3
EAC Notch Sensitivity Analysis on Common Specimens

As previously seen on section 2.3, the linear elastic stress gradient ahead of the notch tip on very large plates loaded by purely tensile stresses has a major influence on the behavior of short cracks that start there. However, most structural components neither have very large residual ligaments compared to the notch dimensions, nor work under pure tensile loads. To quantify how important such effects can be, this chapter applies the notch sensitivity concepts based on the peculiar behavior of short cracks to analyze four standard specimens, M(T), SEN(T), SEN(B), and C(T), modified to have notches instead of cracks, see Fig. 28 to 31.

Figure 28 – Modified C(T) sample based on ASTM E647-13e148.
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Figure 29 – Modified $M(T)$ sample based on ASTM E647-13e148.

Figure 30 – Modified $SEN(B)$ sample based on ASTM E1820-11e261.
In order to guide this numerical study of the propagation behavior of short cracks nucleated at the notch tip in such specimens, some parameters were fixed, as described below:

**Material & Environment**
- Material: Annealed Al 2024;
- Environment: Liquid Gallium;
- Temperature: 35° C.

**Mechanical Properties**
- $K_{IEAC} = 8.5 \text{ MPa}\cdot\sqrt{m}$;
- $S_{EAC} = 45 \text{ MPa}$;
- Yield Strength $S_y = 113 \text{ MPa}$;
- Young’s Modulus $E = 70 \text{ GPa}$;
- Poisson’s Ratio $\nu = 0.33$;
- Free Surface Factor $\eta = 1.12$. 
The mechanical properties \((K_{IEAC}, S_{EAC}, S_y, E \text{ and } \nu)\) used in this work were obtained from the tests mentioned on section 2.5.

**Specimens Geometric Parameters**

- Thickness \((t) = 6\, \text{mm}\);
- Width \((W) = 50\, \text{mm}\);
- Notch semi-axes \((b) = \{5\, \text{mm}, 10\, \text{mm}, 15\, \text{mm}, 20\, \text{mm}\}\)
- Notch tip radii \((\rho) = \{0.1\, \text{mm}, 0.2\, \text{mm}, 0.5\, \text{mm}, 1\, \text{mm}, 2\, \text{mm}, 5\, \text{mm}\}\)

For the M(T) specimens the parameter \(W\) must be multiplied by 2 in order to keep their residual ligaments \(r_l\). Bazant’s coefficient \((\gamma)\) is assumed as equal to 2, in accordance with the model proposed by El-Haddad, Topper and Smith\(^{16-17}\). With these parameters,

\[
a_{0_{EAC}} = \frac{1}{\pi} \cdot \left( \frac{K_{IEAC}}{\eta \cdot S_{EAC}} \right)^2
\]

\[
a_{0_{EAC}} = 9.05\, \text{mm}
\]

Using an analogous equation to eq. (8), the stress intensity factor threshold for EAC \(K_{ith}(a)\) is obtained:

\[
K_{ith}(a) = K_{IEAC} \cdot \left[ 1 + \left( \frac{a_{0EAC}}{a} \right)^{\gamma/2} \right]^{-1/\gamma}
\]
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Figure 32 – SEN(T) specimen: The threshold crack propagation $K_{th}$ (green); $K_{t}$ considering the notch tip stress $\sigma_{ntc}=80$ MPa (Blue); $K_{i}$ (red) considering the nominal stress at the notch tip $\sigma_{n} = S_{Lntc}$ which leads to $\sigma_{ntc} = K_{t} \cdot S_{Lntc} = 136.8$MPa.

Figure 32 shows a comparison between two different load cases: The first one has the notch tip stress $\sigma_{ntc} = 80$ MPa $> S_{EAC}$ (45 MPa) where a non-propagating crack is obtained ($a_{st} = 0.5$ mm) by the intersection $K_{t}$ and $K_{th}$ functions. The second is the limit case ($\sigma_{ntc} = S_{Lntc}$) where $K_{t}$ will never be lower than $K_{th}$.

Five parameters will be highlighted to the study of the notch sensitivity in EAC:

- Crack stop size ($a_{st}$) given a tension at the notch tip ($\sigma_{ntc}$);
- Stress concentrator factor: $K_{t} = \sigma_{ntc}/\sigma_{n}$;
- Nominal tension at the notch tip that causes a propagating crack ($S_{Lntc}$);
- EAC concentrator factor: $K_{tEAC} = S_{EAC}/S_{Lntc}$;
- EAC notch sensitivity: $q_{EAC} = (K_{tEAC} - 1)/(K_{t} - 1)$. 
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For the \textit{a}st analysis, the stress on the notch tip $\sigma_{ntc}$ will be equal to 80 MPa, chosen to be higher than $S_{EAC} = 45$ MPa, in order to provide a comparison between the specimens considering the notch geometric variations.

The analyses were didactically divided considering 3 geometric parameters:

1) Specimen shape;
2) Semi-axis $b$;
3) Tip radius $\rho$.

The notch sensitivity numerical analyses were performed on 10 specimens considering two approaches:

- Fixed value for the notch semi-axis ($b = 20$ mm) and variable notch tip radius ($\rho = 0.1$ mm; 0.2 mm; 0.5 mm; 1 mm; 2 mm; 5 mm);
- Fixed value for the notch tip radius ($\rho = 1$ mm) and variable notch semi-axis ($b = 5$ mm, 10 mm, 15 mm and 20 mm.).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Figure33}
\caption{Geometric parameters of modified C(T) sample and its nominal stress applied at the notch tip.}
\end{figure}
Figure 34 – Geometric parameters of modified M(T) sample and its nominal stress applied at the notch tip.

Figure 35 – Geometric parameters of modified SEN(B) sample and its nominal stress applied at the notch tip.
Two methodologies are presented in this study to obtain the $K_I$ ahead of the notch: the finite element methodology and the mixed methodology. Both have advantages and disadvantages as will be explained in this study.

The finite element methodology uses Quebra2D, which is a friendly numerical tool specially, developed for finite element modeling of cracks propagation in two-dimensional arbitrary geometries (2D). The use of Quebra2D with a considerable accuracy on studies of fatigue short cracks $^{31-32}$, contributes with the confidence to introduce this methodology as calibration model to the mixed methodology.

However, even a friendly numerical tool can be, in some cases, computationally expensive in matters of time or "not so friendly" with some mechanical engineers who can be not familiarized with finite element method or wishing quicker answers than the most accurate answers to their problems.

The mixed methodology is presented in this work to provide a faster tool to the notch sensitivity analysis. The method consists on the use of two distinct
approaches for the calculus of $K_I$ ahead of the notch. The $K_I$ analysis is splitted between the region on the vicinity of the notch influence, using as basis the Creager & Paris\textsuperscript{31} method for the $K_I$ calculus, and the region far away from the notch influence, where $K_I$ is obtained directly by the standards equations.

3.1. Finite Element Methodology for the Stress Intensity Factor $K_I$

In order to obtain the notch sensitivity, the propagation threshold ($K_{Ith}$) and the stress intensity factor on mode I ($K_I$) must be calculated and compared.

The propagation threshold ($K_{Ith}$) is modeled by the eq. (31) and (34), didactically repeated below:

$$a_{0EAC} = \frac{1}{\pi} \cdot \left(\frac{K_{IEAC}}{\eta S_{EAC}}\right)^2$$

$$a_{0EAC} = 9.05 \text{ mm}$$

$$K_{Ith}(a) = K_{IEAC} \cdot \left[1 + \left(\frac{a_{0EAC}}{a}\right)^{\gamma/2}\right]^{-1/\gamma}$$

As it can be seen on equation (34), $K_{Ith}$ is a function of the length of the crack that departs from the notch tip as shown in figure 37.

![Figure 37](image)

**Figure 37** – Length reference, used on the proposed model, for the cracks that depart from the notch tip.

With the data above presented, using Excell\textsuperscript{®} or MathCad\textsuperscript{®}, it can be made a spreadsheet or Cartesian chart for the pair $K_{Ith}(a) \times a$, see Fig. 38.
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Figure 38 – Cartesian chart and Excel® spreadsheet showing $K_{th}$ (a) x a, for the Aluminum 2024 under liquid Gallium attack: $S_{EAC} = 45$ MPa ; $K_{IEAC} = 8.5$ MPa $\cdot \sqrt{m}$; $a_0 = 9.05$ mm.

The above equations, chart and spreadsheet can be replicate for all the samples analyzed on this study, since $K_{th}$ is a function only of the material mechanical properties ($S_{EAC}$ and $K_{IEAC}$).

The stress intensity factor $K_I$ is now necessary in order to provide a comparison with $K_{th}$ and then obtain the notch sensitivity parameters. The $K_I$ calculus was performed with the finite element method, using Quebra2D.

A full description of numerical methods theory is beyond the scope of this work. However, some details of the technique must be introduced to provide the understanding of the methodology used.

There are three main techniques to calculate stress intensity factors SIF using the finite element method\textsuperscript{62-66}, as briefly shown below:

1) Displacement correlation technique, that uses special elements called quarter-point;
2) Potential energy release rate, obtained by modifying the integral that estimates the plastic zones by closing the cracks tips. This technique is called modified crack-closure integral MCC;

3) $J$ integral\(^{67}\) computed on an equivalent area around the crack. This technique is called equivalent domain integral (EDI);

According to Miranda et al\(^{62}\), the three aforementioned methods give different responses when the mesh is not well refined ahead of the crack tip. In this case the EDI method gives better results in comparison with the others techniques. Nevertheless when the finite element mesh is sufficiently refined, the results obtained from all techniques are similar. As Quebra2D refines the mesh for every crack propagation iteration, it can be used any of propagation method without any further loss of accuracy. In this particular study, it was used the displacement correlation technique.

Another important output is the crack propagation direction that can be studied by three distinct approaches: The first one is given by the direction perpendicular to the maximum tensile stress ($\sigma_{\theta_{\text{max}}}$ or $\sigma_{y_{\text{max}}}$), depending on the system coordinates chosen. This first approach was used on this study. The second approach is the direction that maximizes potential energy release rate on the structure $G_{\theta_{\text{max}}}$. The last one is the direction that minimizes the strain energy density $U_{\theta_{\text{max}}}$. To simulate the crack path using any of the above criteria, it must be known the values of $K_I(a_i)$ and $K_{II}(a_i)$, where $a_i$ is the crack length on the increment step $i$. Nevertheless, the SIF calculus for each crack length increment is not an easy task, and usually is performed by sophisticated numerical methods.

Quebra2D was designed to treat, efficiently, these kinds of problem. Among the many features introduced in the software, the following can be highlighted:

- An automatic mesh generator self-adaptive of finite elements, fast and efficient, which evaluates the quality of the elements to maximize their numerical stability;
- Special crack tip element’s mesh, which reproduces the singularity $1/\sqrt{a}$ of the linear elastic fracture mechanics LEFM;
Automatic simulation of crack propagation can be done with user-specified increments;

- The user can choose the technique to calculate $K_I$ and $K_{II}$. The program supports DCT, MCC and EDI.
- The user can choose the prediction’s method for the incremental crack’s direction ($\sigma_{\theta_{\text{max}}}$, $G_{\theta_{\text{max}}}$, $U_{\theta_{\text{max}}}$).

**Figure 39 – Quebra2D:** (a) Stress Analysis on a non-cracked C (T) specimen, (b) Detail of the stress distribution on the vicinity of the notch tip of a non-cracked C (T) specimen, (c) Detail of the stress distribution on a propagating crack initiated at the notch tip of C (T) specimen.

The procedure for obtaining $K_I$ starts with the 2D sketch of the specimens according to the standards. Then the specimen thickness ($t$), the Young’s modulus ($E$) and Poisson’s ratio ($\nu$) are included.

The plane stress state was chosen, given that the analysis for this work considers thin specimens. Analysis on thick specimens and 3D analyses are beyond the scope this study, and must be considered for future works.
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Figure 40 – C(T) specimen based on ASTM E647-13e1\textsuperscript{148} modeled on Quebra2D with the following parameters: \( W=50 \text{mm}, \ b=20 \text{mm}, \ p=2 \text{mm}, \ t=6 \text{mm}, \ E=70 \text{ GPA}, \ \nu=0,33. \)

After input the model geometry and the material parameters, it is necessary to define the loads and constraints applied on the specimen, see fig.41.

Figure 41 – Loads and constraints applied on the specimens. C(T) and SEN(B) specimens are constrained only on \textbf{y-direction}. SEN(T) and M(T) specimens are constrained on \textbf{x-direction} and \textbf{y-direction}. 
The next step is meshing the model. As mentioned before Quebra2D has an automatic mesh generator self-adaptive of finite elements, fast and efficient that uses triangular quadratic quarter-point elements.

In order to calculate the stress concentrator factor ($K_t$), a first mesh is generated with lower refinement, using the Quebra2D default number of nodes per edge and a load of 1 N is applied on the model. Then the notch tip stress is probed, and manually, the mesh is refined (Fig. 42), especially on the notch tip, till the value of $\sigma_{ntc}$ become stable.

Using the analytical expressions shown below, it can be calculated the nominal stress on the notch tip ($\sigma_n$) and finally obtained the stress concentrator value ($K_t = \sigma_{ntc} / \sigma_n$).

\[
C(T) = \frac{P}{t \cdot \left(\frac{2 \cdot W + b}{W - b}\right)^{3/2}} \cdot \frac{1}{\sqrt{b}}
\]

\[
M(T) = \frac{P}{t \cdot W}
\]

\[
\text{SEN(B)} = \frac{3}{2} \cdot \frac{P \cdot (4.5 \cdot W)}{t \cdot W^2}
\]
The nominal tensions calculated in this work are based on the SIF equations provided by the standards. The nominal stress used on the standards are not so clearly detailed on the standard’s description and should be deduced, and sometimes, as in the case of C(T) specimen, must be inferred.

For the M(T), SEN(T) and SEN(B) specimens the notch depth ($b$) is disregarded on the nominal tension calculus. And on the C(T) specimen the nominal tension extracted from the standard is so distinct that is hard to understand what are the considerations for its calculus.

Even considering the nominal stress calculus provided by the standards not appropriate, they shall be used in order to provide a consistent analysis for the notch sensitivity.

In order to perform a double check, all the $K_I$ values were calculated using another finite elements software, ABAQUS®. The results comparison shown differences lower than 0.5% for all specimens, which could be explained by the different mesh applied by each software on the models.
Figure 42 – Quebra2D mesh refinement process: (a) First mesh generated with the default numbers of nodes/edge (5); (b) The first mesh refinement, increasing the number of nodes all around the model; (c) Last mesh refinement increasing the number of nodes on the notch tip; (d) Notch detail showing the mesh refinement.

Figure 43 – $K_t$ result for a C(T) specimen with the following parameters: $W=50\text{mm}$, $b=20\text{mm}$, $p=2\text{mm}$, $t=6\text{mm}$, $E=70$ GPA, $\nu=0.33$: (a) Stress distribution on $y$-direction ($\sigma_y$); (b) Detail indicating higher stresses values and higher stress gradient around the notch tip; (c) $K_t$ obtained by the analytical value of nominal stress on $y$-direction ($\sigma_n$) and the stress on the notch tip ($\sigma_{ntc}$) on $y$-direction obtained with Quebra2D.
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The load value then is increased till the notch tip stress ($\sigma_{ntc}$) becomes equal to 80MPa, as previously defined to provide the astart comparison between the specimens.

The next step of the procedure is to obtain $K_I$, and starts by introducing a tiny crack ($a = 0.01\text{mm}$) on the notch tip. This small crack is represented by a line with five nodes and defined as a crack on Quebra2D model.

**Figure 44** – (a) Stress distribution ($\sigma_y$) around the notch tip on a non-cracked C(T) specimen; (b) Tiny crack (a=0.01mm) introduced on the notch tip to provide a start to the propagation analysis; (c) Mesh created considering the induced crack (with 5 nodes); (d) Stress distribution ($\sigma_y$) due the introduction of the tiny crack.

Then some attributes must be specified on the software to guide it on the crack propagation analysis (figure 45):

- Crack size increment rule;
- Crack’s stopping rule;
- Techniques to calculate stress intensity factors $K_I$;
- Techniques to calculate the crack propagation’s direction;
- Criteria to evaluate de equivalent stress intensity factor $K_{eq}$.
As the interest is only on $K_I$, given the symmetry of the specimens and the applied loads directions, the criteria for evaluating the equivalent stress intensity factor $K_{eq}$ are disregarded on this work.

![Figure 45 – Quebra2D crack’s propagation input parameters: (a) Crack length’s increment; (b) Stopping criteria for the crack’s propagation; (c) Technique to calculate stress intensity factors; (d) Techniques to calculate the crack’s propagation direction.](image)

The threshold $K_{th}$ was defined as lower as possible, given that the interest on the work is to obtain the values of $K_I$, and define a considerable value for $K_{th}$ could lead a non-desirable crack stop.

The increment size was defined as 0.2 mm. The number of elements on each increment was defined as 10. So, each element has a length equal to 0.02mm.

The stopping rule was chosen as the number of steps, in the macro analysis equal to 50, which leads to a final crack size of 1cm (50 steps of 0.2 mm). This final crack size was defined in order to provide a first approximation of $a_{st}$ (fig. 46 and 47).

A second analysis is then performed in order to refine the crack’s discretization, using 50 increment steps of 0.02 mm, resulting on a better $a_{st}$ approximation (fig. 48 and 49).

The crack tip coordinates $(x,y)$ and the values of $K_I$, $K_{II}$, $K_{eq}$ are written by the program on an ASCII file. These information are then treated in order to provide the pair $(K_I \times a)$. 
Figure 46 – $K_{th}(a)$ and $K_i(a)$ on a C(T) specimen: $W=50\, \text{mm}$, $b=20\, \text{mm}$, $\rho=2\, \text{mm}$, $t=6\, \text{mm}$, $E=70$ GPA, $v=0.33$.

Figure 47 – $K_i(a) - K_{th}(a)$ on a C(T) specimen: $W=50\, \text{mm}$, $b=20\, \text{mm}$, $\rho=2\, \text{mm}$, $t=6\, \text{mm}$, $E=70$ GPA, $v=0.33$. 
Figure 48 – Refinement of $K_{th}(a)$ and $K_i(a)$ on a C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70\text{ GPA}$, $\nu=0.33$. This analysis provides a better result for $a_{st}$.

Figure 49 – Refinement of $K_i(a) - K_{th}(a)$ on a C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70\text{ GPA}$, $\nu=0.33$. This analysis provides a better result for $a_{st}$. 
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With the numerical data obtained on Quebra2D, the value of \( a_{st} \) is given through the linearization between the closest values that lead \( K_I - K_{Ith} = 0 \), see table 2.

<table>
<thead>
<tr>
<th>CT (( p = 2 ) mm)</th>
<th>a (m)</th>
<th>( K_I ) (Mpa·Vm)</th>
<th>( K_{Ith} ) (Mpa·Vm)</th>
<th>( K_I - K_{Ith} ) (Mpa·Vm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,21E-03</td>
<td>2,98E+00</td>
<td>2,92E+00</td>
<td>5,97E-02</td>
<td></td>
</tr>
<tr>
<td>1,24E-03</td>
<td>2,99E+00</td>
<td>2,95E+00</td>
<td>3,66E-02</td>
<td></td>
</tr>
<tr>
<td>1,27E-03</td>
<td>3,00E+00</td>
<td>2,98E+00</td>
<td><strong>1,60E-02</strong></td>
<td></td>
</tr>
<tr>
<td>1,30E-03</td>
<td>3,01E+00</td>
<td>3,01E+00</td>
<td><strong>-1,72E-03</strong></td>
<td></td>
</tr>
<tr>
<td>1,33E-03</td>
<td>3,02E+00</td>
<td>3,04E+00</td>
<td><strong>-1,75E-02</strong></td>
<td></td>
</tr>
<tr>
<td>1,36E-03</td>
<td>3,04E+00</td>
<td>3,07E+00</td>
<td><strong>-3,41E-02</strong></td>
<td></td>
</tr>
</tbody>
</table>

In order to calculate the threshold nominal stress on the notch tip (\( S_{Lntc} \)), two assumptions are considered in this work:

1) The analyses are under linear elastic fracture mechanics;
2) The nominal stress on the notch tip is directly proportional to the load applied on the specimen.

Considering the above exposed, the procedure to find \( S_{Lntc} \) starts with the multiplication of \( K_I \) values, obtained on the first analysis, by an adjustment factor (\( Z \)) till obtain the tangency of \( K_I \) and \( K_{Ith} \) functions, see fig. 50. Another approach can be the search of the tangency between \( K_I - K_{Ith} \) function and the abscissa’s axis, see fig.51.

Then to obtain the value of \( S_{Lntc} \), just need to multiply the value of the nominal stress on the notch tip \( \sigma_n \), for the initial condition (\( \sigma_{ntc} = \sigma_n \cdot K_I = 80\text{MPa} \)), by the adjustment factor (\( Z \)).

\[
S_{Lntc} = \sigma_n \cdot Z
\]  

(39)
Then the values of EAC stress concentration factor $K_{tEAC}$ and EAC notch sensitivity $q_{EAC}$ can be calculated by the following equations:

$$K_{tEAC} = S_{EAC}/S_{Lntc}$$  \hspace{1cm} (40)

$$q_{EAC} = (K_{tEAC} - 1)/(K_t - 1)$$  \hspace{1cm} (33)

**Figure 50** – C(T) Specimen $W=50$mm, $b=20$mm, $\rho=2$mm, $t=6$mm, $E=70$ GPa, $v=0.33$: $K_I$ function (Blue) with $\sigma_{ntc} = 80$MPa; $K_I$ function (Red) adjusted in order to obtain $S_{Lntc}$.
3.2. Mixed Methodology for the Stress Intensity Factor $K_I$

The second methodology was called mixed because it treats, separately, the $K_I$ on the notch vicinity, which is totally influenced by the stress gradient disturbed by the notch, and the $K_I$ far from the notch, which is dominated by the crack’s influence, geometry of the specimen and the type of the load applied, i.e. if it is a tensile or bending load.

The stress intensity factor $K_I$ on the notch vicinity was treated using the approach brightly proposed by Creager & Paris\textsuperscript{51}. This interesting and powerful technique circumvents the singularity problem of the Irwin\textsuperscript{68}/Williams\textsuperscript{69} solutions ($r \to 0 \therefore \sigma \to \infty$), see figure 52, and uses the available $K_I$ standard solutions, to calculate $K_I$ and the disturbed stress field on the notch surround.
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Figure 52 – Stress Field based on $K_I$, obtained by Irwin$^{68}$ and Williams$^{69}$, on the vicinity of crack tip.

Creager and Paris showed, in the case of slender and depth notches with tip radius $\rho$, shifting the origin of the coordinate’s axes ($r$ and $\theta$) of $\rho/2$ to inside the notch (figure 53), it would be possible to estimate the stress field on the notch vicinity using the SIF standard solutions of the corresponding cracks.

Figure 53 – Coordinates used on the model proposed by Creager and Paris.
The stress fields ahead of the notches on modes I, II and III are given by:

\[
\begin{align*}
\sigma_x &= \frac{K_I}{\sqrt{2\pi}r} \cdot \frac{\rho}{2r} \left\{ \frac{-\cos(3 \cdot \theta/2)}{\cos(3 \cdot \theta/2)} \right\} + \\
\sigma_y &= \frac{K_I}{\sqrt{2\pi}r} \cdot \frac{\rho}{2r} \left\{ \frac{1 - \sin(\theta/2) \cdot \sin(3 \cdot \theta/2)}{1 + \sin(\theta/2) \cdot \sin(3 \cdot \theta/2)} \right\} + \\
\tau_{xy} &= -\frac{K_I}{\sqrt{2\pi}r} \cdot \frac{\rho}{2r} \left\{ \frac{\sin(3 \cdot \theta/2)}{\cos(3 \cdot \theta/2)} \right\} \tag{41}
\end{align*}
\]

\[
\begin{align*}
\sigma_x &= \frac{K_{II}}{\sqrt{2\pi}r} \cdot \frac{\rho}{2r} \left\{ \frac{\sin(3 \cdot \theta/2)}{\cos(3 \cdot \theta/2)} \right\} + \\
\sigma_y &= \frac{K_{II}}{\sqrt{2\pi}r} \cdot \frac{\rho}{2r} \left\{ \frac{-\sin(\theta/2) \cdot [2 + \cos(\theta/2) \cdot \cos(3 \cdot \theta/2)]}{\sin(\theta/2) \cdot \cos(\theta/2) \cdot \cos(3 \cdot \theta/2)} \right\} + \\
\tau_{xy} &= +\frac{K_{II}}{\sqrt{2\pi}r} \cdot \frac{\rho}{2r} \left\{ \frac{\cos(\theta/2) \cdot [1 - \sin(\theta/2) \cdot \sin(3 \cdot \theta/2)]}{\sin(\theta/2) \cdot \cos(3 \cdot \theta/2)} \right\} \tag{42}
\end{align*}
\]

\[
\begin{align*}
\tau_{xz} &= \frac{K_{III}}{\sqrt{2\pi}r} \cdot \left\{ -\sin(\theta/2) \right\} \\
\tau_{yz} &= +\frac{K_{III}}{\sqrt{2\pi}r} \cdot \left\{ \cos(\theta/2) \right\} \tag{43}
\end{align*}
\]

In that way, to estimate the $K_t$ value of these slender notches it is necessary perform the variable’s substitution $r = \frac{\rho}{2}$ on the above equations. As the interest of this work is $K_I$, and $\sigma_y(r=0, \theta=0) = K_I \cdot \sigma_n$, $K_I$ can be estimated as:

\[
K_t \approx \frac{2 \cdot K_t(a'=b)}{\sigma_n \cdot \sqrt{\pi \cdot \rho}} \tag{44}
\]

To use equation 44, it is important that the crack’s length must be equal to the notch semi-axis ($a' = b$) in the standard’s $K_I$ equations as shown on figure 54 and on the equations described below:
Figure 54 – Coordinates used on the standards for $K_i$ evaluation. $K_i$ is calculated considering $a' = b$.

\[ C(T) \]

\[
K_i(P, a') = \frac{P}{t\sqrt{W}} \cdot \left[ \frac{2 + \frac{a'}{W}}{\left(1 - \frac{a'}{W}\right)^{1.5}} \right] \cdot \left[ 0.886 + 4.64 \cdot \frac{a'}{W} + 14.72 \cdot \left(\frac{a'}{W}\right)^3 - 5.6 \cdot \left(\frac{a'}{W}\right)^3 \right]
\]

\[ M(T) \]

\[
K_i(P, a') = \frac{P}{t} \cdot \frac{\pi \left(\frac{2 - a'}{W}\right)}{2W} \cdot \sqrt{\left[ \frac{\pi - 2 - a'}{2}\right]}
\]

\[ SEN(B) \]

\[
K_i(P, a') = \frac{6.75P}{t\sqrt{W}} \cdot \left[ \frac{\sqrt{\frac{a'}{W}}}{\left[1+2\left(\frac{a'}{W}\right)\right]^{1.5}} \right] \cdot \left[ 1.99 - \frac{a'}{W} \cdot \left(1 - \frac{a'}{W}\right) \cdot \left(2.15 - \frac{3.93\cdot a'}{W} + 2.7 \left(\frac{a'}{W}\right)^2 \right) \right]
\]

\[ SEN(T) \]

\[
K_i(P, a') = \frac{P}{t\sqrt{W}} \cdot \left[ 1.99 \cdot \left(\frac{a'}{W}\right)^{0.5} - 0.41 \cdot \left(\frac{a'}{W}\right)^{1.5} + 18.7 \cdot \left(\frac{a'}{W}\right)^{2.5} + 38.85 \cdot \left(\frac{a'}{W}\right)^{3.5} + 53.85 \cdot \left(\frac{a'}{W}\right)^{4.5} \right]
\]
The disturbed stress ($\sigma_y$) gradient around the notch can be estimated considering $\theta = 0$, the value of $K_I(a'=b)$, and part of the equation 41:

$$\sigma_y(r) = \frac{K_I(a' = b)}{\sqrt{2 \cdot \pi \cdot r}} \cdot \left(1 + \frac{\rho}{2 \cdot r}\right)$$  \hspace{1cm} (48)

Performing the variable’s substitution $r = (\rho/2) + a$:

$$\sigma_y(a) = \frac{K_I (a' = b)}{\sqrt{\pi \cdot (\rho + 2 \cdot a)}} \cdot \left(1 + \frac{\rho}{\rho + 2 \cdot a}\right)$$  \hspace{1cm} (49)

Thus the stress intensity factor close to the notch surround ($K_{iclose}$) can be expressed as:

$$K_{iclose}(a) = \sigma_y(a) \cdot \sqrt{\pi \cdot a}$$

or

$$K_{iclose}(a) = K_I(a' = b) \cdot \frac{\sqrt{\pi \cdot a}}{\sqrt{\pi \cdot (\rho + 2 \cdot a)}} \cdot \left(1 + \frac{\rho}{\rho + 2 \cdot a}\right)$$  \hspace{1cm} (50)

The stress intensity factor $K_I$ far away from the notch, called on this work as $K_{ifar}$, is obtained directly from the standard’s equations presented before on eq. (45), (46), (47) and (30) with the variable’s substitution $a' = b + a$:

$C(T)^{48}$

$$K_{ifar}(P, a) = \frac{P}{t \cdot \sqrt{W}} \cdot \left[2 \cdot \frac{a+b}{W} \right] \cdot \left[0.886 + 4.64 \cdot \frac{a+b}{W} + 
+ 14.72 \cdot \left(\frac{a+b}{W}\right)^3 - 5.6 \cdot \left(\frac{a+b}{W}\right)^3\right]$$  \hspace{1cm} (45)

$M(T)^{48}$

$$K_{ifar}(P, a) = \frac{P}{t} \cdot \sqrt{\frac{\pi \cdot \left(\frac{2 \cdot (a+b)}{W}\right)}{2 \cdot W}} \cdot \sec \left[\frac{\pi \cdot \left(\frac{2 \cdot (a+b)}{W}\right)}{2}\right]$$  \hspace{1cm} (46)
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\[ K_{f_{\text{far}}} (P, a) = \frac{6.75 \cdot P}{t \cdot \sqrt{W}} \left[ \frac{a+b}{W} \right]^{1.5} \cdot \left[ 1.99 + \frac{a+b}{W} \cdot \left( 2.15 - \frac{3.93 \cdot (a+b)}{W} + 2.7 \left( a+b/w \right)^2 \right) \right] \]  

\[ K_{f_{\text{far}}} (P, a) = \frac{P}{t \cdot \sqrt{W}} \cdot \left[ 1.99 \cdot \left( \frac{a+b}{W} \right)^{0.5} - 0.41 \cdot \left( \frac{a+b}{W} \right)^{1.5} \right] + 18.7 \cdot \left( \frac{a+b}{W} \right)^{2.5} + 38.85 \cdot \left( \frac{a+b}{W} \right)^{3.5} + 53.85 \cdot \left( \frac{a+b}{W} \right)^{4.5} \]  

The mixed methodology was applied in this work with MathCad® using the following procedure:

1) Choose the type of specimen: C(T), M(T), SEN(B), SEN(T),
2) Define the geometric parameters of the specimen: W, b, \( \rho \) and t;
3) Define the desirable stress on the notch tip (\( \sigma_{\text{ntc}} \));
   In this work, \( \sigma_{\text{ntc}} \) is equal to 80MPa in order to perform the ast comparison.
4) Calculate \( \sigma_n \) and \( K_t \). Use the equations (35), (36), (37) or (38) for \( \sigma_n \). To calculate \( K_t \), use equation 44 with the \( K_I \) equations (45), (46), (47) and (30);

Nominal Stress \( \sigma_n \)

C(T)

\[ \sigma_{n_{\text{CT}}} (P) = \frac{P}{t} \cdot \left( \frac{2 \cdot W + b}{(W-b)^{3/2}} \right) \cdot \frac{1}{\sqrt{b}} \]  

M(T)

\[ \sigma_{n_{\text{M(T)}}} (P) = \frac{P}{t \cdot W} \]
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**SEN(B)**

\[
\sigma_{n_{SENB}}(P) = \frac{3}{2} \cdot \frac{P \cdot (4.5 - W)}{tW^2}
\]  

(37)

**SEN(T)**

\[
\sigma_{n_{SEN(T)}}(P) = \frac{P}{tW}
\]

(38)

**Stress Concentration Factor** \(K_t\)

\[
K_t \cong 2 \cdot K_1(a' = b) \frac{\sigma_n}{\sqrt{\pi} \cdot \rho}
\]

(44)

**Standards Stress Intensity Factor** \(K_i(a' = b)\)

**C(T)**

\[
K_i(P) = \frac{p}{t \sqrt{W}} \left[ \frac{2 + \frac{b}{W}}{\left(1 - \frac{b}{W}\right)^{1.5}} \right] \cdot \left[0.886 + 4.64 \cdot \frac{b}{W} + 14.72 \cdot \left(\frac{b}{W}\right)^3 - 5.6 \cdot \left(\frac{b}{W}\right)^3\right]
\]

(45)

**M(T)**

\[
K_i(P) = \frac{p}{t} \cdot \frac{\sqrt{\frac{\pi \cdot \left(\frac{2b}{W}\right)}{2W}} \cdot \sec \left[\frac{\pi \cdot \left(\frac{2b}{W}\right)}{2}\right]}
\]

(46)

**SEN(B)**

\[
K_i(P) = \frac{6.75P}{t \sqrt{W}} \left[ \frac{\sqrt{\frac{b}{W}}}{\left[1 + 2 \cdot \frac{b}{W}\right] \left(1 - \frac{b}{W}\right)^{1.5}} \right] \cdot \left[1.99 - \frac{b}{W}\right] \cdot \\
\left(1 - \frac{b}{W}\right) \cdot \left(2.15 - \frac{3.93b}{W} + 2.7 \left(\frac{b}{W}\right)^2\right)
\]

(47)
\[ K_I(P) = \frac{p}{t \sqrt{W}} \cdot [1.99 \cdot \left(\frac{b}{W}\right)^{0.5} - 0.41 \cdot \left(\frac{b}{W}\right)^{1.5} +
+ 18.7 \cdot \left(\frac{b}{W}\right)^{2.5} + 38.85 \cdot \left(\frac{b}{W}\right)^{3.5} + 53.85 \cdot \left(\frac{b}{W}\right)^{4.5}] \] (30)

5) Calculate \( K_{I\text{close}} \) with equation (50) combined with the \( K_I(a'=b) \) equations (45), (46), (47) and (30);

\[ K_{I\text{close}}(a) = K_I(a'=b) \cdot \frac{\sqrt{\pi \cdot a}}{\sqrt{\pi \cdot (\rho + 2 \cdot a)} \cdot \left(1 + \frac{\rho}{\rho + 2 \cdot a}\right)} \] (50)

6) Calculate \( K_{I\text{th}}(a) \) with eq. (31) and (34);

\[ a_{0EAC} = \frac{1}{\pi} \cdot \left(\frac{K_{IEAC}}{\eta S_{EAC}}\right)^2 \] (31)

\[ K_{I\text{th}}(a) = K_{IEAC} \cdot \left[1 + \left(\frac{a_{0EAC}}{a}\right)^\gamma / 2\right]^{-1/\gamma} \] (34)

7) Plot \( K_{I\text{close}}(a) \) and \( K_{I\text{th}}(a) \). The intersection of these functions results on \( a_{st} \). Another method to find \( a_{st} \) is search for the first root of \( K_{I\text{close}}(a) - K_{I\text{th}}(a) = 0 \);
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Figure 55 – $K_{th}(a)$ and $K_{close}(a)$ obtained for the C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70\text{GPA}$, $\nu=0.33$, $\sigma_{ntc}=80 \text{ MPa}$.

Figure 56 – $K_{close}(a) - K_{th}(a)$ for the C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70 \text{ GPA}$, $\nu=0.33$, $\sigma_{ntc} = 80 \text{ MPa}$. 
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8) Calculate the stress intensity factor far from the notch influence ($K_{Ifar}$) using the eq. (45), (46), (47) or (30) with the variable’s change $a' = a + b$. Then plot $K_{Ifar}$ together with $K_{Iclose}$ and $K_{Ith}$.

**Standards Stress Intensity Factor $K_{Ifar}$ ($a' = a + b$)**

1. $C(T)^{18}$

$$K_I(P, a) = \frac{P}{t \sqrt{W}} \cdot \left( \frac{2 \left( \frac{b+a}{W} \right)^{1.5}}{1 - \left( \frac{b}{W} \right)^{1.5}} \right) \cdot \left[ 0.886 + 4.64 \cdot \frac{b+a}{W} + 14.72 \cdot \left( \frac{b+a}{W} \right)^3 - 5.6 \cdot \left( \frac{b+a}{W} \right)^3 \right]$$  \hspace{1cm} (45)

2. $M(T)^{48}$

$$K_I(P, a) = \frac{P}{t} \cdot \frac{\pi}{2W} \cdot \sec \left( \frac{\pi \left( \frac{2 (b+a)}{W} \right)}{2} \right)$$  \hspace{1cm} (46)

3. $SEN(B)^{61}$

$$K_I(P, a) = \frac{6.75P}{t \sqrt{W}} \cdot \left( \frac{\sqrt{b+a}}{W} \right) \cdot \left[ 1.99 - \frac{b+a}{W} \cdot \left( 1 - \frac{b+a}{W} \right) \cdot \left( 2.15 - \frac{3.93 \cdot b+a}{W} + 2.7 \left( \frac{b+a}{W} \right)^2 \right) \right]$$  \hspace{1cm} (47)

4. $SEN(T)^{49}$

$$K_I(P, a) = \frac{P}{t \sqrt{W}} \cdot \left[ 1.99 \cdot \left( \frac{b+a}{W} \right)^{0.5} - 0.41 \cdot \left( \frac{b+a}{W} \right)^{1.5} + 18.7 \cdot \left( \frac{b+a}{W} \right)^{2.5} + 38.85 \cdot \left( \frac{b+a}{W} \right)^{3.5} + 53.85 \cdot \left( \frac{b+a}{W} \right)^{4.5} \right]$$  \hspace{1cm} (30)
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Figure 57 – $K_{ith}(a)$, $K_{close}(a)$, $K_{far}(a)$ obtained for the C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70$ GPA, $\nu=0.33$ and $\sigma_{ntc} = 80\text{MPa}$.

9) The threshold nominal stress on the notch tip ($\sigma_n = S_{Lntc}$) is obtained increasing the load $P$ on the specimen till obtain the tangency of the $K_{far}$ and $K_{ith}$ functions (figure 58) or the tangency between $K_{far} - K_{ith}$ function and the abscissa’s axis (figure 59);

Figure 58 – $K_{ith}(a)$, $K_{close}(a)$ and the adjusted $K_{far}(a)$ in order to obtain $\sigma_n = S_{Lntc}$ for the C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70$ GPA, $\nu=0.33$. 
Figure 59 – $K_{ifar}(a) - K_{ith}(a)$ adjusted in order to obtain $\sigma_n = S_{Lntc}$ for the C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $p=2\text{mm}$, $t=6\text{mm}$, $E=70\text{ GPA}$, $\nu=0.33$.

10) Calculate $K_{EAC}$ and $q_{EAC}$ using the equations (40) and (33);

\[
K_{EAC} = \frac{S_{EAC}}{S_{Lntc}} \tag{40}
\]

\[
q_{EAC} = \frac{(K_{EAC} - 1)}{(K_t - 1)} \tag{33}
\]

It is relevant to notice that the mixed methodology shows a discontinuity between $K_{fclosest}$ and $K_{far}$, as it can be seen on fig. 57 and 58. The graphical comparison (fig.60) between the two proposed methodologies highlights the error of the mixed methodology on the ast calculation.
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Figure 60 – Comparison between finite elements methodology and mixed methodology for the C(T) specimen: $W=50\text{mm}$, $b=20\text{mm}$, $\rho=2\text{mm}$, $t=6\text{mm}$, $E=70$ GPA, $\nu=0.33$ and $\sigma_{ntc}=80\text{MPa}$. Point (2) shows the accurate $a_{st}$ value using the finite element methodology. Points (1) and (3) show incorrect $a_{st}$ values using the mixed methodology.