7. Model and Calibration of Parameters

7.1. Households

We assume a closed economy populated by a representative household who obtains utility by private consumption (*c*), government consumption (g^c) and leisure (*l*). The instantaneous utility function is given by

$$U(c_t, g_t^c, l_t) = \frac{[(c_t + \theta g_t^c)^{\chi} {l_t}^{1-\chi}]^{1-\sigma}}{1-\sigma}, \quad (1)$$

where χ measures the utility derived from consumption relative to leisure, and $\theta \in [0, 1]$ is a parameter scaling the utility derived from government consumption. The specification for the relationship between private consumption and public follows, among others, Barro (1981). σ is the inverse of the intertemporal elasticity of substitution. Given the format for the instantaneous utility and the discount factor, $\beta \in (0, 1)$, the household has preferences over streams of consumption and leisure $\{c_t, g_t^c, l_t\}_{t=0}^{\infty}$ according to

$$\sum_{t=0}^{\infty} \beta^t U(c_t, g_t^c, l_t). \quad (2)$$

In each period, the representative household is limited by a budget constraint given by

$$(1 + \tau_t^c)c_t + k_{t+1} + b_{t+1} \leq (1 - \tau_t^h)w_t h_t + [(1 - \tau_t^k)r_t^k + 1 - \delta]k_t + (1 + r_t)b_t, \quad (3)$$

where *w* is the wage, and *h* the total hours worked. The time endowment is normalized to unity, such as $h = 1 - l$. The family can purchase government bonds, which pay an interest rate *r*. *b* is the public bonds stock held by the household. r^k is the rental rate of private capital and δ stands for the depreciation rate. $\tau_t^c, \tau_t^h, \tau_t^k$ are the tax rates on consumption, hours worked and capital, respectively.

Hence, the representative household chooses paths $\{c_t, h_t, l_t, b_{t+1}, k_{t+1}\}$ such that (2) is maximized subject to the constraint (3) and $h_t = 1 - l_t$, for all *t*.[19]

7.2. Firms

Production is undertaken by a representative firm. In each period, a final good is produced using capital (*k*), hours worked (*h*) and public capital (K^g) with a technology

$$y_t = F\left(k_t, h_t K_t^g\right) = k_t{}^{\alpha} h_t{}^{1-\alpha} (K_t^g)^{\gamma}. \quad (4)$$

The assumption for public capital is the same adopted in the previous chapters, following, for instance, Barro (1990). As we can see, although the production function has constant returns to scale to private inputs, the public capital is included as a positive externality over production, measured by $\gamma > 0$.

The first order conditions for the firm's problem imply that, for all periods,

$$r_t^k = \alpha \frac{y_t}{k_t};\ w_t = (1-\alpha)\frac{y_t}{h_t}. \quad (5)$$

7.3. Government

In each period, the government collects taxes on capital, consumption and hours worked, as well as it may issue public debt (*b*) to finance expenditures. The government constraint is given by

$$g_t^c + g_t^i + r_t b_t = g_t + b_{t+1} - b_t, \quad (6)$$

where $g_t = \tau_t^c c_t + \tau_t^h w_t h_t + \tau_t^k r_t^k k_t$ is the government tax revenue and g_t^i is the public investment.

[19] In the Appendix B, we derive the first order conditions for the representative household's problem.

The tax rates $\tau_t^c, \tau_t^h, \tau_t^k$ can adjust endogenously in each period, responding to the deviations of the public debt to GDP ratio, $s_t = b_t/y_t$, to the steady state value s:

$$\tau_t^j = \tau^j \exp\{\varphi_j(s_t - s)\}, j = c, h, k, \tag{7}$$

where τ^j is the tax rate of steady state. The parameter φ_j is the tax rate semi-elasticity with respect to the deviation of s_t to s. Higher values for φ_j indicate a more aggressive fiscal adjustment. s can be interpreted as a long run target for the ratio of public debt to GDP pursued by the government.

7.3.1. Time-to-Build Modeling

We model the government spending process according to Leeper et al. (2010), as mentioned in previous chapters. Yet, we restate equations due to some subtle differences between the model derived previously and the one presented here.

Considering N as the number of periods required to complete infrastructure projects, the public stock evolves according to

$$K_t^g = (1 - \delta_g)K_{t-1}^g + g_{t-N}^a, \tag{8}$$

where g^a is the approved investment.

We set $\{\phi_0, \phi_1, \phi_2, \dots, \phi_N\}$ as the spend-out rates, defined to be between the period when funds are obligated (0) to the period when the project is concluded (N), being incorporated to the stock of public capital for the next period.

Thus, we define public investment in t as

$$g_t^i = \sum_{n=0}^{N-1} \phi_n g_{t-n}^a, \tag{9}$$

where $\sum_{n=0}^{N-1} \phi_n = 1$.

7.3.2. Fiscal Regimes

We set the government consumption and approved investment to be determined as exogenous fractions of the output:

$$g_t^a = \pi_0 y_t, \tag{10}$$

and

$$g_t^c = \pi_1 y_t, \tag{11}$$

where π_0 e π_1 measure the weight of government consumption and approved investment in the economy.

7.4. Market Clearing Condition

Finally, we close the model with a goods market clearing condition:

$$c_t + k_{t+1} - (1-\delta)k_t + g_t^c + g_t^i = y_t. \tag{12}$$

7.5. Calibration of Parameters

7.5.1. Tax Rates

The calibration of parameters tries to match key Brazilian data between 2000 and 2006 adjusted to a quarterly basis. Tax rates on consumption, hours worked and capital are obtained by revenues collected by the federal, state and local governments.[20]

τ^c is calculated by the ratio of consumption tax revenues to aggregate consumption, both in terms of GDP. We get an average of τ^c = 0.23 between 2000 and 2006. Taxes basically include IPI, ICMS, Taxes on Imports (*Imposto de Importação* – II), ISS and COFINS.

In general, the methodology applied to this work produced tax rates on capital and hours worked higher than those found in other papers modeling the

[20] Data is provided by IBGE.

Brazilian economy, especially since 2004.[21] Due to the difficulties concerning the correct estimation of capital and labor incomes in Brazil, we chose to use the least tax rates calculated for both inputs between 2000 and 2006, in order to adopt values similar to those used in the recent Brazilian empirical literature, as in Paes and Bugarin (2006).

In this way, we set τ^k = 0.14, based on the ratio of capital taxes (IRPJ, IRRF on capital, CSLL, IPTU, IPVA and CPMF) to gross profits, both calculated as fractions of GDP. Finally, for the tax rate on hours worked, we divide the collected labor taxes (IRPF, social security contributions, FGTS, PIS/PASEP and IRRF on labor) by labor income in the National Accounts, both again in terms of GDP, getting $\tau^h = 0.22$.

7.5.2. Parameters on Technology and Preferences

Using annual data on stocks of private and public capital, we found depreciation rates of δ = 0.014 e δ_g = 0.009 on a quarterly basis (implying annualized rates of 5.6% and 3.5%, respectively).[22]

With respect to γ, as mentioned previously, there is no consensus about an appropriate value for this parameter. In Brazil, Ferreira and Maliagros (1999) found long run elasticities even higher than 0.4. However, due to the uncertainty surrounding this parameter, we continued to pick up a conservative value (γ = 0.10). As a robustness check, we chose a very high value for γ (γ = 0.35), as to reflect infrastructure constraints affecting some sectors of the Brazilian economy.

Turning to parameters regarding preferences, we set $\beta = \frac{1}{1+r}$ = 0.986, assuming a real interest rate on public debt of 6% (equivalent to a quarterly rate of r = 0.015). The discount rate above is equal to 0.95 in an annual basis. With these values, we found r^k = 0.0327 (or an annualized rental rate on private capital of 13.4%). Using the average of capital-GDP ratio for the private sector between 2000 and 2006, K/Y = 3.1, as well as the rental rate, we get α = 0.40. Finally, we

[21] The average tax rates for capital and hours worked are τ^k = 0.18 e τ^h = 0.23. The calibration approach of this work differs from the previous literature, which sets values for parameters based on a single particular year (for instance, Ferreira e Pereira (2009)).

[22] The value found for the depreciation rate on private capital are similar to Ferreira e Nascimento (2005), who set δ = 0.066 for 2004, and Paes e Bugarin (2006), who calibrate δ = 0.055 for 2002.

chose χ so that one quarter of the time endowment is spent on labor services in steady state (h = 0.25). The value is consistent with the empirical evidence for Brazil, as in Paes e Bugarin (2006) and Gonzaga, Machado and Machado (2003).

With respect to the intertemporal elasticity of substitution, $1/\sigma$, the empirical literature in Brazil is very scarce. Thus, we set σ = 3, a usual value in literature.[23] Finally, we calibrate the government consumption weight on the household utility, θ, to θ = 0.5. We also consider two extreme cases, of θ = 0 and θ = 1.[24] In the first one, the government consumption is purely waste, whereas, in the second, private and government consumption are perfect substitutes.

7.5.3. Fiscal Policy and Time-to-Build Parameters

Lastly, we calibrate fiscal policy parameters. First, we set s = 2.04, the average of the net public debt between 2000 and 2006.[25] Second, using monthly public investment data adjusted for a quarterly basis (2002:Q1 e 2006:Q4), we calculate π_1 = 0.018.[26] Moreover, given the Brazilian tax burden of 32% (average of 2000 to 2006), we calibrate $\pi_0 = g/y - \pi_1 - rs$ by the government budget in steady state. We get π_0 = 0.266. Since π_0 is a residual parameter, the value found was higher than the average of public consumption to GDP ratio around 20%.

For a matter of simplicity, we also assume that the sequence $\{\phi_n\}_{n=0}^{N-1}$ of spend-out rates follows $\phi_0 = 0$ and $\phi_n = 1/(N-1)$ for all n = 1, ..., N – 1. We consider in the analysis three scenarios for implementation delays: one quarter delay (N = 1, $\phi_0 = 1$), two years (N = 8, $\phi_0 = 0$ and ϕ_n = 1/7 for n = 1, ..., 7), three years (N = 12, $\phi_0 = 0$ and ϕ_n = 1/11 para n = 1, ..., 11) and four years (N = 16, $\phi_0 = 0$ and ϕ_n = 1/15 for n = 1, ..., 15). Therefore, we assume the government does not spend any amount of obligations in the period when they are granted, and, afterwards, distributes investment expenditures evenly over time. A similar procedure is used in the seminal paper of Kydland and Prescott (1982) as well as in Leeper, et. al (2010). As we will see later, the model fits public

[23] Issler and Piqueira (2000) estimated a value of 0.25 for $1/\sigma$. The authors, however, adopted a cautious stance regarding the estimation results.
[24] For θ = 0 and θ = 1, we adjust χ = 0.33 and χ = 0.41, respectively, so that h = 0.25 in both cases.
[25] The value of s = 2,04 corresponds to the public debt to GDP ratio on a quarterly basis, instead of annual.
[26] Public investment data were recently published by IPEA in the work of Santos et. al (2011), who calculated high frequency series for federal, state and local governments.

investment data more accurately when the time-to-build process reaches three years. In this case, the government spends less than 10% of obligations each quarter. In the following table, we summarize the calibrated parameters.

Table 4: Calibrated Parameters.

Calibrated Parameters							
Preferences		Technology		Government			
β	0.986	α	0.40	τ^c	0.23	π_0	0.018
χ	0.38 / 0.33 / 0.4	γ	0.10	τ^h	0.22	π_1	0.266
σ	3	δ	0.014	τ^k	0.14		
θ	0.5 / 0 / 1	δ_g	0.009				

7.5.4. Calibration of Different Fiscal Adjustment Scenarios

Lastly, we calibrate the φ_j parameters, $j = c, h, k$. We consider three alternative fiscal adjustment schemes. In each one, the government chooses whether the increase in expenditures is financed by higher taxes on consumption, capital or hours worked. This approach implies that, in each adjustment scheme, only one of the three tax rates τ^c, τ^h, τ^k can vary over time to stabilize the public debt to GDP ratio in the long run. Moreover, for each one of the three distinct schemes, the parameter φ_j is calibrated for three scenarios.

In the first one, φ_j is chosen to ensure that the deviation between s_t and s is less than 5% in only five years. In the second, the procedure is analogous, but the 5% target for the deviation ($s_t - s$) is to be achieved in a horizon of ten years. And, finally, in the last scenario, the target can be attained in an even larger period, of fifteen years. Therefore, we assess how tight and flexible fiscal adjustments affect macroeconomic variables in the short and long run.

7.5.5. The Time-to-Build Process for the GAP

In the model, we simulate a temporary shock on the approved public investment, which shifts from $\pi_1 = 0.018$ to $\pi_1 = 0.026$ for four years, and, then, returns to the steady state value of $\pi_1 = 0.018$.

Table 5: Quarterly public investment to GDP ratio (Ig/Y) – model and data

	Model N=12 (three years)	**Data Quarterly Ig/Y**	**Ratio Model and Data**
2007/Q1	0.018	0.018	0.99
2007/Q2	0.018	0.018	1.00
2007/Q3	0.019	0.017	1.08
2007/Q4	0.019	0.017	1.14
2008/Q1	0.020	0.019	1.04
2008/Q2	0.021	0.021	1.01
2008/Q3	0.022	0.023	0.92
2008/Q4	0.022	0.025	0.89
2009/Q1	0.023	0.021	1.12
2009/Q2	0.024	0.023	1.04
2009/Q3	0.025	0.023	1.05
2009/Q4	0.025	0.024	1.04
2010/Q1	0.026	0.025	1.03
2010/Q2	0.026	0.028	0.91
2010/Q3	0.026	0.029	0.89
2010/Q4	0.026	0.027	0.97

Public investment data is deseasonalized. Source: IPEA.

Table 6: Annual public investment to GDP ratio (Ig/Y) – model and data

	Model N=12 (three years)	**Data Annual Ig/Y**	**Ratio Model and Data**
2007	0.019	0.018	1.04
2008	0.021	0.022	0.95
2009	0.024	0.023	1.04
2010	0.026	0.028	0.94

Source: IPEA.

Simulations will report impulse response functions (obtained by a shooting-algorithm method) for implementation delays of two, three and four years. As mentioned previously, the time-to-build process of three years has the best fit to quarterly and annual data jointly.[27] We also present simulations for the case of a one quarter lag (the usual assumption in macroeconomic models). In

[27] We selected the appropriate time-to-build process according to a simple criterion: we found the lag parameter, N, that minimizes $\sum_{t=1}^{4}(ig_t^{anual} - \widehat{ig}_t^{anual})^2 + \sum_{t=1}^{16}\left(ig_t^{trimestral} - \widehat{ig}_t^{trimestral}\right)^2$, where ig_t and $\widehat{ig}_t$ represent the public investment to GDP ratios observed in the data and generated by the model, respectively. The superscript refers to the corresponding periodicity.

Tables 5 and 6, we compare public investment to GDP ratios generated by the model (with a three-year time-to-build process) to those actually observed in the data.[28] As we can see, the model is able to provide a satisfactory fit in spite of its simplicity. The fit remains reasonable even if we compare the model predictions in a higher frequency basis.[29]

Considering the concern of the federal government with delays in the GAP projects, the three-year lag may be viewed as conservative, and, in fact, an implementation delay of four years may be easily conjectured. In this last case, the government spends little more than 6% of obligations each quarter.

[28] The public investment to GDP ratios reported in the tables correspond to a particular model: The tax rates remain constant over time, and the government adjusts its budget through lump-sum taxes. In this case, there is no public debt. In the Appendix B, we present the impulse response functions for this specific case.

[29] In the Appendix B, we report public investment to GDP ratios concerning the delays of two and four years. In the first case, the model tends to overpredict the actual ratios, both in the quarter and in the annual basis. In the second case, the opposite occurs, since the model underpredicts the observed data. However, the fit is better when we set a four-year implementation lag.