information and informal workers can take their employers to court.[5] Bargain et al. (2012) also develop a model

[3]Bargain & Kwenda (2011) find these results in fixed-effects estimations using data from Brazil, Mexico and South Africa. Botelho & Ponczek (2011) reach the same conclusion using Brazilian data with different specifications (also using panel data sets), and observe that the formal wage premium decreases as workers become older and more educated. Lehman & Pignatti (2007) find similar results for the Ukrainian labor market. The idea of a two-tiered informal sector goes back at least to Fields (1990). Günther & Launov (2012) develop an econometric model of selection to test the hypothesis of heterogeneity inside the informal sector. They find that there are two distinct groups in the informal sector in Côte d'Ivoire.

[4]Many authors, including all references in footnote 3, have used the term "segmentation" to describe the bottom tier of the informal sector. By that, they mean that wages are not fully determined by individual productivity and compensating differentials. This interpretation, which is replicated theoretically in models such as Fields (1975), Rauch (1991) and our own, is different from the original concept of segmented labor markets, as described in Dickens & Lang (1985) or Cain (1976). In the case we discuss, increases in education (or, more generally, productivity) can lead every worker to better jobs, a view that contrasts sharply with that of labor market dualists. In addition, the significant flow of workers in and out of the informal sector, particularly among the lower-skilled, undermines the hypothesis of strong non-economic barriers of entry into the so-called primary sector.

[5]Ulyssea & Barros (2010) discuss some statistical properties that can be used in economic

to account for heterogeneity in income gaps between formal and informal sectors, but focus instead on self-employed workers.

To the best of our knowledge, this is the first search and matching model of informality that combines worker heterogeneity, firm heterogeneity, minimum wages, mandated benefits, and explicit compliance decisions by both workers and firms. We draw upon many search and matching models from the informality literature, but differ from them in important aspects. Boeri & Garibaldi (2007) propose a simple model where the most productive workers sort themselves into the formal sector. Many of their results turned out to be a common feature of search and matching models of informality, such as the proposition that repression of informal activities may lead to higher unemployment. However, institutional details such as mandated benefits and minimum wages are missing, and the model does not allow workers to search simultaneously for both formal and informal jobs.[6] Albrecht et al. (2009) avoids both problems, but assumes strong structural differences between sectors, with no compliance decision on the firm side. The informal sector is simply an exogenous subsistence sector where there is no wage dispersion, regardless of worker productivity.

The models in Ulyssea (2010), Bosch & Esteban-Pretel (2012) and Meghir et al. (2012) have more sophisticated compliance decisions and are better equipped in institutional details, but forgo worker heterogeneity. Regarding the firm's compliance decision, Ulyssea (2010) still assumes many structural differences between sectors: formal and informal goods are not perfect substitutes, informal firms are less productive, and entry costs into the formal sector are higher. Bosch & Esteban-Pretel (2012) and Meghir et al. (2012) take a different route. In their models, formal and informal firms only differ in that the former must abide to labor regulations, while the latter face higher job turnover or informality costs related to firm size. These assumptions are closer to what Perry et al. (2007) have called the *exit view* of firm informality: entrepreneurs rationally balance the costs and benefits of non-compliance in their sectoral choice with no significant barriers to entry. Under this light, the lower productivity in informal firms is the result of self-selection, as opposed to structural differences between sectors.[7] The problem of the firm in our model

models that are sufficient to replicate these wage patterns.

[6] In Boeri et al. (2011), the authors add a minimum wage to a simplified version of the model in Boeri & Garibaldi (2007). This results in a change in the sorting behavior: the least skilled workers now find it more profitable to search for formal jobs instead of informal ones, increasing the average productivity in the informal sector.

[7] This perspective has been supported by the experiment in De Mel et al. (2013) and also by other empirical evidence showing that firms change their compliance decision in response to changes in tax rates (Monteiro & Assunção (2012), Fajnzylber et al. (2011)) or in the intensity of enforcement of labor regulation (Almeida & Carneiro (2012)).

also follows this interpretation.

Regarding institutional details, the three models above feature undirected search – that is, workers search for both formal and informal jobs simultaneously – and formality costs related to labor law. Ulyssea (2010) and Meghir et al. (2012) also include the valuation of mandated benefits in the problem of the worker, but they restrict attention to unemployment insurance and severance payments. In our model, we allow for other important formal benefits such paid vacations and the thirteenth salary (an extra annual salary present in the Brazilian labor regulation and also in that of many other developing and developed countries). Finally, Meghir et al. (2012) is the only model that explicitly accounts for minimum wages, making it the closest to our work. Since workers are ex-ante identical in their model, they resort to on-the-job search to generate intra-sector wage dispersion.

We use a search and matching model of labor markets to be able to account endogenously for unemployment. Still, the competitive model developed by Amaral & Quintin (2006) is an important reference to our work. In that model, households are heterogeneous both in entrepreneurial talent and in their educational productivity, and may choose whether to set up firms, to offer unskilled labor, or to invest in education and then offer skilled labor. The model explains many features of the informal sector – smaller firms, workers with lower schooling and wages, lower capital-to-labor ratios – without resorting to any kind of barriers to entry or intrinsic differences between formal and informal firms. The structure of worker heterogeneity in our framework is partially based on their model, particularly regarding capital-skill complementarity in the production function.

The remainder of this paper is organized as follows. The next section briefly describes the main stylized facts of the Brazilian labor market and explains why increased formalization is a puzzle under existing models of informality. The third section presents the model and discusses some of its characteristics. The fourth section explains the quantitative calibration of the model. The fifth section analyzes the evolution of labor market outcomes in Brazil using the calibrated model and presents our policy experiments. The sixth section concludes the paper.

II
Empirical Context

The term "informality" is used to describe many different aspects of non-compliance with regulations. In this paper, we focus on the decision by firm and worker not to comply with labor law when contracting with each other, thus excluding self-employed and domestic workers from our analysis. We also follow the bulk of the literature and restrict our attention to urban informality.

In the Brazilian labor market, a salaried job position is usually considered formal if the worker's "labor card" (*carteira de trabalho*) is signed by the employer. This is the definition we use henceforth. An employee with a signed labor card is entitled to social protections such as severance payments, pensions and unemployment insurance, while the employer is obliged to pay social security contributions and payroll taxes. Appendix A contains a thorough description of benefits available to formal workers and costs associated with formal employment.

With a clear definition of informality, we can turn to the data. First, we discuss some aspects of the Brazilian labor market as of October 2003, the baseline for our quantitative exercises. In particular, we highlight specific patterns that underlie our modeling choices. Following, we analyze the trend in labor informality up to 2012 and relate it to other changes in the labor market during that period. Most of the data we use in this paper come from the Monthly Employment Survey (PME), a household survey run in the six largest metropolitan areas in Brazil that collects information on workers and their employment status. The period of our analysis is restricted by the availability of data from the PME under a consistent methodology.

The average informal worker in Brazil earns a lower wage, is less educated, and works in a smaller firm than her formal counterpart. The first claim is evident in the top row in Table II.1. While the average formal wage was 1,261 Brazilian Reais in 2003, the average informal wage was 35% lower, at 820 Reais. The latter two claims can be seen in Table II.2, which shows the distribution of workers across sectors, firm sizes, and educational categories. By comparing the totals along rows for each sector, the differences in average schooling become apparent: 40% of informal employees have less than 8 years of schooling, but

this number reduces to less than 28% in the formal sector. The differences in firm size can be seen in the column totals. While only a minority (roughly 1/16) of formal employees work in firms with 5 people or less, this fraction is over one third for informal employees. These facts are consistent with many papers that discuss the empirical aspects of informality in the developing world, such as La Porta & Shleifer (2008) and Maloney (2004).

These aggregate patterns have been interpreted as evidence that informality is circumscribed to low-earning, unskilled workers, but a closer look reveals that this assertion is not accurate. Table II.1 shows that the informality rate among workers with at least an undergraduate degree is 17.4%, not dramatically lower than the overall rate of 27.8%. Moreover, informal workers with an undergraduate degree earn almost three times as much as the average formal employee.[1] These observations also suggest that there is no labor market segmentation in the traditional sense: as a worker becomes more educated, she is more likely to be employed formally, and also more likely to receive higher wages if she stays in the informal sector. Finally, the fact that some informal firms are willing to pay high wages for skilled workers shows that the technology used by these firms displays substantial returns to human capital, contradicting many depictions of informality where informal firms are structurally different from formal firms.

We can use the data on firm size in Table II.2 to infer the distribution of skill across firms in both sectors. Comparisons between different columns in the same sector show that, as firm sizes increase, the proportion of more educated workers also increases. In other words, a larger firm is more likely to hire a larger fraction of educated workers. The most important takeaway is that this pattern is observed for workers in both sectors, again suggesting that the technologies used by formal and informal firms are not structurally different. In particular, these observations are consistent with the hypothesis of capital-skill complementarity in the production function of both formal and informal firms.

Now we turn to the evolution of informality in Brazil since the 1990's. Figure II.1 shows that the rate of informal salaried workers was rising up to 2002, but then declined sharply.[2] In Appendix B, we show that the decline was widespread in the economy and not driven by workforce reallocation (i.e., by economic activities that are intrinsically more formal). What makes this

[1]Note that these individuals are not self-employed professionals defaulting on taxes or social security contributions, since we have restricted our sample to wage earners. Thus, they are comparable to the average formal employee.

[2]In Figure II.1, we use data from the National Household Survey (PNAD) instead of the PME because of methodological changes in the latter survey in 2002.

Table II.1 – Labor market outcomes, October 2003 - October 2012

	Informality		Unemployment		Wage Gap		Wage Growth		Group Share	
Sample	2003	2012	2003	2012	2003	2012	Formal	Informal	2003	2012
Whole workforce	27.8%	17.7%	12.9%	5.2%	35%	25%	21%	39%	100.0%	100.0%
By schooling:										
Less than 8 years	35.4%	26.5%	12.6%	4.2%	29%	20%	27%	42%	35.2%	22.1%
8 to 10 years	30.6%	22.6%	17.6%	7.2%	30%	22%	23%	36%	20.1%	17.7%
Highschool and college dropouts	22.8%	14.1%	13.4%	5.9%	24%	16%	10%	23%	33.8%	43.3%
Undergraduate or more	17.4%	12.1%	3.9%	2.7%	12%	8%	-5%	-1%	10.9%	16.9%

Source: IBGE/PME, author's calculations.

Notes: Data is presented for October 2003 and October 2012. Informality is fraction of salaried workers in the private sector with a signed work card. Wage gap is the difference between informal and formal average wages as a fraction of formal wages. Wage gain is the relative increase in average wage from 2003 to 2012.

Table II.2 – Educational distribution of workers by sector and firm size

	Formal workers, by employer size				Informal workers, by employer size			
Education	2 - 5	6 - 10	11+	Average	2 - 5	6 - 10	11+	Average
Less than 8 years	36%	30%	27%	28%	49%	37%	33%	39%
8 to 10 years	24%	23%	20%	20%	25%	23%	22%	23%
Highschool and college dropouts	37%	41%	42%	41%	24%	35%	36%	32%
Undergraduate or more	4%	6%	12%	11%	2%	5%	9%	6%
Total	1,133	1,226	13,937	16,296	2,363	731	3,196	6,290

Source: IBGE/PME, author's calculations.

Notes: Salaried workers only. Employer size is reported by the worker in the household survey. The percentage values sum to one along columns. Data from October 2003.

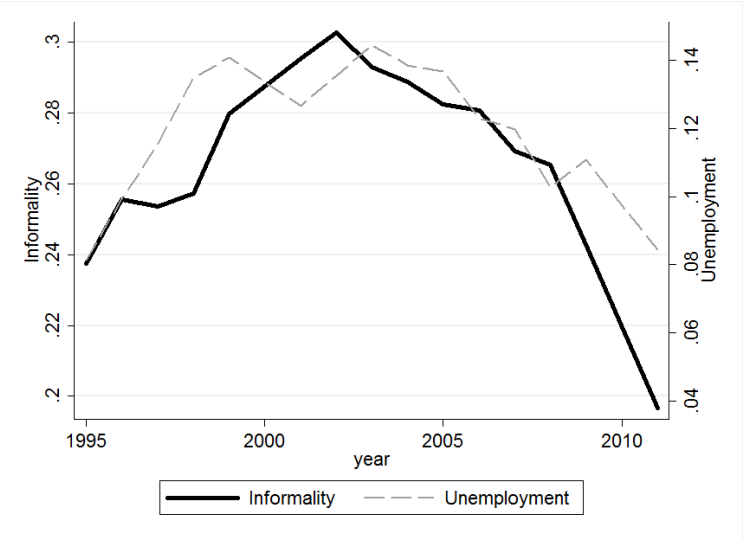

Figure II.1 – Evolution of informality (salaried workers only) and unemployment for the Brazilian workforce.
Source: IBGE/PNAD, author's calculations. The sample is restricted to the six metropolitan regions surveyed in the IBGE/PME.

intriguing is the observation that, while the upward trend has been credited to increasing costs of formal employment during the 1990's, these costs continued to rise even after the reversal.[3] In particular, the minimum wage was rising through the whole period, accumulating real gains of 60% from early 1995 to the end of 2003 and another 61% up to October 2012.

Changes in overall productivity and in enforcement of regulation could explain part of the decline in informality, but they cannot account for other important shifts in labor market outcomes. The close relationship between the two series in Figure II.1 suggests that informal employment has a countercyclical component, as proposed by Boeri & Garibaldi (2007) and Bosch & Esteban-Pretel (2012). In addition, there is some evidence that the enforcement of labor regulation in Brazil has become more efficient, a factor that could also bring down both the unemployment and informality rates.[4] However, the evolution of wage patterns shown in Table II.1 is difficult to reconcile with any of these hypotheses. While average wages increased in both sectors from 2003 to 2012, the gains accrued primarily to the less educated

[3]Barros & Corseuil (2001)explain how the passing of the 1988 Constitution significantly raised employment costs (payroll and firing costs and mandated benefits). Bosch et al. (2007) claim that these changes were the most important factor behind the increase in informality during the 1990's. We present a brief discussion of changes in labor legislation and tax rates after 2003 in Appendix A.

[4]The effect of enforcement over unemployment is ambiguous in most models, and quantitative analyses show diverging results. While Boeri & Garibaldi (2007) and Ulyssea (2010) find that increased enforcement leads to higher unemployment, Bosch & Esteban-Pretel (2012) and Meghir et al. (2012) reach the opposite conclusion.

workers. In addition, the wage increases were larger in the informal sector, suggesting that the minimum wage was not the cause behind this heterogeneous pattern. It is hard to rationalize why an increase in overall productivity would not result in higher wages for the more educated workers. It is even harder to conciliate the wage patterns with increases in enforcement: simulations in Bosch & Esteban-Pretel (2012) and Meghir et al. (2012) predict that the gap between formal and informal wages should increase as a consequence of more enforcement, contrasting sharply with the data.

Improved educational outcomes, evident in the Participation columns in Table II.1, may also have contributed to the patterns described, despite being rarely identified in the literature as an important determinant of informality. Three intuitive arguments can hint at the potentially important role played by schooling. First, since unemployment is much lower among educated workers, increases in the share of workers in this group would lead to a decline in unemployment simply due to a compositional effect. Second, a similar argument can be made to explain a decline in informality. In fact, both Mello & Santos (2009) and Barbosa Filho & Moura (2012) find that changes in workforce composition, particularly in schooling, can statistically account for a major part of the reduction in informality rates from 2002 to 2007. Third, the reduction in the wage gap between schooling levels is consistent with increases in the relative supply of educated workers.

In the next section, we develop a model to assess the quantitative relevance of each of the factors discussed before.

III
The Model

In this section, we develop a continuous time model of labor markets with search frictions, firm and worker heterogeneity, informality, a minimum wage and mandated benefits. There is a continuum of measure 1 of infinitely-lived, income-maximizing workers with identical preferences. Workers can be either skilled or unskilled, and the fraction η of skilled workers in the population is exogenous. The measure of firms in this economy is m, and all of them behave as risk-neutral profit-maximizer agents. They use both kinds of labor in producing the single consumption good. Our specification of labor market imperfections and the problem of the firm follows Pissarides (2000).

In our model, the compliance decision is related to labor informality, and not firm informality. Although these concepts are highly correlated in the data, there are some important differences which are reflected in our modeling choices. For instance, we focus on payroll taxes, leaving taxes over sales or profits outside the model. Moreover, we do not consider the possibility of an intensive margin of informality within firms, as proposed in Ulyssea (2011). Instead, firms make one single formality decision encompassing all of its job relations. From now on, we use the term "informal firm" or "formal firm" to refer to establishments that decide to set up informal or formal job relations, respectively.

III.1 Labor Markets

There are two separate labor markets, one for each level of skill. Firms need to post vacancies in order to find workers, paying a fixed cost ξ for each vacancy per unit of time. The number of matches taking place per unit of time is given by a matching function $M(V_i, U_i)$, where V_i and U_i are the measures of open vacancies and unemployed workers in job market $i \in \{s, u\}$ (for skilled and unskilled workers, respectively). We make the standard assumptions that $M(\cdot)$ is increasing in its arguments, concave and has constant returns to scale. This enables us to use the more convenient form $q(\theta_i)$ for the instantaneous probability of a vacancy being filled. This means that over a short time interval

δt, the probability that a vacancy gets matched to an unemployed worker is $q(\theta_i)\delta t$. θ_i is the labor market tightness in market i, that is, the ratio of open vacancies to unemployed workers: $\theta_i = \frac{V_i}{U_i}$, $i \in \{s, u\}$. The probability that an unemployed worker finds a job in a small interval δt is given by $\theta_i q(\theta_i)\delta t$.

In this formulation, we make no distinction between formal and informal firms in the search process. The aggregate $V_i = V_i^{for} + V_i^{inf}$ is the sum of all vacancies posted by formal and informal firms, and unemployed workers search simultaneously in both sectors. After a worker is matched to a vacancy, the probability that this vacancy is offered by a formal firm is given by $\phi_i = \frac{V_i^{for}}{V_i}$, which is simply the fraction of vacancies posted by formal firms in the market i. The probability of a firm finding a suitable worker, given its search effort (i.e., the number of vacancies posted) and the labor market tightness, is not affected by whether the firm intends to set up formal or informal job relations. In this assumption, as in many others, we try to reduce to a minimum the number of structural differences between the formal and informal sectors. Our modeling of the search process is most similar to that in Bosch & Esteban-Pretel (2012), where firms decide to comply or not comply with regulations after it is matched to the worker. Other models with undirected search, such as Ulyssea (2010) and Meghir et al. (2012), assume exogenous differences in the visibility of vacancies by sector.

III.2 Problem of the Firm

All firms share the same production function $F(k, n_s, n_u)$, where inputs n_s and n_u are units of skilled and unskilled labor employed in production. The term k is an ex-ante productivity parameter that is distributed in the population of firms according to an exogenous distribution $G(k)$ that is fixed over time. We assume that $F(\cdot)$ is concave and has decreasing returns to scale in (n_s, n_u). Moreover, we assume that $\sigma_{k,n_s} < \sigma_{k,n_u}$, where $\sigma_{i,j}$ denotes the partial elasticity of substitution between inputs i and j (see Hamermesh (1993), chapter 3). This means that, as k increases, firms employ increasing fractions of skilled workers.

Our preferred interpretation for the parameter k is that it represents the firm's capital endowment, and henceforth we call this parameter capital. In this case, the conditions on the partial elasticities of substitution become capital-skill complementarity, and the hypothesis on the distribution $G(k)$ being fixed is a reduced form for financial imperfections that lead to an heterogeneous distribution of financial resources in equilibrium. This interpretation would be problematic if compliance decisions could change a firm's access to finance, as proposed in Amaral & Quintin (2006), but recent evidence suggests

otherwise.[1] An alternative interpretation would view the parameter k as entrepreneurial talent or skill, as in Lucas (1978), with an additional hypothesis regarding the elasticities of substitution (e.g., entrepreneurs cannot efficiently manage a large number of skilled workers if they are not highly talented themselves).

The hypothesis of capital-skill complementarity, or the alternative hypothesis of entrepreneurial talent being a better complement to skilled labor, is necessary to replicate the hiring patterns described in Chapter II. The third chapter in Hamermesh (1993) reviews many micro studies in industrialized countries and argues that there is strong evidence for capital-skill complementarity in production functions. However, it is not clear that his conclusions can be ported to small, informal firms in developing countries. Our discussion in Chapter II helps bridge this gap, since we show that the correlation between higher proportions of highly educated workers and larger firms is also seen in the informal sector.

Because of search frictions, firms cannot directly choose the amount of labor inputs employed in production. Instead, the control variable is the number of vacancies posted at each instant, $v_s(t)$ and $v_u(t)$. The firm also decides whether to comply with labor regulations or not. For simplicity, we assume that this decision is taken at the beginning of time and cannot be changed. If the firm complies, it must pay taxes τ over its total payroll. If the firm instead chooses to hire workers informally, it avoids payroll taxes but incurs in an informality penalty $\rho(n)$, where n is the total number of workers hired by the firm. We assume that $\rho(n)$ is strictly increasing and convex.

Along with capital-skill complementarity, the size-based informality penalty is the ingredient behind the aggregate differences between sectors. First, it is clear that this penalty induces larger firms to formalize. Once we take into account that firms with incentives to hire more workers are the ones with a larger capital endowment, it becomes clear that average capital in the formal sector will be greater than in the informal sector. Because of capital-skill complementarity, this leads to a higher proportion of skilled workers in formal firms. Still, there will be some skilled workers in the informal sector, as we see in the data. As in Meghir et al. (2012), we do not specify how the informality

[1] In the experiment in De Mel et al. (2013), firms that are randomly induced to formalize do not display significant changes in behavior, including regarding the use of external finance. Anecdotal evidence from the websites of Brazilian private and public banks, such as the borrowing guide in BNDES (2013), show that credit lines for small entrepreneurs generally make few requirements regarding the legal status of labor relations. When they exist, the requirements are limited to workers with a labor card – if the entrepreneur employs workers without a labor card, it is unlikely that the bank will know about their existence. Finally, informal entrepreneurs may access finance as individuals.

penalty emerges. In general, it can be seen as the product of the probability of being caught by labor inspectors and the monetary value of the sanction. It can also encompass the lack of access to public goods available to formal firms, such as courts.

We can state the instantaneous profit function of the firm with capital endowment k, according to its compliance decision j, as:

$$\pi_k^j(n_s, n_u, v_s, v_u) = \begin{cases} F(k, n_s, n_u) - \left(n_s w_s^{for}(k) + n_u w_u^{for}(k)\right)(1+\tau) - (v_s + v_u)\xi & \text{if } j = for \\ F(k, n_s, n_u) - n_s w_s^{inf}(k) - n_u w_u^{inf}(k) - \rho(n_s + n_u) - (v_s + v_u)\xi & \text{if } j = inf \end{cases}$$

where $w_i^j(k)$ is the wage that this firm needs to pay for workers of type i if it decides to set up a job relation of type j. We describe how these wages are determined in the next subsection – for now, it suffices to say that wages are not a function of firm size or vacancy posting behavior. From the left to the right, instantaneous profits are total production, minus total payroll, minus payroll taxes (in the case of formal firms) or the informality penalty (for informal firms), minus the costs of vacancy posting.

Job relations are destroyed at exogenous rates λ^j, which vary according to the compliance decision. This captures the empirical fact that informal firms have a much higher labor turnover than their formal counterparts.[2] The dynamics of labor quantities inside each firm is then:

$$\dot{n}_i(t,k) = v_i(t,k)q(\theta_i(t)) - \lambda^{j(k)} n_i(t,k), \quad i = s, u$$

The instantaneous variation in the number of workers of type i is equal to the number of vacancies multiplied by the probability of each vacancy being filled, minus the rate of job destruction in that firm. In this equation, we implicitly assume that every match turns into a job relation. Later, in the Nash bargaining section, we show that all job offers are accepted in equilibrium.

We are ready to state the problem of the firm:

$$\Pi(k) = \max_{j \in \{for, inf\}} \Pi^j(k)$$

[2]See the turnover analysis in Gonzaga (2003) and Bosch & Maloney (2010), and also the calibration results in Bosch & Esteban-Pretel (2012) and Meghir et al. (2012). The existence of high dismissal costs in the formal sector provides strong incentives to keeping an employee. Albrecht et al. (2009) formally develop this argument, using a search and matching model with endogenous job destruction and an informal sector. Moreover, as mentioned in the introduction, our target equilibrium is the one in which the minimum wage is binding for unskilled workers, who strictly prefer formal employment. Thus, the formal employee also has more incentives to keep the job relation than the informal one. It would be interesting to use a model with endogenous separation rates, but we do not believe that the gains offset the additional complexity in our case.

$$\begin{aligned}\Pi^j(k, n_{s0}, n_{u0}) \quad &= \quad \max_{v_s(t), v_u(t)} \int_0^\infty e^{-rt} \pi_k^j\Big(n_s(t), n_u(t), v_s(t), v_u(t)\Big) dt \\ &\text{s.t.} \quad \dot{n}_i(t) = v_i(t) q\left(\theta_i(t)\right) - \lambda^j n_i(t), \quad i = s, u, \quad t \in [0, \infty) \\ &\qquad n_s(0) = n_{s0},\ n_u(0) = n_{u0}\end{aligned}$$

For a firm with capital endowment k and a given compliance decision, the total present value of profits is the discounted sum of all instantaneous profits, assuming vacancies posted at each instant are optimally chosen. The discount rate r is the same for all firms. Given its initial conditions and capital endowment, the firm makes the compliance choice that maximize total profits.

We are interested in steady-state solutions. Solving the optimal control problem,[3] we can find expressions equivalent to first order conditions in the regular firm problem, conditional on the compliance decision:

$$\text{Formal FOC's:} \qquad F_{n_i}(\cdot) - w_i(1+\tau) - \frac{(r+\lambda^{for})\xi}{q(\theta_i)} = 0, \quad i = s, u$$

$$\text{Informal FOC's:} \qquad F_{n_i}(\cdot) - w_i - \rho'(\cdot) - \frac{(r+\lambda^{inf})\xi}{q(\theta_i)} = 0, \quad i = s, u$$

We denote by $n_i^j(k)$ the quantities of labor that solve the FOC's above, given the compliance decision j. In steady-state equilibria, the quantities of labor in each firm are constant and, thus, there is a direct mapping between the number of employees at each instant and the number of vacancies posted:

$$v_i^j(k) = \frac{n_i^j(k)\lambda^j}{q(\theta_i)}, \quad i = s,\ u, \quad j = for,\ inf$$

In general, the steady-state equilibrium is not unique, in the sense that different initial conditions on the state variable (labor) might lead to different outcomes for the same parameter set. To see that, consider the following example. Suppose that there are two firms with capital k: the first starts with labor quantities $n_s^{inf}(k)$ and $n_u^{inf}(k)$, while the second starts with $n_s^{for}(k)$ and $n_u^{for}(k)$.

[3]The Pontryagin sufficient conditions for optimality conditional on the firm choosing to be formal are:

$$\begin{aligned} -e^{-rt}(-\xi) + \mu_i(t) q\left(\theta_i(t)\right) &= 0, & i = s, u \\ -e^{-rt}\left[F_{n_i}(\cdot) - w_i(1+\tau)\right] - \mu_i(t)\lambda^{for} &= -\dot{\mu_i}(t) & i = s, u \end{aligned}$$

For the informal firm the conditions are analogous. To find the steady-state solution, we assume that the tightnesses $\theta_s(t)$ and $\theta_u(t)$ are constant in the expressions above. The transversality conditions are satisfied trivially, as the optimal vacancies $v_i(t)$ are constant in t and, thus, $\mu_i(t) = \frac{-\xi e^{-rt}}{q(\theta_i)}$ converge to zero.

Also assume that:

$$\Pi_k^{for}\left(n_s^{for}(k), n_u^{for}(k)\right) > \Pi_k^{inf}\left(n_s^{inf}(k), n_u^{inf}(k)\right)$$

It is clear that firm 2 will decide to be formal and maintain the optimal level of employees. It is also natural to think that firm 1 would consider being formal: at the optimal level of employment, formal profits are higher, given its capital endowment. However, since it did not start with the optimal labor quantities for a formal firm, it will pay an additional vacancy posting cost at $t = 0$ should it decide to be formal. If this adjustment cost is higher than the difference in lifetime profits between formal and informal firms for this capital level, this firm will choose the informal sector and keep its labor inputs at their initial value. Then, we would have a steady-state equilibrium with both formal and informal firms for the same level of k. To avoid these concerns, we focus on the steady-state equilibrium in which a firm with capital k is informal if and only if $\pi_k^{inf}\left(n_s^{inf}(k), n_u^{inf}(k), v_s^{inf}(k), v_u^{inf}(k)\right) \leq \pi_k^{for}\left(n_s^{for}(k), n_u^{for}(k), v_s^{for}(k), v_u^{for}(k)\right)$ – that is, there is no firm in the situation of firm 1 above.

III.3 Wage Determination

We follow Pissarides (2000) and model wage determination through bargain. A successful match in the labor market generates positive rents which must be split between firm and worker. The fraction of those rents earned by the worker, σ, is an exogenous parameter in the model. We assume that neither firms nor workers can choose to destroy the job relation, so that the job destruction rate is exogenous.

Define by $E_i^j(w)$ the value that workers of type $i \in \{s, u\}$ place on holding a job position of type $j \in \{for, inf\}$ which pays wage w. Also, define by U_i the opportunity cost of the worker – that is, the expected present value of being unemployed. This value is assumed to be the same for all workers of type i, regardless of whether he was previously employed in the formal or informal sector. The equivalence between unemployed workers regardless of the previous employment status is achieved by including unemployment benefits in the expressions for $E_i^{for}(w)$ instead of in U_i, as in Ulyssea (2010). We use the quantity $E_i^j(w) - U_i$ as a measure of the rent earned by the worker when he accepts a job offer. Finally, define by $J_i^j(k, w)$ the value that a marginal worker of type i being hired with wage w can add to a company of type j with capital k, once the match has occurred. This is the rent earned by the firm in the bargain, which we assume to be independent of firm size.

Wages are determined by Nash bargaining between the worker and the firm, and potentially by the minimum wage. First, let's assume that the minimum wage is not binding. If the bargaining power of workers is given by σ, we can solve for the schedules of wages $w_i^j(k)$ using the following expression:

$$(1-\sigma)\left(E_i^j\left(w_i^j(k)\right)-U_i\right)=\sigma J_i^j\left(k,w_i^j(k)\right), \quad i=s,\,u, \quad j=for,\,inf, \quad \forall k$$

If a match creates non-negative rents, the axiomatic approach imposes that $E_i^j\left(w_i^j(k)\right) \geq U_i$. This means that holding a job is better than staying unemployed and thus workers will always accept job offers. If there were no rents to share, then the firm wouldn't make the job offer in the first place.

Differently from many other models of informality, such as Ulyssea (2010) and Bosch & Esteban-Pretel (2012), we do not allow formal and informal workers to have different bargaining power. Adding this degree of freedom can be a straightforward way to create a formality wage premium, but it is difficult to justify empirically. In our model, worker heterogeneity and minimum wages fill this role in a more intuitive way, while also allowing for richer patterns of wage dispersion.

Before we proceed, it is important to discuss the hypothesis that $J_i^j(\cdot)$ is not a function of firm size. The major source of value derived from the match is the marginal productivity of that worker. Unless we impose constant marginal productivities for each type of labor for a given k, the decision to hire or not to hire an additional worker will change the marginal productivities of all other workers currently employed by the firm, and also of workers hired in the future. As shown in Stole & Zwiebel (1996), the solution of this problem in a context of individual bargaining without commitment requires including those externalities in the valuation of the marginal worker, which in our case requires solving a non-linear system of differential equations for each level of k. For simplicity, we assume that those externalities are not taken into account in the bargaining process. Regardless of actual firm size, the marginal productivities used for bargaining purposes are those of a firm at the optimal level of employment. This hypothesis can be seen as a form of wage stickiness: there might be institutions, such as collective agreements or labor court rulings, that prevent wages from changing on a marginal basis if firms deviate from their optimal labor level.

Now we show that, as long as the FOC's of the problem of the firm are valid, the four wage schedules $w_i^j(k)$ are in fact constant for all values of k.

First, we state the flow value equations that define $J_i^j(k,w)$:

$$\begin{aligned} rJ_i^{for}(k,w) &= F_{n_i}\left(k, n_s(k), n_u(k)\right) - w(1+\tau) - \lambda^{for} J_i^{for}(k,w) \\ rJ_i^{inf}(k,w) &= F_{n_i}\left(k, n_s(k), n_u(k)\right) - w - \rho'\left(n_s(k)+n_u(k)\right) - \lambda^{inf} J_i^{inf}(k,w) \end{aligned}$$

The return of the marginal worker to the firm in each instant is equal to its marginal product minus the wage paid to her, minus taxes paid or the increase in the informality cost (depending on whether the firm is formal or not), minus the loss incurred if the job relation is destroyed in that instant.

Now from the FOC's obtained in the last subsection, we know that in a steady state solution, when all firms are making optimal choices of firm size, the following relations are true:

$$\begin{aligned} F_{n_i}(\cdot) - w_i^{for}(k)(1+\tau) &= \frac{(r+\lambda^{for})\xi}{q(\theta_i)}, \quad i = s,u \\ F_{n_i}(\cdot) - w_i^{inf}(k) - \rho'(\cdot) &= \frac{(r+\lambda^{inf})\xi}{q(\theta_i)}, \quad i = s,u \end{aligned}$$

Replacing the values in the left hand side of these equations in the expressions defining $J_i^j(k,w)$, we find that:

$$J_i^{for}\left(k, w_i^{for}(k)\right) = J_i^{inf}\left(k, w_i^{inf}(k)\right) = \frac{\xi}{q(\theta_i)}, \quad i = s,u$$

which does not depend on k. In a steady state equilibrium, the value added by the marginal worker of type i is the same across the distribution of firms. Since $E_i^j(w)$ is not a function of k – the worker is concerned about the wages and benefits, not the capital endowment of the firm he is employed –, it is clear that the solutions to the Nash bargaining equations are functions only of the formality status and of the skill level of the employee, but not of the capital stock or the size of each firm. In other words, there are only four wage values in this economy: w_s^{for}, w_u^{for}, w_s^{inf} and w_u^{inf}.

The intuition behind this result is that, regardless of the capital stock, all firms adjust the number of workers so as to equate the search cost to the marginal productivity minus direct labor costs (wages, taxes and informality penalty). Since the search cost is assumed to be the same for all firms, the value added by the marginal worker in equilibrium is the same across the capital distribution. Finally, the assumption that worker's bargaining power is not related to firm size or capital stock guarantees that the solution to the Nash bargaining does not vary with k.[4]

[4]This result simplifies the model's numerical tractability and its interpretation. However,

Now we introduce the minimum wage $\bar{w}$. Only formal firms are subject to this restriction. If the solution to the Nash bargain in formal firms results in a wage which is lower than $\bar{w}$, then the minimum wage is binding. In this case, $w_i^{for} = \bar{w}$ and the Nash bargain restriction is an inequality, favoring the worker side.

Putting all together, we can define the wages in this economy to be the solution of the following system of equations and potentially inequations:

$$\begin{aligned}
(1-\sigma)\left(E_i^{for}(w_i^{for}) - U_i\right) &\geq \sigma\frac{\xi}{q(\theta_i)}, \quad i = s,u, \quad > \text{only if } w_i^{for} = \bar{w} \\
(1-\sigma)\left(E_i^{inf}(w_i^{inf}) - U_i\right) &= \sigma\frac{\xi}{q(\theta_i)}, \quad i = s,u
\end{aligned}$$

To solve this system, we need functional forms for $E_i^j(w_i^j)$ and U_i. They are defined by the flow relationships below:

$$\begin{aligned}
rE_i^{inf}(w) &= w + \lambda^{inf}\left[U_i - E_i^{inf}(w)\right] \\
rE_i^{for}(w) &= w + benefits_i\left(w, \lambda^{for}\right) + \lambda^{for}\left[U_i - E_i^{for}(w)\right] \\
rU_i &= \theta_i q(\theta_i)\left[\phi_i E_i^{for}(w_i^{for}) + (1-\phi_i)E_i^{inf}(w_i^{inf}) - U_i\right]
\end{aligned}$$

These expressions are similar to those used by Ulyssea (2010) and Meghir et al. (2012).[5] As defined earlier, ϕ_i is the ratio of formal vacancies to total vacancies in job market i. The term $benefits\left(w, \lambda^{for}\right)$ encompasses institutions such as social security, unemployment insurance and others, which are only available if the worker holds (or used to hold) a formal job. In the calibration exercise, we propose a functional form for this term that incorporates many regulations in Brazilian labor law.

one might wish to relate to the firm size wage premium literature, particularly as it concerns informality. A simple way to account for a firm size wage premium that persists after controlling for the worker's characteristics and formality status would be to assume that the bargaining power of workers increases with k, as a result of greater worker organization. See Pratap & Quintin (2006) and Badaoui et al. (2010) for a discussion of the relationship between the formality wage premium and the firm size wage premium.

[5] We can use the flow value equations to find direct expressions for $E_i^j(w_i^j)$ and U_i, which can then be used to solve the Nash bargaining restrictions:

$$E_i^{for}(w_i^{for}) = \frac{w_i^{for} + \lambda^{for}U_i + benefits(w,\lambda^{for})}{r+\lambda^{for}} \qquad E_i^{inf}(w_i^{inf}) = \frac{w_i^{inf} + \lambda^{inf}U_i}{r+\lambda^{inf}}$$

$$U_i = \frac{\theta_i q(\theta_i)\left[\frac{\phi_i\left(w_i^{for} + benefits\left(w_i^{for},\lambda^{for}\right)\right)}{r+\lambda^{for}} + \frac{(1-\phi_i)w_i^{inf}}{r+\lambda^{inf}}\right]}{r + \theta_i q(\theta_i)\left[1 - \frac{\phi_i\lambda^{for}}{r+\lambda^{for}} - \frac{(1-\phi_i)\lambda^{inf}}{r+\lambda^{inf}}\right]}$$

III.4 Compensating Differentials

From the final Nash bargaining equations, we can show that:

$$E_i^{for}\left(w_i^{for}\right) \geq E_i^{inf}\left(w_i^{inf}\right), \quad i = s,\, u$$

This expression is an equality if the minimum wage is not binding for skill level i. In this case, we can use the definition of $E_i^j(w_i^j)$ in footnote 5 to show that:

$$w_i^{inf} = \frac{r+\lambda^{inf}}{r+\lambda^{for}}\left(w_i^{for} + benefits\left(w_i^{for}, \lambda^{for}\right)\right) - \frac{rU_i\left(\lambda^{inf}-\lambda^{for}\right)}{r+\lambda^{for}}$$

If the minimum wage is not binding and jobs in both sectors have the same expected duration ($\lambda^{for} = \lambda^{inf}$), then the difference between formal and informal wages is equal to the value that workers attribute to the mandated benefits. If the expected duration in the formal sector is larger, as we see in the data, then the wage differentials should be even higher to compensate for that. If the minimum wage is binding, then this equation is no longer valid: the informal wages are smaller than the value needed to make workers indifferent between sectors, and formal jobs are strictly preferred. However, workers still accept informal job offers, since it is too costly to defer the offer and keep looking for a good job. In this case, formal jobs are rationed in equilibrium.

III.5 Equilibrium

An equilibrium in our model is defined as wages w_s^{for}, w_u^{for}, w_s^{inf} and w_u^{inf}, schedules of firm decisions $j(k)$, $n_s(k)$ and $n_u(k)$, and labor market tightnesses θ_s and θ_u such that:

1. Taking wages, tightnesses and compliance decisions $j(k)$ as given, the labor schedules $n_s(k)$ and $n_u(k)$ solve the firm's FOC's;

2. The compliance decisions $j(k)$ are the ones that provide the greatest profits to each firm, if it could choose initial conditions;

3. The labor market tightnesses are consistent with their definition after we aggregate the measures of vacancies and employment;

4. The four wages solve the system given by the Nash bargaining restrictions.

We solve the model numerically using the definitions above. Details are available in Appendix C.

IV
Calibration

In this section, we calibrate the model to replicate the state of the labor market in Brazil as of October 2003. We choose this date because it is close to the reversal of the informality trend, as shown in Figure II.1, and because this is the month when the second wave of the Informal Urban Economy survey (*Economia Informal Urbana*, ECINF) was run by the Brazilian Bureau of Statistics (IBGE). The ECINF targeted small urban firms, most of which were unregistered, thus providing us with an estimate of the number of informal firms in the economy. We also use the survey's micro data in the next section to assess some of the hypotheses we make in the model. However, since the ECINF is relatively small and was not repeated after 2003, this survey is not our main source.

Most of the data we use comes from the Monthly Employment Survey (*Pesquisa Mensal do Emprego*, PME), also run by IBGE. The PME is a household survey that provides information on employment, wages, occupational choice, formality status and other characteristics of the workforce, including educational attainment. We use three other data sources from IBGE: the CEMPRE, a register of formal firms; the National Account System (*Sistema de contas nacionais*, SCN); and estimations of the size of the workforce.

It is important to explain how we map between observed traits in the data and worker skill in the model. If we observe intra-sector wage dispersion and a binding minimum wage in equilibrium, then the minimum wage must bind only for unskilled workers. This gives us an interpretation of skill for the quantitative exercises. Unskilled workers in the model represent workers in the data who receive exactly the minimum wage when employed in the formal sector. If they receive more than the minimum wage in formal jobs, then they are labeled as skilled workers in the model. Note that this definition suggests that most workers are skilled – at least in the informal sector, where only 10.33% of workers receive the minimum wage (see table IV.3 below).

The downside of this definition is that we can't observe skill in the data for all workers, thus requiring the estimation the fraction of skilled workers in the calibration procedure and the use of an arbitrary proxy for the changes in

this structural parameter from 2003 to 2012. By definition, we only observe the skill status of formal workers in the data. We would like to infer the skill status of unemployed and informal workers using observable characteristics such as age or education, but it is impossible to make any such connection. Although older and more educated workers are less likely to earn exactly the minimum wage, this relationship is far from deterministic: there are many uneducated workers receiving more than the minimum wage in the formal sector, and also some highly educated workers in the opposite situation. More generally, a probabilistic regression of skill using schooling, age and race has poor predictive power. Thus, in the calibration, we remain agnostic about the parameter η (the measure of skilled workers in the population) and let it be selected by a minimum distance algorithm, along with other unobserved parameters of the model. In Subsection V.1(e), we describe how we proxy the change in workforce composition from October 2003 to October 2012.

IV.1 Functional Forms

The production function is given by:

$$F(k, n_s, n_u) = A\left[Bk{n_s}^{\alpha\gamma} + (1-B){n_u}^{\beta\gamma}\right]^{\frac{1}{\gamma}}$$

The parameter A is a common productivity factor. We restrict the exponents α and β to be smaller than one, so that the function has decreasing returns to scale for any given k. This production function implies that an entrepreneur with zero capital is productive, but will only use unskilled labor. We also restrict the parameter γ to the interval $(0, 1]$ to ensure the desired property of capital-skill complementarity. In the limiting case, where $\gamma = 1$, increases in the capital endowment only raise the productivity of skilled labor. If $\gamma \in (0, 1)$, then there is some degree of complementarity between capital and unskilled labor: unskilled workers will be more productive in a firm with more capital and more skilled workers.[1]

The capital endowment follows a Generalized Pareto distribution, to account for the fact that the majority of firms are small but a large part of the workforce is employed in big firms (see Table 2 in IBGE (2005)). We set the location parameter to zero, so that the smallest firms have zero capital. Also, we normalize the scale parameter to $1 - T$, where T is the shape (tail) parameter, so that the average capital endowment is always one.[2] Increases

[1] If $\gamma = 0$, the production function collapses to a Cobb-Douglas format and the elasticity of substitution between any two pairs of inputs will be the same. If $\gamma < 0$, then unskilled labor would be a better complement to capital than skilled labor.

[2] Allowing for other values for the scale parameter would not add information to the

in T are thus mean-preserving spreads that add probability mass to extreme values of capital endowment. The cumulative distribution of capital is given by:

$$G(k) = 1 - \left(1 + \frac{Tk}{1-T}\right)^{-\frac{1}{T}}$$

For computational purposes, we divide this distribution in 500 atoms with equal probability mass.

Since the informality penalty must be increasing and convex, we use a quadratic function, $\rho(n) = Cn^2$. This choice results in significant computational gains because of the linearity of the first derivative. In the specification of the matching technology, we follow the literature and use a Cobb-Douglas function. We thus have $q(\theta) = D\theta^{-E}$, where D is the matching scale and E is the matching elasticity.

Finally, the valuation of formal benefits by workers takes the form:

$$benefits_i\left(w_i^{for}, \lambda^{for}\right) = \left(b_i^F + \lambda^{for} b_i^D\right)\bar{w} + b_i^V w_i^{for}$$

We allow for three categories of benefits to better reflect the Brazilian labor law. The term b_i^V encompasses benefits that vary according to the worker's nominal wage, such as the thirteenth salary or severance payments received in the case of dismissal. It also includes compulsory contributions discounted from the nominal wage. The term b_i^D is the present value of unemployment insurance, measured in multiples of the minimum wage. Finally, b_i^F are transfers received by the worker that are also measured in multiples of the minimum wage. The details of the computation of these parameters are provided in Appendix A.

IV.2 Parameters

Table IV.1 shows a first subset of the model's parameters, along with the values we use in the calibration. We estimate the measure of firms using the total number of salaried workers and the number of firms, both formal and informal.[3] The job destruction rates are taken from the estimates of the duration of employment spells in Gonzaga (2003). The values for the

model, since the changes in the scale of k could be offset by changes in the parameters A, B and γ in the production function.

[3]The PME asks unemployed workers what was the nature of the last employment. We use this information to proxy the fraction of unemployed workers who are looking for salaried jobs. We estimate that salaried workers account for 73% of the workforce, either employed or unemployed. We multiply the total size of the workforce in 2003 calculated by IBGE to get the number of salaried workers. We take the number of formal firms from the CEMPRE and the number of informal firms from the ECINF, excluding the self-employed workers. The measure m is the ratio of firms to salaried workers.

Table IV.1 – Parameters set according to the data or literature

Parameter	**Value**	**Source**
m (measure of firms)	0.0905	Ratio of number of firms to workforce
λ^{for} (formal hazard rate)	0.030	Gonzaga (2003).
λ^{inf} (informal hazard rate)	0.082	Gonzaga (2003).
τ (payroll tax rate)	0.7206	Appendix A.
b_s^F, b_u^F (fixed benefits)	0.0057, 0.05	Appendix A.
b_s^V, b_u^V (variable benefits)	0.235, 0.306	Appendix A.
b_s^D, b_u^D (unemp. insurance)	7.48, 4.00	Appendix A.
r (discount rate)	0.08	Real interest rate.
D (matching scale)	0.30	Ulyssea (2010).
E (matching elasticity)	0.50	Ulyssea (2010).
σ (worker bargaining power)	0.45	Ulyssea (2010).

payroll tax rate and benefits are calculated in Appendix A according to the methodology in Souza et al. (2012). The discount rate for workers and firms is assumed to be the real interest rate. Finally, we take the parameters of the matching function and the bargaining power of workers from Ulyssea (2010).

We use a minimum distance procedure to set the remaining nine parameters displayed in Table IV.2. The algorithm, described in detail in Appendix C, minimizes the norm of a vector where each element is the relative distance between the model outcomes and the nine targets listed in Table IV.3. The targets were defined as follows. The first two, the unemployment and informality rates, are directly observable in the PME data set. To determine the average wage of skilled workers in the formal sector, we identify which of them are skilled according to whether they receive more than the minimum wage. The wage for formal, unskilled workers is naturally the minimum wage. We also use the fraction of formal workers receiving the minimum wage as a target.

To set the target for the informal unskilled wage, we refer to the quantile panel regressions in Table 3 in Bargain & Kwenda (2011). Using the same PME data set, they find that, for workers at the quantile 0.2 of the wage distribution, the wage penalty associated with informality is of 7.8%. We also measure the average wages for informal workers in the data and use it as a target. However, since we cannot distinguish between skilled and unskilled workers in the informal sector, we cannot ascertain what is the informal, skilled wage.

The eighth target, labor share of income, is the fraction of total production (net of search costs and informality penalties) that is not firm profits nor government surplus in the model. We calculate the empirical counterpart of this measure using the National Accounts System. This number may vary

from 39.5% to 50.1% depending on whether one considers the self-employed as labor or capital. The number we use, 44.2%, is found by simply ignoring the self-employed in the computation. The last target, the fraction of firms with 10 or fewer employees, is essentially a means to set the shape parameter of the capital distribution. We use 10 workers as the threshold to match one of the categories in the distribution of firm sizes in the CEMPRE report.[4]

Table IV.3 shows that the model can replicate all of the targeted patterns with reasonable accuracy. Before we proceed to the next subsection, it is convenient to study our baseline specification and verify some properties of the model. First, the estimated fraction of skilled workers is 0.67, more than half of the total population of salaried workers. A breakdown per sector shows the expected aggregate patterns discussed in Chapter II: while 90% of the formal employees are skilled, this fraction drops to 29% in the informal sector. Among the unemployed, the fraction of skilled is also 29%.

Next, we can analyze how firms differ along the capital distribution. Each row in Table IV.4 describes firms in one of the 500 quantiles. For instance, the top row shows the smallest firms in the model, while the bottom row shows the largest. The columns show the capital endowment, the number of employees, the fraction of skilled employees, the compliance status and the average wage. As expected, firm sizes and the fraction of skilled workers increase with the capital endowment. Interestingly, the smallest firms in the model have a little more than one employee, as we would expect, even though we do not target this moment in the calibration.

An important feature of the model is the non-monotonicity of average wages along the firm distribution. Within each sector, wages are monotonically increasing with capital because of the compositional effect of a larger fraction of skilled workers. However, at the margin between informality and formality, average wages decrease. This happens because skilled workers receive more when working in informal firms to compensate for the lack of mandated benefits. This discontinuity is a better estimator of the average wage gap between formal and informal workers conditional on individual productivities than simple differences between average wages in each sector.

[4]Note that we use not only the CEMPRE report in IBGE (2005) to set this target, but also the number of informal firms estimated by the ECINF.

Table IV.2 – Minimum distance results

Parameter	**Value**
A (productivity)	10.6447
B (technology bias)	0.6208
α (skilled exp.)	0.5771
β (unskilled exp.)	0.5669
γ (CES param.)	0.6743
η (measure of skilled)	0.6705
C (informality cost)	0.0398
ξ (search cost)	1.3088
T (firm dist. shape)	0.2064

Table IV.3 – Calibration results

Outcomes	**Target Value**	**Model Value**
Unemployment	12.90%	12.87%
Share informal workers	27.80%	27.85%
Formal skilled wage	3.530	3.565
Formal unskilled wage	1	1
Share formal workers MW	10.33%	10.30%
Informal unskilled wage	0.922	0.929
Avg. informal wage	2.125	2.093
Labor share of income	44.2%	43.1%
Firms w/ 10 or fewer emp.	93.4%	92.4%

Note: Wages in model units (one model unit is equivalent to the minimum wage in October 2003).

Table IV.4 – Firms in the model

Quantile	**Capital**	**Size**	**Fraction Skilled**	**Formal?**	**Average Wage**
1	0,0	1,3	0%	No	0,93
250	0,6	1,9	8%	No	1,24
350	1,1	2,9	19%	No	1,72
450	2,3	7,9	53%	No	3,11
470	3,0	12,2	68%	No	3,71
480	3,6	23,7	71%	Yes	2,81
490	4,7	51,8	79%	Yes	3,02
500	13,6	1677	96%	Yes	3,45

Note: Wages in model units (one model unit is equivalent to the minimum wage in October 2003).

V
Quantitative Results

V.1 The Formalization of Brazilian Labor Markets

In this subsection, we study to which extent our model can shed light on the changes in informality observed in Brazil from 2003 to 2012. First, we examine how each of the main institutional changes observed in this period would individually affect the labor market. Then, we verify whether these changes can jointly explain the evolution of informality, unemployment and wages.

Throughout the analysis, we often refer to Table V.1, where each row describes changes in a particular labor market outcome. In the first column, it describes how the Brazilian labor market changed from October 2003 to October 2012 using the same data sets and definitions we have used in the calibration. Each of the other columns shows how changes in one or more parameters would affect the labor market outcomes in the model, by comparing the baseline calibration with a new steady-state equilibrium where only those parameters are different.

In the period we study, the unemployment rate fell by 7.7 percentage points (from 12.9% to 5.2%), while the informality rate dropped by 10.2 points (from 27.8% to 17.6%). The average wage has increased by 28.2%, but, as pointed in Chapter II, the gains were larger for the low-skilled formal and informal workers. Since we cannot identify the type of workers in the informal sector, it is not possible to disentangle the increases among the skilled and unskilled in that sector. However, informal wages as a whole have increased by 39%, significantly more than what was observed for formal skilled workers. Finally, the share of formal workers that receive the minimum wage has increased by 4.8 percentage points in the data, from 10.3% to 15.1%.

Table V.1 – Quantitative experiments

Outcomes	(1) Data	(2) Minimum wage	(3) Payroll tax	(4) Benefits	(5) Inf. cost	(6) Fraction skilled	(7) All but prod/ty	(8) Prod/ty	(9) All changes	(10) No change in skill
Unemployment (p.p.)	-7.7	1.3	0.0	0.0	0.2	-5.1	-3.2	-2.2	-4.6	-4.6
Informality (p.p.)	-10.2	3.9	-0.4	0.0	-2.9	-13.6	-9.9	0.1	-9.1	-9.1
Wages (%):										
Average	28.2	1.0	0.2	0.0	-0.8	1.2	3.2	23.2	28.2	102.8
Formal, skilled	22.6	-0.8	0.3	0.0	-0.1	-7.8	-8.2	24.9	14.5	113.6
Formal, unskilled	61.2	61.2	0.0	0.0	0.0	0.0	61.2	0.0	61.2	61.2
Informal, average	39.0	-3.0	-0.7	0.0	-7.6	27.4	17.9	33.0	56.8	82.8
Informal, skilled	-	2.2	0.3	0.0	-0.1	-7.1	-4.6	22.7	16.2	106.1
Informal, unskilled	-	-8.6	0.4	-0.1	-2.7	57.5	36.7	32.0	79.8	116.1
Share f. workers MW (p.p.)	4.8	-5.4	0.2	0.0	0.8	-0.1	-5.3	3.1	-3.7	8.0
Product[b] (%)	27.0	-0.8	0.1	0.0	0.2	9.2	8.3	24.9	35.2	116.8
Govt. net revenues (%)[c]	-	-24.2	-1.0	0.6	3.3	10.6	-20.0	39.8	24.7	191.8

Notes: [a]Change from 2003 to 2007 (IBGE/SCN is only data available up to 2007). [b]For column 1, product is GDP per capita. For the remaining columns, product is total production in the model net of search costs and the informality penalty. [c]In the baseline calibration, the government appropriates 8.4% of total production. Numbers in this line represent relative changes over this baseline amount, not changes in the participation.

(a) Minimum Wage

The minimum wage has increased by 61.2% from 2003 to 2012. The effects of a change of this magnitude over the baseline calibration are shown in column 2 of Table V.1. Both unemployment and informality increase, as expected. Wages for skilled workers in both sectors increase because of a substitution effect: as the price of unskilled labor goes up, firms choose higher proportions of skilled workers, tightening that labor market. The opposite happens for informal unskilled workers, whose wage falls by almost 9%. The reason for this decline is the decreased demand for unskilled labor by formal firms, which increases unemployment and lowers the outside option of workers being hired by informal firms.

Aggregate production decreases by 0.8%, but the share of income appropriated by workers increases by one percentage point. This means that, on an aggregate perspective, workers are better off. However, unskilled workers who are not hired by formal firms are strictly worse off because of higher unemployment and lower informal wages. Government revenues fall by 24% because many benefits, such as unemployment insurance, are indexed by minimum wages.

Note that the model predicts that the fraction of formal workers receiving the minimum wage falls. Although this seems counter-intuitive, this is a consequence of the simplifying assumption of only two skill levels. If there were many levels of skill, then an increase in the minimum wage could make the minimum wage bind for a larger share of the workforce. With only two levels, this channel is blocked: the minimum wage always binds for exactly 33% of the workforce, unless the increase is so large that it eliminates any dispersion in formal wages (which is not the case). Formal firms then hire a higher proportion of skilled workers for two reasons. First, the increase in informality means that the firms who remain in the formal sector have, on average, a larger capital endowment. Second, unskilled labor becomes relatively more expensive for these firms.

It is also interesting to note that the negative effects of the minimum wage on employment are small, in line with many empirical assessments such as Card (1992) or Card & Krueger (1995). In our simulations, the increase of 61% in the minimum wage merely results in a reduction in employment of 1.5% for all workers, or 5.2% for unskilled workers. The effect for skilled workers is null. The mechanism behind this result is the steep decline in informal wages for unskilled workers, which partly offsets the reduction in formal employment. This is in line with the traditional view that the informal sector is, for some workers (in our model, the unskilled), an alternative to unemployment, as

stated in Fields (1975), Rauch (1991) and Boeri & Garibaldi (2007).

(b) Payroll Taxes

The only change in the costs of formal employment from 2003 to 2012 was the phasing out of a temporary additional contribution to the worker's severance payment fund (FGTS). As described in Appendix A, we estimate that this change has decreased the total payroll tax rate only slightly, from 72.06% of the nominal wage to 71.43%. Column 3 shows that, as standard models would predict, informality falls. Wages rise for all workers, except for the ones who receive exactly the minimum wage. This is a consequence of the axiomatic bargaining approach, through which workers receive part of the increased profits by firms. Product rises and government revenues decline. Still, the effects described above are quantitatively minor.

(c) Mandated Benefits

The only changes in labor regulations that affected how workers value formal jobs were in the calculation details of income tax and social security contributions, which are both deducted from the wage of formal employees. However, on average, they did not result in significant changes in the size of the deductions. When we recalculate the parameters of the $benefits$ expression using 2012 data (Appendix A), we find that the differences are negligible. Hence, they do not cause any important effects in labor market outcomes, as is evident in column 4 of Table V.1.

(d) Enforcement of Regulation

We use data from the Ministry of Labor to estimate changes in enforcement of regulation from 2003 to 2012. Reports of the aggregate results of labor inspections, available in MTE (2013), show that the number of workers targeted by inspectors rose during the last decade both in absolute terms and as a fraction of the workforce.[1] We use the relative increase as a proxy for increases in enforcement of regulation in the model. We find that the fraction of the workforce that was inspected rose by about 39% from 2003 to 2012. Thus, we raise the parameter C by the same proportion.

The fifth column shows how this change would impact our baseline calibration. First, informality decreases, as expected. We argued in Chapter II that the effects of increased enforcement over unemployment are ambiguous in

[1] Other indicators, such as total revenues from fines, were also rising. For a thorough discussion of enforcement of regulation in Brazil, see Cardoso & Lage (2005).

many models, and this is also true for ours. There is an extensive margin effect because firms who change their compliance decision hire more workers, and also an intensive margin effect because the remaining informal firms hire fewer workers. For our calibration, the quantitative result is that unemployment would rise marginally with a change in enforcement of this magnitude. The only significant change in wages is a steep decline in earnings among the informal, unskilled workers. Thus, our model replicates the results found in Bosch & Esteban-Pretel (2012) and Meghir et al. (2012). Government revenue increases, but we must be cautious about this result since we do not take into account the costs of increasing enforcement.

(e) Workforce Composition

Since we cannot observe skill as defined in the model, we must find a proxy for the change in workforce composition during the period. We use the change in the fraction of workers with incomplete primary education (less than 8 years of schooling) as an estimate of the change in the fraction of unskilled workers. Our reasoning is that, since unskilled workers in the model represent a group of low wage individuals, we associate them with a group of low-education workers of similar size in the data. As Table II.1 shows, this group accounted for 35.2% of the workforce in 2003, a similar number to the 33% of unskilled workers in the baseline calibration. Since that group decreased its participation by 13.1 percentage points over these 9 years, we use this difference as the measure of increase in the share of skilled workers in the model.

We find that the predicted changes agree with our discussion in Chapter II. Both unemployment and informality decrease sharply as a consequence of a more skilled workforce. Wages for the informal unskilled workers increase, while they decrease for the skilled workers in both sectors. To a large extent, this is a consequence of the relative increase in the supply of skilled workers. The labor market for skilled individuals becomes less tight (and the reverse happens for unskilled workers), with direct effects on wage bargain. In addition, because firms hire more skilled labor in the new equilibrium, the marginal product of unskilled work increases. The combination of a tighter labor market and greater productivity is behind the steep increase in the informal, unskilled wage. Wages for the unskilled formal workers do not rise because the minimum wage remains binding; despite the increase in informal wages, formal jobs are still strictly preferred by these workers. The formal-informal wage gap is substantially reduced.

(f) Estimating Changes in Productivity

Now we assess the model's performance when the five changes discussed above are put together. The results are shown in column 7. Note that, when compared with column 1, the fall in informality predicted by the model is almost as large as the one observed in the data. However, the decline in the unemployment rate is smaller. It is also important to note that increases in average wages are also smaller than the real changes observed in the data. Finally, the growth in total production in the model is less than a third of the growth in GDP per capita from 2003 to 2012.

These observations suggest that there might have been an increase in the overall productivity of the economy in this period. To estimate the productivity gains, we verify by how much we have to increase parameter A in order to match the empirical increase in average wages. We find that productivity was 23.79% higher in 2012 than in 2003.

Before we assess the performance of the model with all of the above changes plus the increase in productivity, we study the individual effects of the productivity gains. Column 8 shows that unemployment declines and wages rise uniformly in the new equilibrium. However, informality increases marginally. Most models find that increases in productivity lead to less informality, but in our model this effect is ambiguous. On the one hand, firms have an incentive to hire more workers, leading to increased cost of informality. However, wages will also rise. Note that the informality penalty is based on firm sizes, not on wages, while the costs of formal employment are proportional to the nominal wage. Thus, it is possible that the increase in total payroll taxes following a rise in productivity more than offsets the increase in the informality size penalty for the marginal firm, resulting in an increase in informal labor.

(g) Explaining the Evolution of Labor Market Outcomes

In column 9, we consider changes in minimum wages, taxes, benefits, enforcement, education and productivity together. First, the model does a good job in explaining the decline in informality. It also predicts a decline in unemployment of 4.6 percentage points, which is in the correct direction but falls short of the observed decline of 7.7 points. Predictions regarding wages are close to the empirical patterns, though the model overestimates the gains of informal workers. The only dimension where the model prediction conflicts with the empirical observations is in the share of formal workers receiving the minimum wage. This is caused by the simplifying assumption of only two levels of skill in the model, as argued in section VIII.1. Overall, the model is able to

Table V.2 – Individual contribution of each factor

		All changes, except:					
Outcomes	All Changes	Minimum wage	Payroll tax	Benefits	Inf. cost	Fraction skilled	Prod/ty
Unemployment (p.p.)	-4.6	-7.0	-4.5	-4.6	-4.6	0.0	-3.2
Informality (p.p.)	-9.1	-19.8	-9.0	-9.4	-7.1	3.0	-9.9
Wages (%):							
Average	28.2	22.9	27.8	28.1	29.2	24.9	3.2
Formal, skilled	14.5	15.0	14.1	14.5	14.7	24.1	-8.2
Formal, unskilled	61.2	15.0	61.2	61.2	61.2	61.2	61.2
Informal, average	56.8	89.9	56.5	56.0	64.3	20.5	17.9
Informal, skilled	16.2	13.7	15.8	16.2	16.3	24.9	-4.6
Informal, unskilled	79.8	102.9	79.6	80.3	82.8	15.2	36.7
Share f. workers MW (p.p.)	-3.7	6.2	-3.8	-3.7	-4.1	-3.4	-5.3
Product[b] (%)	35.2	36.6	35.2	35.2	35.0	23.9	8.3
Govt. net revenues (%)[c]	24.7	60.1	26.8	24.1	21.9	15.6	-20.0

Notes: [a]Change from 2003 to 2007 (IBGE/SCN is only data available up to 2007).
[b]Product is total production in the model net of search costs and the informality penalty.

explain the main outcomes of the Brazilian labor market within a reasonable degree of quantitative precision.

We can refer back to the discussion in Chapter II and determine which factor was main driver behind the declines in informality and unemployment. In Table V.2, we show what happens when all but one of the changes is taken into account. We find that, for instance, the declines in both unemployment and informality would have been considerably larger if the minimum wage had not increased. We can verify that changes in workforce composition were indeed the most important cause for the fall in informality: without a larger fraction of skilled workers, the informality rate would have increased by three percentage points in our simulations, instead of declining by nine. The relevance of enforcement is secondary. Without changes in this parameter, the decline in informality would have been only two percentage points smaller.

In our discussion in the empirical section, we have argued that it is difficult to explain the decline in informality and unemployment in Brazil exclusively with changes in enforcement and productivity. In the exercise above, we have shown that changes in workforce composition are fundamental to explain the observed patterns. However, the reader might not be persuaded by our quantitative results, since the changes in enforcement, productivity and workforce composition are not directly observable and had to be proxied arbitrarily. In particular, it is possible that the effects of schooling on skill are significantly smaller than what we assumed, and thus our quantitative results overestimate the role of education. To strengthen our argument and show that

changes in workforce composition are necessary for replicating the observed patterns, we propose another exercise.

Suppose that we want to explain the evolution of labor market outcomes in Brazil without resorting to changes in the fraction of skilled workers. We are free to choose the values of productivity and enforcement that would lead to the same declines in informality and unemployment as the specification in column 9. We find that productivity should have more than doubled from 2003 to 2012, while the costs of informality should have increased by 325%. The impact of these changes in labor markets are shown in column 10 of Table V.1. In this scenario, wages would have gone up by more than 100% on average, and so would total product. In addition, the wage increases would be roughly similar across sectors. These results are clearly at odds with the data, suggesting that changes in workforce composition should be taken into account when assessing labor market outcomes in Brazil during the last decade.

V.2 Policy Experiments

One major policy concern in developing countries has been how to bring down informality rates without incurring in increases in unemployment and poverty. In this subsection, we use the model to assess the effectiveness of labor market policies under this criterion, while also keeping track of the fiscal burden they impose on the government.

The first labor market policy we consider is a reduction in payroll taxes for low wage workers. In the last subsection, we learned that a lower payroll tax rate might lead to a decline in informality, with no adverse effect on unemployment (see column 3 in Table V.1). On the other hand, it also leads to a drop in government revenues that is not small relative to the decline in informality. However, we know that the smaller, informal firms are more intensive in unskilled labor than formal firms. In addition, only a small fraction of government revenues comes from payroll taxes of low-skilled workers, since their wages are low and they account for a small fraction of formal employment. Thus, it might be optimal for the government to subsidize the employment of low wage formal workers using a progressive payroll tax policy, in which tax rates are smaller for the low wage workers.

In Table V.3, we examine the progressive payroll tax policy using as a starting point the model as of October 2012 (column 9 in Table V.1). In the first column, we show the result of simply decreasing the overall tax rate by 1.43 percentage points (to 0.70) as a reference. As argued above, although this reduction could lead to positive effects in unemployment and informality, there would be significant costs for the government. In columns 2 to 5, we assess

Table V.3 – Policy experiments

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Progressive payroll tax, $\tau_s = 0.7143$						$\tau = 0.7343$,
Outcomes	$\tau = 0.70$	$\tau_u = 0.70$	$\tau_u = 0.50$	$\tau_u = 0.30$	$\tau_u = 0.09^b$	$b_u^F = 0.085$	$b_u^F = 0.085$
Unemployment (p.p.)	-0.1	0.0	-0.3	-3.0	-3.6	0.0	0.1
Informality (p.p.)	-0.7	-0.3	-2.6	-18.7	-18.7	-0.3	0.4
Wages (%):							
Average	0.5	-0.1	-0.2	-4.9	-4.1	0.0	-0.8
Formal, skilled	0.8	0.0	0.2	0.0	0.0	0.0	-1.1
Formal, unskilled	0.0	0.0	0.0	0.0	12.7	0.0	0.0
Informal, average	-0.7	-0.3	2.0	-	-	-0.2	0.3
Informal, skilled	0.7	0.0	0.1	0.0	0.0	0.0	-1.0
Informal, unskilled	0.7	0.6	8.3	46.0	74.6	0.8	-0.1
Share formal workers MW (p.p.)	0.2	0.1	1.9	12.4	12.8	0.1	-0.2
Product[a] (%)	0.1	0.0	0.3	1.9	1.9	0.0	-0.1
Govt. net revenues (%)	-3.1	0.3	1.2	13.5	0.2	-4.7	-0.2

Notes: In all columns, the reference is the model as of October 2012, with $\tau = 0.7143$.[a]Product is total production net of search costs and the informality penalty.

similar policies where the reduction in payroll tax rates is restricted to workers who earn close to the minimum wage (that is, the unskilled workers in the model). The program achieves similar results in employment and formalization, but government revenues actually increase. The formalization induced by lower taxes among low-skilled workers is sufficient to induce marginal firms to comply, and thus enlarges the tax base. The taxes raised from skilled jobs in firms that formalize more than offset the earnings forgone from low-skilled workers in supra-marginal firms. In addition, wages increase substantially for unskilled workers in the informal sector because of a tighter labor market. Thus, this policy is likely to have positive effects on poverty alleviation.

It is interesting to note that, in our simulations, the government revenues keep rising until all firms are formal, which is achieved when the tax rate for low wage workers is around 30%.[2] At this point, government revenues are 13.5% higher than they would be without the policy, and total product (net of search costs) is 1.9% higher. It is possible to further diminish the tax rate until the program roughly breaks even with the baseline scenario. In this case, there is a substantial increase in wages for unskilled workers as a consequence of a tighter labor market. However, at this point the minimum wage is not binding anymore, so our characterization of the tax discount is not clear and results should be interpreted with caution.

Next, we consider a relatively similar policy in which the government subsidizes low-skilled formal employment by increasing benefits available to these workers, instead of by reducing tax rates. In column (6), we assess the consequences of increasing the fixed payments by the government to low-skilled workers from 5% of the minimum wage to 8.5%[3]. We find that there is a reduction in informality, although not a large one relative to the costs incurred by the government. If the payroll tax is raised by 2 percentage points so that the program breaks even, the positive results vanish (see column 7). In the new equilibrium, total welfare is lower and both unemployment and informality rise.

The second policy is ineffective because of the binding minimum wage. In an unrestricted scenario, the formal, unskilled wage would drop after the increase in benefits, because of rent sharing between worker and firm. This would create incentives for the posting of more formal, unskilled vacancies, and the results would be similar to the previous program. In the case we study,

[2]We examined all values of the tax rate for low wage employees in a grid from 0 to 0.70, with 0.05 increments. In this grid, government revenues were maximized at $\tau_u = 0.30$. In addition, we observed that the change in government revenues was negative for $\tau_u = 0.05$ and $\tau_u = 0$. We refined the grid between 0.05 and 0.10 to 0.005 increments and found that the government roughly breaks even when $\tau_u = 0.09$.

[3]This policy is equivalent to augmenting a current program in Brazilian labor markets called "abono salarial" (see Appendix A).

wages cannot adjust downward, so the supply of formal vacancies remains unchanged. The only channel left for lowering informality is the increase in informal wages, which results from an increase in the outside option of unemployed unskilled workers when bargaining (because formal jobs become even better).

Three important caveats should be made regarding our progressive payroll tax results. First, our model assumes that every firm hires both skilled and unskilled workers. This enables the government to increase its revenues by inducing firms to formalize through lower taxes for unskilled workers. If firms instead hire a single type of worker – either all skilled or all unskilled –, then there would be far less potential to increase revenues with this policy. The second limitation is the assumption that there is a single compliance decision for all workers. If firms are free to make individual compliance decisions for each worker, then the policy would merely result in the formalization of low wage workers, while high wage employees would remain informal. Third, there is a possibility of under-reporting of wages in the formal sector, which is not taken into account in our model.

We believe that these concerns are not enough to overturn our qualitative analysis, though the quantitative results in Table V.3 should not be taken at face value. To assess the relevance of the first two issues, we examine the ECINF survey. For each of the small firms surveyed, we have information on the number of employees, whether they hold a signed labor card, wages and their schooling levels. To ascertain the relevance of the first concern, we examine the degree of wage dispersion within firms of the informal sector. In 64% of the informal firms with five employees, the highest paid worker received at least 50% more than the lowest paid worker.[4] In 20% of them, the highest paid worker received more than three times the wage paid to the lowest earning worker. The data also shows that, in most of these firms, workers belong to different educational categories (as listed in Table II.1). This evidence suggests that there is a substantial degree of worker heterogeneity within small, informal firms, as assumed in the model.

Regarding the second concern, we concede the existence of an intensive margin of informality, as suggested by Ulyssea (2011). Still, the formalization of low wage workers should increase the probability of formalization of high

[4]The ECINF survey targeted firms with up to five employees. For consistency, we do not count the owner(s) or unpaid workers as employees. Likewise, we define firms as informal if none of their employees possess a signed labor card. There are 99 informal firms with exactly five employees in the data set. If we look this measure of wage dispersion in smaller firms, we find that the fraction of them with wage gaps of 50% becomes smaller, but remain significant (51% of firms with four employees, 38% of firms with three employees, and 24% of firms with two employees).

wage workers for two reasons. If firms formalize a fraction of their workforce, they become more visible to labor inspectors and thus the cost of keeping informal workers increases. Also, the existence of formal ties to some workers may make it easier for others to take the employer to court, thus strengthening the argument in Araújo & Ponczek (2011). The data supports the view that most firms will either hire all workers formally or all informally. Among firms in the ECINF data set with five employees, 32% hire all workers informally, while 46% hire all of them with a signed labor card. Only 22% of them have both formal and informal employees. This number is even lower for smaller firms.

Finally, although we acknowledge that this policy would increase incentives to under-report wages, there are already large incentives for firm owners to under-report under current labor law. In addition, since the value of many mandated benefits is based on the contracted nominal wage, the employee's incentives are on the opposite direction. Thus, we do not believe that the implementation of the progressive payroll tax would dramatically increase under-reporting of wages.

VI
Concluding Remarks

This paper studies how the interplay between workforce composition and labor market institutions, particularly minimum wages, can affect informality, unemployment and wages. The framework we develop allows for worker heterogeneity, search frictions and more institutional details than most other models in the literature. In addition, we model the compliance decisions by firms and workers so that it reflects recent evidence suggesting that, while firms and highly educated workers choose between formality and informality without any significant barriers to entry, some of the least educated workers are rationed out of the formal sector and must accept inferior jobs to avoid unemployment.

The model is used to study the decline in informality rates in Brazil from October 2003 to October 2012. In the calibration exercises, we show that the model is able to replicate important features of real labor markets, particularly wage patterns and the rates of unemployment and informality. Then, we show that the model can explain with reasonable quantitative precision the evolution of labor market outcomes in that country, using the estimated changes in tax rates, benefits, minimum wage, enforcement of regulation, workforce composition and productivity. The increase in schooling levels is the most important factor behind the sharp decline in informality among salaried workers.

We also perform additional experiments to test the implications of two policies aimed at reducing informality. First, we show that decreasing the payroll tax rate for low wage workers can have positive effects on both employment and formalization, while at the same time increasing government revenues. On the other hand, a subsidy of formal unskilled labor implemented by a direct transfer from the government to low-skilled workers is not cost-effective. The discrepancy between the two approaches is caused by the binding minimum wage, which prevents downward adjustments of formal wages in the second case and thus precludes the creation of more formal jobs.

Bibliography

J. Albrecht, L. Navarro and S. Vroman. **The effects of labour market policies in an economy with an informal sector**. *The Economic Journal*, 119(539):1105–1129, 2009.

R. Almeida and P. Carneiro. **Enforcement of labor regulation and informality**. *American Economic Journal: Applied Economics*, 4(3):64–89, 2012.

P. S. Amaral and E. Quintin. **A competitive model of the informal sector**. *Journal of Monetary Economics*, 53(7):1541–1553, 2006.

L. Araújo and V. Ponczek. **Informal wages in an economy with active labor courts**. *Unpublished manuscript*, January 2011.

O. Bargain and P. Kwenda. **Earnings structures, informal employment, and self-employment: New evidence from brazil, mexico and south africa**. *Review of Income and Wealth*, 57:S100–S122, 2011.

E. E. Badaoui, E. Strobl and F. Walsh. **The formal sector wage premium and firm size**. *Journal of Development Economics*, 91(1):37–47, 2010.

R. Barros and C. H. Corseuil. **The impact of regulations on brazilian labor market performance**. *Inter-American Development Bank Research Network Working paper*, R-427, 2001.

O. Bargain, E. E. Badaoui, P. Kwenda, E. Strobl and F. Walsh. **The formal sector wage premium and firm size for self-employed workers**. *IZA Discussion Paper*, 6604, 2012.

F. H. Barbosa Filho and R. L. Moura. **Evolução recente da informalidade no brasil: Uma análise segundo características da oferta e demanda de trabalho**. *Texto para discussão IBRE-FGV*, 17, 2012.

BNDES. **Mais bndes - encontre o financiamento mais adequado ao seu negócio**, 2013. Available at http://www.bndes.gov.br/SiteBNDES/bndes/bndes_pt/Orientador/index.html.

T. Boeri and P. Garibaldi. **Shadow sorting**. *NBER International Seminar on Macroeconomics*, 2005:125–163, June 2007.

T. Boeri, P. Garibaldi and M. Ribeiro. **The lighthouse effect and beyond**. *Review of Income and Wealth*, 57:S54–S78, 2011.

M. Bosch, E. Goni and W. Maloney. **The determinants of rising informality in brazil: Evidence from gross worker flows**. *The World Bank Policy Research Working Paper*, 4375, 2007.

M. Bosch and W. F. Maloney. **Comparative analysis of labor market dynamics using markov processes: An application to informality**. *Labour Economics*, 17(4):621–631, Aug. 2010.

M. Bosch and J. Esteban-Pretel. **Job creation and job destruction in the presence of informal markets**. *Journal of Development Economics*, 98(2):270 – 286, 2012.

F. Botelho and V. Ponczek. **Segmentation in the brazilian labor market**. *Economic Development and Cultural Change*, 59(2):437–463, Jan. 2011.

G. G. Cain. **The challenge of segmented labor market theories to orthodox theory: A survey**. *Journal of Economic Literature*, 14(4):1215–1257, Dec. 1976.

D. Card. **Do minimum wages reduce employment? a case study of california, 1987-89**. *Industrial and Labor Relations Review*, 46(1):38–54, Oct. 1992.

D. Card and A. B. Krueger. **Time-series minimum-wage studies: A meta-analysis**. *The American Economic Review*, 85(2):238–243, May 1995.

A. Cardoso and T. Lage. **A inspeção do trabalho no brasil**. *Revista de Ciências Sociais*, 48(3):451–490, 2005.

A. P. Souza, S. P. Firpo, V. Ponczek, E. Zylberstajn and F. Ribeiro. **Custo do trabalho no brasil: Proposta de uma nova metodologia de mensuração**. FGV/EESP, 2012.

S. De Mel, D. McKenzie and C. Woodruff. **The demand for, and consequences of, formalization among informal firms in sri lanka**. *Forthcoming in the American Economic Journal: Applied Economics*, 2013.

W. T. Dickens and K. Lang. **A test of dual labor market theory**. *The American Economic Review*, 75(4):pp. 792–805, 1985.

P. Fajnzylber, W. F. Maloney and G. V. Montes-Rojas. **Does formality improve micro-firm performance? evidence from the brazilian simples program**. *Journal of Development Economics*, 94(2):262 – 276, 2011.

G. S. Fields. **Rural-urban migration, urban unemployment and underemployment, and job-search activity in ldcs**. *Journal of Development Economics*, 2(2):165 – 187, 1975.

G. S. Fields. **Labour market modelling and the urban informal sector: Theory and evidence**. In D. Turnham, B. Salomé and A. Schwarz, editors, *The informal sector revisited*, pages 49–69. Organisation for Economic Cooperation and Development, 1990.

G. Gonzaga. **Labor turnover and labor legislation in brazil**. *Economía*, 4(1):165–222, Oct. 2003.

I. Günther and A. Launov. **Informal employment in developing countries: Opportunity or last resort?** *Journal of Development Economics*, 97(1):88–98, 2012.

IBGE. **Estatísticas do cadastro central de empresas 2003**. 2005. Available at http://www.ibge.gov.br/home/estatistica/economia/ cadastroempresa/2003/default.shtm.

D. Hamermesh. **Labor demand**. Princeton University Press, 1993.

H. Lehman and N. Pignatti. **Informal employment relationships and labor market segmentation in transition economies: Evidence from ukraine**. *IZA Discussion Paper*, 3269, 2007.

J. Lucas, Robert E. **On the size distribution of business firms**. *The Bell Journal of Economics*, 9(2):508–523, Oct. 1978.

MTE. **Resultados da fiscalização do trabalho - nível brasil - 2003 a 2012**, 2013. Available at http://portal.mte.gov.br/fisca_trab/estatisticas.htm.

W. F. Maloney. **Informality revisited**. *World Development*, 32(7):1159–1178, July 2004.

C. Meghir, R. Narita and J.-M. Robin. **Wages and informality in developing countries**. *Unpublished manuscript*, September 2012.

R. F. Mello and D. D. Santos. **Aceleração educacional e a queda recente da informalidade**. *Boletim Mercado de Trabalho IPEA*, 39:27–33, 2009.

J. C. Monteiro and J. J. Assunção. **Coming out of the shadows? estimating the impact of bureaucracy simplification and tax cut on formality in brazilian microenterprises**. *Journal of Development Economics*, 99(1):105 – 115, 2012.

G. Perry, W. F. Maloney, O. Arias, P. Fajnzylber, A. Mason and J. Saavedra. **Informality: Exit and exclusion**. World Bank, 2007.

C. A. Pissarides. **Equilibrium unemployment theory**. MIT Press, 2000.

R. La Porta and A. Shleifer. **The unofficial economy and economic development**. *Brookings Papers on Economic Activity*, 2008(2):275–363, 2008.

S. Pratap and E. Quintin. **Are labor markets segmented in developing countries? a semiparametric approach**. *European Economic Review*, 50(7):1817–1841, Oct. 2006.

J. E. Rauch. **Modelling the informal sector formally**. *Journal of Development Economics*, 35(1):33 – 47, 1991.

F. Schneider and D. H. Enste. **Shadow economies: Size, causes, and consequences**. *Journal of Economic Literature*, 38(1):77–114, 2000.

L. A. Stole and J. Zwiebel. **Intra-firm bargaining under non-binding contracts**. *The Review of Economic Studies*, 63(3):375–410, July 1996.

L. Tornarolli, D. Battistón, L. Gasparini and P. Gluzmann. **Exploring trends in labor informality in latin america, 1990-2010**. *Unpublished manuscript*, March 2012.

G. Ulyssea and R. P. Barros. **On the empirical content of the formal-informal labor market segmentation hypothesis**. *Brazilian Review of Econometrics*, 30(2):289–310, 2010.

G. Ulyssea. **Regulation of entry, labor market institutions and the informal sector**. *Journal of Development Economics*, 91(1):87 – 99, 2010.

G. Ulyssea. **Formal and informal firms dynamics**. *Unpublished manuscript*, 2011.

VII
Appendices

Appendix A: Costs of Formal Labor and Valuation of Benefits by the Formal Employee

In this Appendix, we calculate the cost of formal employment and the valuation of mandated benefits by formal workers based on the methodology of Souza et al. (2012). In each subsection, we first show the results for the baseline calibration in October 2003. Then, we discuss the changes in regulations from 2003 to 2012 and calculate the parameters for October 2012.

In order to correctly reflect labor regulations and the differences between formal and informal jobs, it is important to have a clear grasp of what we call wage in the model and how it relates to the data. In the data set we use (PME), workers are asked to report their nominal monthly wages. If they are formal, they are asked not to include annual contributions such as the thirteenth salary. On the other hand, they report gross wages before formal deductions (such as income tax or social security contributions). However, if workers are informal, such concerns are irrelevant and the reported wage is actually what is being paid by the employer and received by the worker. On the employer side, a similar distinction must be made: while the cost of informal employment is essentially the reported wage, for formal workers the cost might be much higher once all contributions and mandated benefits are taken into account.

In the model, wages should reflect the reported wage in the PME data set, and the payroll tax (τ) and the $benefits$ term are used to adjust the costs of formal employment and the valuation of formal jobs by employees, respectively. Thus, for the purposes of the model, the payroll tax rate must encompass everything that a formal employer must pay but a informal employer must not, as a multiple of the reported wage. Likewise, the term $benefits$ is the difference between the valuation of formal jobs and reported wage. In principle, this term can be either positive or negative, depending on whether the advantages of formal employment (e.g., thirteenth salary, vacations) are quantitatively more important than the social security and income tax deductions. In the calculations below, we show that all parameters of the $benefits$ term are

positive, meaning that formal jobs are preferred to informal jobs for a given reported wage.

Cost of Formal Employment

Under Brazilian labor laws, contributions paid by employees are fixed fractions of the base salary. Thus, the payroll tax rate is the same regardless of the type of worker in the model. Later, we discuss that this is not true regarding the valuation of formal jobs by employees; for instance, highly paid workers are subject to income tax, but low wage workers are not.

Table VII.1 shows our calculations of the cost of formal employment in October 2003. For simplicity, we normalize the base salary to 100. Formal workers are entitled to a thirteenth salary annually and an additional stipend of 1/3 of the monthly wage when they leave for vacation. In addition, if they are dismissed, the employer must notify them at least 30 days earlier. During that period, the employee is entitled to use up to 25% of its work time in job search. As discussed in Gonzaga (2003), the advance notification is in practice an additional severance payment, since workers are not expected to devote much effort to their tasks during that month and the employer cannot rely on them.

Now we turn to the contributions that the employer is obliged to pay. These are levied over not only the nominal monthly wage, but also the additional payments described above (thirteenth salary, vacation stipend and advance notice). The first item is the monthly contribution of 8% of the wage to the worker's severance payment fund (FGTS). In the following row, we state the expected balance of this fund after 33.24 months, which is the expected duration of formal employment in the model. This information is used to calculate the severance payment, which is 50% of the total FGTS balance at the time of dismissal. Note that, of the 50% payment, 40% go to the dismissed employee and the remaining 10% are appropriated by the government. In addition, there was an additional temporary contribution to the FGTS fund of 0.5%, which expired in December 2006.

The largest cost that formal employers face is the social security contribution (INSS), which accounts for 20% of the nominal wage. Finally, there are some other smaller contributions, including mandatory insurance and contributions that are specific to the activity developed by the firm. We use Souza et al. (2012) as a reference in listing those contributions.

After all contributions are taken into account, we find that formal employers pay 57.7% more than the nominal monthly wage to each worker. However, this calculation does not take into account that formal employees are

entitled to paid vacations of one month per year. Thus, although the employer pays for the 12 months in the year, each employee is only productive in 11 of them. In other words, for each 11 workers that the firm wants to use in production, 12 must be hired, because 1 in every 12 is expected to be in vacation at each time. After making the corresponding adjustments, we find that the total cost for each worker that the firm wants to use in production is 72.06% of the nominal wage in October 2003.

We then proceed to the calculation of the cost of formal employment in October 2012. The only change in regulations that affected the cost paid by the employer was the phasing out of the temporary FGTS contribution. When we exclude that contribution, we find that the equivalent payroll tax rate in October 2012 was 71.43% of the nominal wage.

Valuation of Mandated Benefits

In this subsection we account for all characteristics of formal employment that can make it more or less attractive to workers when compared with informal employment. Differently from the previous section, some of the items we consider affect low wage and high wage workers differently, such as the income tax. Thus, we have separate valuations for low wage workers and high wage workers. Low wage workers are those who earn exactly the minimum wage. The high wage worker is a representative agent for all other formal employees.

Table VII.2 shows our calculations of the value attributed to benefits and contributions that calculated as fractions of the base salary. When taken together, these regulations compose the variable benefits parameters in the $benefits$ expression, b_s^V and b_u^V. The first five rows are similar to those in Table VII.1: formal workers receive not only the nominal monthly wage, but also the thirteenth salary, the vacation stipend and the advance notification in case of dismissal. Two items are then deducted from the raw total wage: the social security (INSS) deduction and the income tax (IRPF). For the low wage workers, we use the lowest brackets: zero income tax in both years and social security deductions of 7.65% (in 2003) or 8.00% (in 2012). For the high wage workers, we calculate the deductions for each individual worker in the PME data set that receives more than the minimum wage, using the corresponding tax rates and brackets in each year. Then, we calculate the average deduction per worker.

The next four items are benefits that are valuable to formal workers. The first is the FGTS fund. Workers can withdraw money from their accounts in the FGTS fund, but only in a few special occasions: dismissal, retirement and

Table VII.1 – Cost of formal employment in October 2003

Item	Rationale	Value
Nominal wage (A)		**100.00**
13th salary (A.1)	1/12 of A	8.33
Vacation stipend (A.2)	0.33/12 of A	2.78
Advance notice	(A+A.1+A.2) x prob. dismissal	3.34
Raw total wage (B)		**114.45**
FGTS contribution (B.1)	8% of B	9.16
FGTS balance on dismissal (B.2)	*B.1 x average duration*	*304.36*
Severance payment	50% of B.2 x prob. dismissal	4.58
FGTS temporary extra	0.5% of B	0.57
Employer INSS contribution	20% of B	22.89
SAT, INCRA, S system	5.3% of B	6.07
Total with contributions (C)		**157.72**
Vacation adjustment	1/11 of C	14.34
Total cost		172.06
Payroll tax rate (τ)		**0.7206**

when buying a house. In addition to being illiquid, resources in the fund are also less valuable than a direct payment because their returns are lower than the market interest rate. Souza et al. (2012) consider two extreme scenarios in their exercise: one in which the valuation of FGTS funds is 100% of the nominal balance, and other where workers do not value resources in the fund at all. They then report the valuation of benefits as a range. We take an intermediate route and assume that the value of deposits in the worker's FGTS account is 50% of the employer's actual disbursement.

The remaining benefits are the severance payment, the compulsory work accident insurance (SAT) and vacations. The first two items are calculated in a similar manner as in the previous subsection, when assessing the costs of formal employment. To input the valuation of vacations by workers, we use exactly the same value calculated as the cost of vacancy for employers. In this sense, vacations can be regarded as a transfer from firm to worker. Thus, if we calculate the difference between aggregate total payroll taxes and aggregate benefits, vacations and other transfers, such as the thirteenth salary, are canceled out, and we can use the result as government surplus in the model. We find that the net valuation of variable benefits is around 30% of the base salary for low wage workers, and around 23% for high wage workers.

The fixed benefits parameters (b_s^F, b_u^F) reflect a program called "abono salarial", which is an annual stipend equal to the minimum wage paid to low wage workers (those who receive up to two times the minimum wage per month). To be eligible for this benefit, the employee must have been employed formally for at least five years (not necessarily in the same firm). We use the PME data set and estimate that 60% of formal employees who earn less than two minimum wages are entitled to the abono salarial. We thus find $b_u^F = 0.05$ ($0.6 \cdot 1/12$). Only 40% of workers defined as high wage employees earn less than twice the minimum wage in the data. Thus, we set $b_s^F = 0.02$.

Finally, we calculate the unemployment insurance parameters (b_s^D, b_u^D). Unemployed workers who were previously employed formally for at least six months are entitled to unemployment benefits. Although the size of the monthly payments vary according to the wage in the last employment, there are caps on the minimum and maximum values paid. Low wage workers will always receive exactly one minimum wage, while most others will receive the maximum value of 1.87 times the minimum wage. The number of payments may vary from 3 to 5, according to the duration of all formal jobs in the last 36 months. For simplicity, we assume that the expected present value of these payments is equivalent to four times the value of each payment. Thus, $b_s^D = 4 \cdot 1.87 = 7.48$ and $b_u^D = 4$.

Table VII.2 – Valuation of variable benefits

		October 2003		October 2012	
Item	Rationale	Low wage	High wage	Low wage	High wage
Nominal wage (A)		**240.00**	**848.00**	**622.00**	**1680.47**
13th salary (A.1)	1/12 of A	20.00	70.67	51.83	140.04
Vacation stipend (A.2)	0.33/12 of A	6.67	23.56	17.28	46.68
Advance notice	(A+A.1+A.2) x prob. dismissal	8.02	28.35	20.79	56.17
Raw total wage (B)		**274.69**	**970.57**	**711.90**	**1923.36**
INSS deduction	7.65%/7.93% (03) or 8.00%/8.27% (12) of B	-21.01	-76.97	-56.95	-159.06
Income tax (IRPF) deduction	0%/5.90% (03) or 0%/5.60% (12) of B	0.00	-57.26	0.00	-107.96
Valuation of FGTS fund	50% of employer contribution	10.99	38.82	28.48	76.93
Severance payment	40% of FGTS balance x prob. dismissal	8.79	31.06	22.78	61.55
Work accident insurance (SAT)	2% of B	5.49	19.41	14.24	38.47
Total with contributions (C)		**278.95**	**925.63**	**720.45**	**1833.29**
Vacation adjustment	Equal to the cost of vacation paid by employer	34.41	121.59	88.86	240.07
Total valuation		313.36	1047.22	809.30	2073.36
Variable benefits parameter		**0.306**	**0.235**	**0.301**	**0.234**

Appendix B: Informality Trends by Economic Activity

In this Appendix, we show that the decline in the informality rate in Brazil was widespread in the economy, and also that it was not caused by reallocation of workers across sectors. In the PME survey, workers report the economic activity to which their main job belongs, choosing one of 60 categories. In Table VII.3, we list 15 economic activities with the largest number of workers. Together, they account for 76% of the workforce in 2003, and 78% in 2012. For each activity, we compute the formality rates in 2003 and 2012, and also the share of the workforce employed therein. Note that, since the PME targets workers in large metropolitan areas, few of them are employed in agricultural or extractive activities.

The first important observation is that formality increased in all economic activities listed. The share of formal workers increased more in activities that were initially more informal, but even the automotive and chemical industries experienced important gains in formalization. However, it is still possible that part of the decline was caused from workers migrating from less formal activities to others that are intrinsically more formal. To test this hypothesis, we decompose the contribution of each sector for the increase in formalization in the following way:

$$\begin{aligned} \text{Total contribution}_i &= F_{i,2012}P_{i,2012} - F_{i,2003}P_{i,2003} \\ \text{Within contribution}_i &= P_{i,2003} \cdot (F_{i,2012} - F_{i,2003}) \\ \text{Between contribution}_i &= F_{i,2012} \cdot (P_{i,2012} - P_{i,2003}) \end{aligned}$$

where $P_{i,t}$ and $F_{i,t}$ denote the share of the workforce in and the formality rate of activity i in year t, respectively. The sum of the within contributions describe what would happen if the share of workers in each activity remained constant from 2003 to 2012, but the formality rates within each activity changed. The sum of between contributions accounts for the part of the decline in informality that can be attributed to changes in the size of each activity, given the formality rates in 2012. As can be seen in the bottom row of Table VII.3, the decline in informality can be accounted for almost exclusively with

Table VII.3 – Informality trends per economic activity

Economic activity	Formality rate			Share of workforce			Decomposition		
	2003	2012	Change	2003	2012	Change	Within	Between	Total
Construction	55.0	73.6	18.6	7.0	8.1	1.1	1.3	0.8	2.1
Leisure, culture, sports	55.3	65.7	10.4	2.5	2.1	-0.4	0.3	-0.2	0.0
Vehicle trading and repairs; fuel retail	60.2	73.5	13.3	4.3	3.9	-0.4	0.6	-0.3	0.3
Hospitality industry, restaurants	64.3	73.8	9.5	5.3	5.2	-0.2	0.5	-0.1	0.4
Trade and repair of personal/household objects	70.3	83.2	12.8	17.7	17.3	-0.4	2.3	-0.3	1.9
Education	72.6	81.6	9.0	4.4	4.2	-0.2	0.4	-0.1	0.3
Leather industry (including shoe crafting)	73.6	84.0	10.3	2.2	1.5	-0.8	0.2	-0.7	-0.4
Other activities	74.2	82.2	8.1	23.4	21.9	-1.5	1.9	-1.2	0.7
Terrestrial transportation	76.2	85.0	8.8	5.6	5.5	-0.1	0.5	-0.1	0.4
Food industry	77.2	86.1	8.9	2.7	2.6	-0.1	0.2	-0.1	0.1
Services for businesses	77.7	87.2	9.5	9.9	13.9	4.0	0.9	3.5	4.4
Metal crafting, including machines and equipment	78.7	83.9	5.2	2.4	1.9	-0.5	0.1	-0.4	-0.3
Health and social services	79.1	86.6	7.5	5.2	5.4	0.2	0.4	0.1	0.5
Real estate	80.8	84.2	3.4	3.5	2.6	-0.9	0.1	-0.7	-0.6
Chemical industry	88.5	92.9	4.4	2.3	1.8	-0.5	0.1	-0.5	-0.4
Automotive industry	93.1	95.9	2.8	1.5	2.1	0.7	0.0	0.6	0.7
Whole workforce	72.2	82.3	10.1	100.0	100.0	0.0	9.9	0.2	10.1

Notes: Informality is defined as proportion of workers without a signed labor card. Data does not include domestic workers, public servants or self-employed workers.

changes within each activity.

The facts we show in this Appendix suggest that idiosyncratic shocks are unlikely to be the cause behind the formalization of the Brazilian labor market. This is the reason why we focus on factors that influenced the whole workforce, such as educational trends, enforcement policy and labor regulation.

Appendix C: Numerical Solution and Minimum Distance Procedure

This Appendix briefly describes the computational methods we used in this paper. In the first subsection, we explain how an equilibrium can be found. In the second subsection, we describe the minimum distance procedure used in the calibration section.

Solving for the Equilibrium

First, we create a routine (*solveFirmProblem*) that takes wages and labor market tightnesses as given and solves the firm problem for each of the atoms in the capital distribution. The output of this routine are vectors with optimal labor hiring and vacancy posting decisions by each firm. Next, we create a second routine (*solveTightnessesGivenWages*) that takes only wages as parameters and, using the first routine, finds the labor market tightnesses that are consistent with their definition (that is, the tightnesses that solve $\theta_i = \frac{V_i}{U_i}$, $i \in \{s, u\}$).

Once we have defined this routine, the task of finding an equilibrium is that of finding wages that solve the Nash bargaining restrictions, given that tightnesses are found using the *solveTightnessesGivenWages* routine. The algorithm we use is:

1. Guess initial values for the four wages.

2. If w_s^{for} or w_u^{for} are lesser than or equal to the minimum wage, assume that the minimum wage is binding for the corresponding group of workers.

3. Find the zeros of the system of 2, 3 or 4 Nash bargain equations (according to whether the minimum wage is binding for both workers, for only unskilled workers, or for neither).

   (a) If the wages that solve the system above do not include any formal wage below the minimum wage AND the Nash bargain inequalities (if any) are satisfied, then the equilibrium is found.

(b) If the solution contains a formal wage below the minimum wage, go back to step 2.

(c) If any of the Nash bargain inequalities is not satisfied, then the minimum wage should not be binding for that type of workers. Guess a higher value for the corresponding wage and go back to step 2.

The numerical challenges are related to solving systems of equations (the tightness definition equations and the Nash bargain equations) that involve a large number of computations. Since there are important non-linearities in the model (related to the compliance decision, the minimum wage, and the non-smoothness of the capital distribution), it is not guaranteed that derivative-based methods can find a solution. Our algorithms use more than one method to increase robustness without sacrificing performance. We start with a derivative-based method for finding zeros. If the first method fails, we switch to using binary search iteratively in each of the equations in the system until a solution has been approximated. Then, the algorithm switches back to the derivative-based method to approximate the solution faster, up to the desired precision.

Minimum Distance Procedure

In the minimum distance procedure, we select nine parameters in order to minimize a loss function related to the nine desired targets. For any set of parameters Θ for which we can calculate a steady-state equilibrium, the loss function is given by:

$$L(\Theta) = \sum_{i=1}^{9} \left(\frac{r_i(\Theta) - r_i^*}{r_i^*} \right)^2$$

where $r_i(\Theta)$ is the outcome i in the steady state equilibrium where the parameters are those in Θ, and r_i^* is target i in Table IV.3.

It is difficult to find the solution of this minimization problem for three reasons: the number of choice variables is large, the equilibrium calculation is time consuming and the loss function is not smooth in the parameter space, which makes derivative-based methods perform poorly. To account for the third problem, we design a minimization procedure that combines features of pattern search and line search. In addition, before using this procedure to find the optimal set of parameters, we approximate the solution using our numerical procedure to minimize an alternative loss function. Basically, we add wages and labor market tightnesses as choice variables and include the residuals of the tightnesses definitions and Nash bargaining equations in the loss function.

The computation of the alternative loss function only requires solving the problem of the firm, being thus much faster. However, the minimization of the alternative loss function does not imply the minimization of the main loss function. This would only be true if a zero was found, which does not happen in our exercises.