Josue Cox Lescano

Fiscal Multipliers in Times of War and Peace

DISSERTAÇÃO DE MESTRADO

DEPARTAMENTO DE ECONOMIA

Programa de Pós–graduação em Economia

Rio de Janeiro
March 2014

Pontifícia Universidade Católica do Rio de Janeiro

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Dissertação de Mestrado

Dissertation presented to the Programa de Pós-Graduação em Economia of the Departamento de Economia, PUC-Rio as partial fulfillment of the requirements for the degree of Mestre em Economia.

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Prof. Eduardo Zilberman
Advisor
Departamento de Economia — PUC–Rio

Prof. Carlos Viana de Carvalho
Department of Economics — PUC–Rio

Prof. Tiago Couto Berriel
Department of Economics — PUC–Rio

Prof. Diogo Guillen
Gavea Investimentos

Prof. Monica Herz
Coordenadora Setorial do Centro de Ciências Sociais – PUC–Rio

Rio de Janeiro, March 26th, 2014

Josue Cox Lescano

B.A., Economics, Universidad de Lima, 2005-2010 (Lima, Peru).

Bibliographic data

Cox, Josue

Fiscal Multipliers in Times of War and Peace / Josue Cox Lescano; advisor: Eduardo Zilberman. — Rio de Janeiro : PUC–Rio, Departamento de Economia, 2014.

v., 58 f: il. ; 29,7 cm

1. Dissertação (Mestrado em Economia) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento De Economia.

Inclui Bibliografia.

1. Economia – Dissertação. 2. fiscal multipliers. 3. Markov switching. 4. price and wage rigidities. 5. first and second order approximations. I. Zilberman, Eduardo. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. III. Título.

CDD: 330

Acknowledgments

I want to thank my family for the unconditional support during these years. To my parents, Edmundo and Vasty, for their constant teaching. To my grandparents, Edwin and Carmen, and my aunt Dora for all the effort they put into my career.

I would like to thank my adviser, Prof. Eduardo Zilberman, for all the support and guidance.

I am actually in debt with Felipe, Gustavo, Pepeu, and Tomas for making the experience of living in Rio de Janeiro an amazing time.

To all my other colleagues and professors from PUC-Rio, I am deeply grateful for all the effort we have put into our work together over the past couple of years. I am also indebted with Carlitos for his help with the final presentation of this work

Finally, I want to thank Carmen for these two years and what they have meant. I know how difficult it has been and how much we have learned from this experience. This have showed me that your are the love of my life. Thanks for making all these things meaningful.

Financial support from CAPES and PUC-Rio is also gratefully acknowledged.

Abstract

Cox, Josue; Zilberman, Eduardo (Advisor). **Fiscal Multipliers in Times of War and Peace**. Rio de Janeiro, 2014. 58p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Fiscal multiplier literature uses war episodes as exogenous variation to achieve identification. However, the economy behaves differently in war episodes *vis-à-vis* peace periods; and hence the multiplier obtained using war episodes may be different from the multiplier prevailing in peace periods. To assess this assertion we use a calibrated New Keynesian model. We use these model to assess how different multipliers are in times of war and peace. We then turn to a Markov switching model in which government spending process is regime dependent, and we take into account this dependence when solving the model. We can obtain a difference of order four between war and peace multipliers using different definitions. In addition, we obtain a similar difference when we allow the presence of news in the model. These results shed light on the possible bias of the multipliers presented in fiscal literature.

Keywords

fiscal multipliers; Markov switching; price and wage rigidities; first and second order approximations;

Resumo

Cox, Josue; Zilberman, Eduardo. **Multiplicadores Fiscais em Tempos de Guerra e Paz**. Rio de Janeiro, 2014. 58p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

A literatura do multiplicador fiscal usa episódios bélicos como variação exógena para alcançar identificação. No entanto, a economia se comporta de forma diferente em episódios de guerra vis-à-vis em episódios de paz; e, portanto, o multiplicador obtido usando episódios de guerra pode ser diferente do multiplicador prevalecente em períodos de paz. Para avaliar essa afirmação, usamos um modelo novo keynesiano calibrado. Usamos este modelo para avaliar como os multiplicadores são diferentes em tempos de guerra e paz. Logo, é utilizado um modelo de mudança de regimen de Markov em que o processo do governo é dependente, e levamos em conta esta dependência do regime ao resolver o modelo. Podemos obter uma diferença de ordem quatro entre os multiplicadores da guerra e da paz que utilizam diferentes definições. Além disso, obtém-se uma diferença semelhante quando permitimos a presença de notícias no modelo. Estes resultados lançam luz sobre o possível viés dos multiplicadores apresentados na literatura fiscal.

Palavras–chave

multiplicadores fiscais; Markov switching; rigidez de preços e salario; aproximação de primeira e segunda ordem;

Contents

1
Introduction

The recent global crisis has brought a strong interest in assessing the effectiveness of fiscal policy in mitigating negative shocks on the economy. There is a vast literature dedicated to identifying the impact of government intervention on key macro variables assuming that they do not depend on the state of the economy. A growing area of research attempts to show that fiscal multipliers are state dependent; for instance, they will vary depending on whether the economy is in expansion or recession, interest rates are near the zero lower bound, etc. This investigation uses this approach to find out if the government purchase multiplier is different depending on whether the economy is at peace or war[1].

As suggested by Hall (2009), the exogenous variation for the government process is of huge importance for identifying the effect of government intervention in the economy. This variation is obtained from two important episodes: (i) World War II and (ii) the Korean War. However, there are considerable differences in economic behavior during peace periods and conflict events such as World War II and the Korean War (McGrattan and Ohanian, 2010). Hence, extrapolating the multipliers obtained using this exogenous variation to peace periods may introduce a bias to the real effect of government purchases during peace times. To explore the existence of bias in these estimates, we use a DSGE model to assess whether multipliers are different if we assume that the government process is different in war and in peace episodes.

Our model includes two key components: (i) real rigidities with the inclusion of capital adjustment costs and variable capital utilization; and (ii) nominal rigidities with the inclusion of price and wage rigidities. In this kind of model, as in the standard RBC model, fiscal policy operates through the canonical wealth effect channel. This channel was explored by Baxter and King (1993). It states that when government spending increases, agents in the model have to pay higher taxes (negative transfers in our model). This causes a decrease in consumption, which is mitigated by an increase in hours worked. The increment in labor causes real wages to decrease. Our model does not include any other mechanism for government purchases to affect the economy apart from this channel.

[1]By war we mean large war episodes, for instance World War II or the Korean War. By peace we mean peace periods and other lesser conflicts.

To asses how different multipliers are, we use two approaches. In the first one we use the model with a different calibration for the government spending process (each one identifying peace and war behavior for the fiscal authority) as if we were handling two separate economies. We solve the model using a first and a second order approach. With the first order solution we explore the role of government purchases' persistence and steady state government expenditure to GDP ratio. We perform this analysis for two reasons: (i) to isolate the effect of the government persistence in the magnitude of the multipliers and (ii) to gauge the effect of government expenditure to GDP ratio in steady state. As we will explain later, the effect of the steady state values is not significant and can be neglected. This is an important result because in the Markov model we do not perform steady-state changes due to technology limitations. Then, we explore the role of the government process variance using a second order approach methodology. There is some evidence of precautionary behavior against fiscal rule volatility; however, given our preference specification, the magnitude is not large.

Then, we introduce a switching government rule in the model and take into account this regime dependence in the solution approach. This approach is similar to Davig and Leeper (2006, 2011), who introduce an estimated Markov switching Taylor rule in a DSGE model. This model uses the output of a Markov switching estimation for the government rule. We estimate this process using the methodology in Hamilton (1989), allowing the persistence parameter and the variance of the process to vary across the regimes. We assume that the underlying state variable is subject to a two-regime process, which we identify as war and peace episodes. We estimate a rule that assumes that government purchases follow an AR(1). In general, we find that the persistence parameter in times of peace is higher than in times of war, but the volatility of the process is higher in times of war.

Both approaches (the standard one and the Markov DSGE) predict that the government multiplier for war episodes is higher than the peace multiplier at impact. However, the war multiplier decreases quickly after the government shock. The gap between these two multipliers increases as time passes. Present value multipliers are also higher during peace episodes. The consumption multiplier is higher during the war episodes because the negative wealth effect is stronger when the government process is more persistent (peace episodes). The model predicts that investment decreases more in response to a more persistent government process. This result is very interesting since government purchases are financed with lump-sum taxes; therefore, agents want to save more to pay future higher taxes; as a result, investment should be higher for the more persistent government process.

The paper is organized as follows. In the second chapter we present the estimation of the Markov switching process for government purchases which is a key input in our model. In the third chapter we introduce the DSGE model and the method used to solve it under the assumption of regime dependence. The fourth chapter presents multipliers when we use first and second order approaches to solve the model. The fifth chapter presents the results of the Markov DSGE model and shows how the model behaves when news are introduced — in the spirit of Ramey (2011). Chapter six concludes.

1.1 Literature Review

The Literature on fiscal multipliers can be divided in two broad groups. Research works in the first one use empirical methods to infer fiscal multipliers from the data. Authors in this group generally use reduced form vector autoregression (VAR) models and make structural assumptions to recover the effect of a structural government shock. Works in the second group use general equilibrium models and calibrate or estimate them to obtain the impact of government intervention in the economy. In recent years both research agendas have used the assumption that the impact of economic policy varies depending on the state of the economy.

Empirical literature on fiscal multipliers has addressed the estimation problem of endogeneity by using military spending as an exogenous source of variation to measure the impact of government purchases on GDP (see for example Barro 1981, Hall 2009, Barro and Redlick 2011, and Ramey 2011). These authors state that military buildups are least likely to respond to economic activity; hence the problem of reverse causality is solved. Typically, they obtain impact and cumulative multipliers between 0.4 and 1.2 for temporary military spending shocks, depending on the sample and on the identification approach. In the case of Ramey, a new variable, constructed using news from articles in *Business Week* from future military spending, is used to identify the VAR.

The methodology suggested by Ramey (2011) has interesting implications. It states that from the point of view of the statistician and the people in the economy, the information set is dramatically different. The statistician will identify an increase in military spending as an unexpected shock; however, people in the economy have information of the possibility of future wars prior to the increase in military spending. As a result, this future increment will be internalized in their expectations well before it takes place. Therefore, the unexpected shock observed by the statistician will be an anticipated shock for the people in the economy.

Barro and Redlick (2011) estimate annual equations to obtain the contemporaneous effect on GDP from temporary military government purchases. They allow for interactions between government purchases and other variables (for instance unemployment) to see if the fiscal multiplier differs depending on the state of the economy. Depending on the data sample, these authors estimate that contemporaneous fiscal multipliers are around 0.44 - 0.68. Intertemporal effects in their regressions are important; then one lag for government military purchases is included. The total effect converges to 0.64 - 0.73 for periods including World Wars I and II. In the case of the Korean War it is not significant. In addition, if Ramey's news variable is included (to measure the time t increase due to expected higher government purchases) to obtain a permanent effect of fiscal intervention, the multipliers converge to values around 0.6 - 0.8 (They find similar results if government defense spending is replaced by total government purchases; see tables II and VI in their paper).

Blanchard and Perotti (2002) use a structural VAR and identify it using institutional information about tax and transfer systems, which permits them to identify fiscal shocks. They use a VAR specification including three variables: taxes, government spending, and GDP. In the identification, authors assumed that tax and government shocks depend on the response to unexpected movements in GDP and the response to structural shocks on spending and taxes. The sample covers the period 1960-1997 (no large war episodes are included). The multipliers in the case of the difference stationary assumption vary from 0.45 to 1.13 depending on the horizon, and the peak multiplier is 1.29. In the case of the stationary trend assumption, multipliers are between 0.55 and 0.90 depending on the horizon, and the impact multiplier is 0.90.

Barro and Redlick (2011) show that the war episodes are the only periods in which there is the necessary exogenous variation for identifying government-induced structural shocks. They also state that there is not enough information in the years after the Korean War to obtain accurate fiscal multipliers. They also argue that the VAR approach used in Fair (2010) and Blanchard and Perotti (2002) is correct for war-driven spending, but is not satisfactory for other types of spending and for samples where this spending is almost unchanged. When the Korean War is included, Blanchard and Perotti show that for the case of first-difference VAR, the multipliers are very similar to the sample without including it. However, in the case of the stationary trend VAR, the effect on GDP is less persistent.

On the other hand, there is a strong interest in investigating the channels through which fiscal policy affects the economy. For this purpose, many authors have relied on general equilibrium models. The neoclassical model[2] is considered the starting point for assessing the impact of government spending on the economy. Baxter and King (1993) is a comprehensive reference for this model. This paper (as well as Barro and King 1984, and Aiyagari, Christiano and Eichenbaum 1992) stresses the importance of wealth effects and intertemporal substitution when fiscal shocks affect the economy. The idea is that when the government spends more, households become poorer and have to reduce consumption and leisure; hence a feature of these types of models is an increase in labor supply that generates an increase in capital utilization (because capital becomes more productive due to the presence of diminishing returns in labor). This creates an increase in GDP, which, however, in many cases is near unity. Baxter and King explain that the multiplier is reduced when the increase in government purchases is temporary and when distortionary taxes are used to finance spending.

McGrattan and Ohanian (2010) use World War II as a laboratory for assessing how well the neoclassical model replicates the main variables of the economy in the context of a large military buildup. They conclude that this model seems to be a good starting point to assess fiscal impact on product and other variables.

In the New Keynesian approach, Galí et. al. (2007) use a model with sticky prices, "rule of thumb" consumers, and a non-competitive labor market to obtain multipliers well above 1.5 for GDP, 1 for consumption, and almost zero for investment. In this case they need a rule of thumb share of 0.5. When they assume a competitive labor market, multiplier for GDP is bellow 1 and for consumption it is negative. Hence they need very strong assumptions to obtain the positive effect observed in empirical models.

There is an alternative approach besides the rule of thumb assumption, whereby the model can predict a positive response of consumption and GDP multipliers above 1. As shown by Zubairy (2009, 2014), deep habits introduced in Ravn, Schmitt-Grohe, and Uribe (2006) generate a demand function with a price-elastic and a perfectly-inelastic component. An increase in government purchases increases the share of the price-elastic component which induces firms to reduce the markup over marginal cost. Hence, labor demand and wages go up, which generates an increase in labor supply and consumption.

[2]As explained by McGrattan and Ohanian (2010), the neoclassical or RBC model is a one-sector optimal growth model with a production function homogeneous of degree one using as inputs labor and capital, a simple law of motion for capital stock, preferences over consumption and leisure, and a constraint that divides output among consumption, investment, and government spending. In addition perfect competition is assumed in all the markets.

Zubairy uses Bayesian methods to estimate a model with this mechanism and obtains multipliers around 1 using a sample covering the period 1954-2008.

Our model is calibrated using a Markov estimation. This approach was used in Davig and Leeper (2006) and Davig and Leeper (2011). In the latter they use a Markov switching model to estimate the parameters of two regime switching policy rules: one for the monetary policy rule (interest rate) and one for the fiscal policy rule (lump-sum taxes). They define a two-state process, where the states are defined as active and passive policy, respectively. Since there are two states and two policy rules, their model reports four different scenarios. In our model we do not have money and distortionary taxation; therefore, only government spending is subject to the Markov process. In addition, they use projection methods to solve their model, whereas we use perturbation methods; i.e., we use a first-order log linear model.

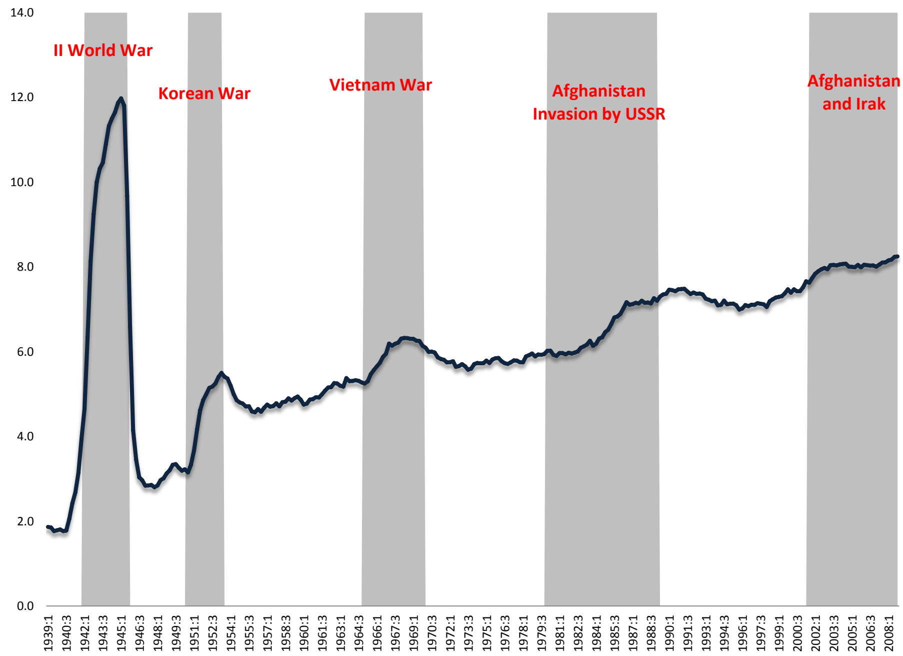
Along these lines, Bianchi (2013) estimates a model with a Markov switching process using Bayesian methods. The author proposes a variation of the Kalman filter to estimate models with Markov switching processes. A direct extension for our paper is to use this proposed filter to estimate the model.

However, all the investigations discussed above rely on the assumption of constant multiplier. Implicit in these works is that government process is not regime dependent. We depart from this assumption and assume that the government rule behaves different across regimes. This allow us to obtain multipliers that depend on whether the economy is in the peace or war regime which gives us an idea of the possible bias when using an invariant instrument. In addition, part of the fiscal multiplier literature attempts to measure the magnitude of the fiscal multiplier to argue (or not) for fiscal intervention when the economy is in a contractionary phase. Our approach is different in that we are not interested in the magnitude of the multiplier but in the difference between multipliers in war and peace periods. This is in line with Andolfatto (2010), who shows (theoretically) the difference between war and peace multipliers in a neoclassical framework. However, his approach is different from ours, as spending buildup in a war episode is defined by an increase in the weight of government consumption in the utility function and a spending buildup in a peace episode is defined by a decline of the productivity parameter in the production function.

2
Empirical Evidence

Figure 2.1 shows government spending from 1939:1 to 2008:4. We have shadowed periods of wars and conflicts involving the U.S. There are (at least) five major episodes of military tension: (i) World War II, which spans from 1942:1 to 1945:3; (ii) the Korean War, which spans from 1950:3 to 1953:2; (iii) the Vietnam War, covering from 1965:1 to 1969:4; (iv) the Afghanistan invasion by the USSR from 1980:1 1989:1; and (v) The Afghanistan and Irak War, starting in 2001:3.

Figure 2.1: Government spending in per capita terms

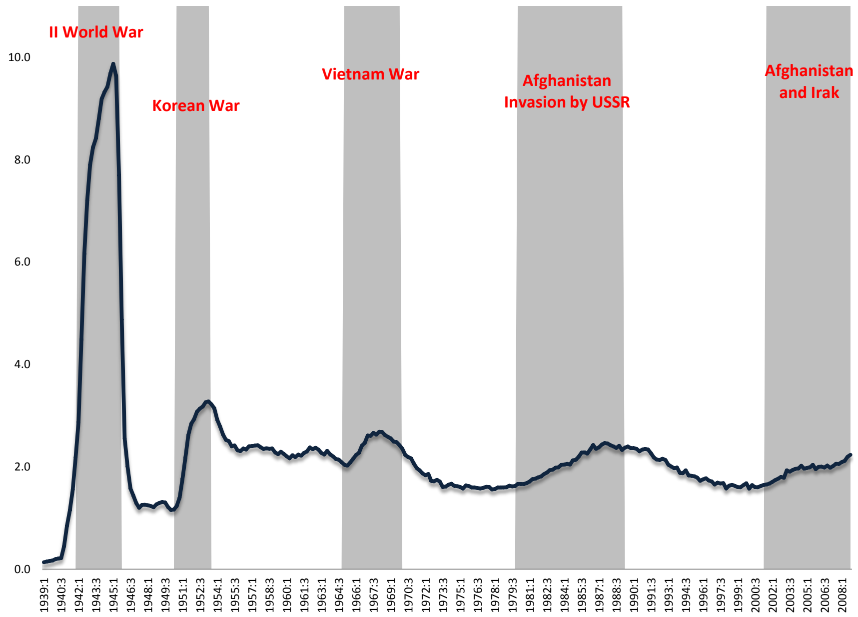


Note: The figure presents the government spending in per capita terms. We use government series in NIPA tables and population over 16 years.

These episodes, however, are heterogeneous. For example, during World War II, the U.S. government introduced several restrictions, including nominal wage and price controls, rationing of necessity goods (through ration coupons), and rationing of some durable goods, as suggested by McGrattan and Ohanian (2010). This did not happen, for example, in the most recent war. Another source of heterogeneity is the magnitude of military spending during these periods. Ramey (2011) stresses the magnitude of military spending during the first two episodes relative to the other ones.

Figure 2.2 shows the path for military spending in the same period. Note that World War II and the Korean War are the most important episodes. We can distinguish these two episodes as large war episodes as, for example, the increment in military spending during World War II was almost five times as large as during the Vietnam War, the Afghanistan invasion, and the Irak war. In addition, military spending during the Korean War was almost twice as large as during the Afghanistan invasion and the Irak war.

FIGURE 2.2: DEFENSE SPENDING IN PER CAPITA TERMS



Note: The figure presents the defense spending in per capita terms. We use government series in NIPA tables and population over 16 years.

In this section we present a Markov Switching estimation, which will be used as an input in the calibration of our model.

2.1 Switching model

We estimate a government policy behavior implied by simple rules as used in the existing literature. In addition, we allow this rule to depend on a non-observable state variable that governs the regimes in this model. We allow the parameters of this rule to depend on these non-observable variables and, hence, to switch depending on the regime. The methods developed in this section were proposed by Hamilton (1989) and Kim and Nelson (1999). We use the algorithm suggested by Perlin (2012).

The algorithm uses a maximum likelihood approach to estimate the persistence parameters and the standard deviation of the process. Briefly, we can summarize this method as follows:

1. Find the parameters that maximize the following likelihood function:

$$\ln L = \sum_{T}^{t=1} \ln \sum_{j=1}^{S} \{ f\left(g_t | S_t = j, \Theta\right) Pr\left(S_t = j\right) \}$$

where Θ is the set of parameters to be estimated. Note that $Pr\left(S_t = j\right)$ is not directly observable.

2. Use Hamilton's filter to estimate $Pr\left(S_t = j\right)$. This is an iterative process which updates the probabilities using the following equation

$$Pr\left(S_t = j\right) = \frac{f\left(g_t | S_t = j, \Omega_{t-1}\right) Pr\left(S_t = j | \Omega_{t-1}\right)}{\sum\limits_{j=1}^{S} f\left(g_t | S_t = j, \Omega_{t-1}\right) Pr\left(S_t = j | \Omega_{t-1}\right)}$$

where Ω_{t-1} is the matrix containing the information until $t-1$.

We consider a government rule that follows an AR(1) process as in Zubairy (2009) and Bianchi (2012). This rule makes government process g_t to follow:

$$g_t = \rho_g(S_t) g_{t-1} + \sigma(S_t) \varepsilon_{g,t} \tag{2-1}$$

where S_t is the government purchase regime, which evolves according to a Markov chain with transition matrix P. g_t stands for government purchases in per capita terms. We detrended this variable, since all the variables in this model are stationary. We allow for two states, which will be interpreted as big war episodes and small war or peace episodes. Therefore, P is a 2×2 matrix.

Results: To estimate this model we use the time series for government purchases used by Ramey (2011), g_t is detrended to ensure stationarity. Data are from NIPA tables and span from 1939:1 to 2008:4.

Parameter estimates for the first process are presented in the following table. We also present the estimated transition matrix. Standard errors are in parentheses.

TABLE 2.1: MS estimation result for process in (2-1)

State	$\rho_g(S_t)$	$\sigma(S_t)$
Peace	0.9929	0.0057
	(0.0063)	(0.0001)
War	0.8900	0.1102
	(0.1372)	(0.0002)

The transition matrix for this process is:

$$P = \begin{bmatrix} 0.99 & 0.01 \\ 0.17 & 0.83 \end{bmatrix} \tag{2-2}$$

Note that the state named as *peace* is characterized by a persistent coefficient $\rho_g(S_t) = 0.9929$ and a low standard deviation $\sigma(S_t) = 0.0057$. Note in Figure B.1 of Appendix B that smoothed probabilities in this estimation imply that the *peace* regime is active in all periods except for major war episodes: World War II and the Korean War. On the other hand, the *war* regime is characterized by a lower persistence coefficient $\rho_g(S_t) = 0.89$ and a higher standard deviation $\sigma(S_t) = 0.1102$.

The transition probability matrix is defined in (2-2). Note that the *peace* regime is more persistent, since the probability of being in the first regime, given that in the previous period the peace regime prevailed, is 0.99. The second regime, which we interpret as a *war* regime, is not as persistent, with a 0.83 probability of remaining in it. The peace and war regimes have an expected duration of 79.9 and 6 periods (quarters), respectively.

3
Model

We use a version of the model presented in Christiano et al. (2005) and Fernández-Villaverde (2010). The economy consists of a large number of identical households who buy a final good in the final good market and hires capital and labor to intermediate firms. They set wages in a monopolistic labor market. A continuum of firms produce differentiated goods being sold in monopolistic goods markets. They hire labor and capital from households. Government purchases follow an exogenous Markov process.

Households in the model are heterogeneous in two ways. First, they consume and save in different amounts for they provide particular quantities of labor. To overcome this characteristic of the model we assume complete markets which guarantee that households consume and save the same. Second, households charge different wages; nevertheless, Calvo's (1983) scheme assures the possibility to express optimal wages as an average of past inflation and current optimal wages.

3.1
Households

The economy is populated by a continuum of households of measure one indexed by $j \in (0, 1)$. Preferences of the j-th household are represented by the following utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U \left\{ c_t - \Phi c_{t-1}; \frac{m_{j,t}}{p_t}; h_{j,t} \right\} \tag{3-1}$$

where β is the discount factor, Φ is an external habit parameter which is introduced to obtain consumption dynamics similar to what is found in VAR impulse responses (hump - shaped responses); c_t denotes consumption in period t, $h_{j,t}$ is the labor supply of j-th household, and $\frac{m_{j,t}}{p_t}$ are real money balances (p_t is the price level which, as we show later, is a price index).

We assume that the momentary utility is separable in all of its arguments:

$$U = \log(c_t - \Phi c_{t-1}) + \upsilon \log \frac{m_{j,t}}{p_t} - \psi \frac{l_{j,t}^{1+\gamma}}{1+\gamma} \tag{3-2}$$

where γ is the inverse Frisch labor supply elasticity. Note that this specification implies that the marginal relation of substitution between labor and consumption is linear; therefore, we can obtain a balanced growth path with constant hours.

We assume complete markets, for this reason households trade Arrow-Debreu assets indexed by household j (isiosyncratic wage-adjustment risk) and by time t (aggregate risk). $a_{j,t+1}$ denotes the amount of assets that pay one unit of consumption in event $\omega_{j,t+1,t}$ purchased by household j at time t at the real price $q_{j,t+1,t}$. Households hold an amount b_{jt} of government bonds that pay a nominal interest rate R_t. The household budget constraint is:

$$\begin{aligned} c_t + i_t + \frac{m_{j,t}}{p_t} + \frac{b_{j,t+1}}{p_t} + \int q_{j,t+1,t} a_{j,t+1} d\omega_{j,t+1,t} = \\ = w_{j,t} l_{j,t} + (r_t u_{jt} - a[u_{jt}]) k_{j,t-1} + \frac{m_{j,t-1}}{p_t} + R_{t-1} \frac{b_{j,t}}{p_t} + a_{jt} + T_t + F_t \end{aligned} \quad (3\text{-}3)$$

where $w_{j,t}$ is the real wage set by household j in period t. We assume that households rent labor services in a monopolitic labor market using the framework proposed by Calvo (1983). Therefore, the unique source of heterogeneity between households comes from wage setting and labor supply. T_t is lump sum transfer and F_t are the profits of the firms in the economy. In addition, k_t is the physical capital stock that households rent to the intermediate firms. u_t denotes utilization rate of capital which is a control variable for the household. $a[u_t]$ denotes the cost in consumption units of a given utilization rate u_t. Note that this introduce a trade off between utilization rate and the cost it implies. The higher u_t the costlier the utilization of k_t. This function is increasing, convex, and equal to zero in the steady-state, i.e. when $u_t = u = 1$, $a[1] = 0$, a' and a'' are both positives.

The stock of installed capital is owned by the household and is subject to the following law of motion:

$$k_{j,t} = (1-\delta) k_{j,t-1} + \left(1 - S\left(\frac{i_{j,t}}{i_{j,t-1}}\right)\right) i_{j,t} \quad (3\text{-}4)$$

where i_t is period t investment and $S(\cdot)$ introduces adjustment cost. This function is such that $S(1) = 0$, $S'(1) = 0$, and $S''(\cdot) > 0$. As in Christiano et al. (2005), Fernández-Villaverde (2010), and Zubairy (2009), we will define $S(\cdot)$ explicitly.

Household maximizes with respect to c_{jt}, b_{jt}, u_{jt}, k_{jt}, and i_{jt}. It is not necessary to take derivatives with respect to Arrow-Debreu assets since, with the specification of utility (separable in consumption and labor), their demand ensures that consumption does not depend on idiosyncratic shocks. In addition, we do not take derivative with respect to real money balances because its maximization comes from the budget constraint.

We will define the marginal Tobin's Q as $q_{j,t} = \frac{Q_{j,t}}{\lambda_{j,t}}$ (the value of capital in terms of its replacement cost). Where $Q_{j,t}$ is the Lagrangian multiplier on the investment-capital constraint and $\lambda_{j,t}$ is the Lagrangian multiplier on the household budget constraint. Using this concept, the equilibrium condition of the households are:

$$(c_{j,t} - \Phi c_{j,t-1})^{-1} - \Phi\beta E_t(c_{j,t+1} - \Phi c_{j,t})^{-1} = \lambda_{j,t}$$
$$\lambda_{j,t} = \beta E_t \left\{ \lambda_{j,t+1} \frac{R_t}{\Pi_{t+1}} \right\}$$
$$r_t = a'[u_{j,t}]$$
$$q_{j,t} = \beta E_t \left\{ \frac{\lambda_{j,t+1}}{\lambda_{j,t}} ((1-\delta)q_{j,t+1} + r_{t+1}u_{j,t+1} - a[u_{j,t+1}]) \right\}$$
$$1 = q_{jt} \left(1 - S\left(\frac{i_{j,t}}{i_{j,t-1}}\right) - S'\left(\frac{i_{j,t}}{i_{j,t-1}}\right)\frac{i_{j,t}}{i_{j,t-1}}\right) + \beta E_t q_{j,t+1} \frac{\lambda_{j,t+1}}{\lambda_{j,t}} S'\left(\frac{i_{j,t+1}}{i_{j,t}}\right)\left(\frac{i_{j,t+1}}{i_{j,t}}\right)^2$$

Wage decision

Household j is a monopolistic supplier of differentiated labor services $l_{j,t}$. These services are aggregated by a representative competitive firm that hires the labor supplied by each household using the following technology:

$$l_t^d = \left(\int_0^1 l_{jt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{3-5}$$

where $0 \leq \eta < \infty$ is the elasticity of substitution among different types of labor. In addition, we denote the aggregate labor demand with l_t^d. Aggregating firm seeks to maximize profits by choosing l_{jt}:

$$v_h = \max_{l_{jt}} w_t l_t^d - \int_0^1 w_{jt} l_{jt} dj$$
$$\text{s.t} \quad l_t^d = \left[\int_0^1 l_{jt}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}$$

where w_{jt} and w_t are time t wage of the labor services j and the aggregate wage. The first order condition of this problem implies a demand for labor j that is decreasing in its relative price and increasing in aggregate labor l_t^d.

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\eta} l_t^d \tag{3-6}$$

Wage index w_t is obtained by noting that profits of the aggregating firm are zero, using (3.6) in v_h:

$$w_t = \left[\int_0^1 w_{jt}^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{3-7}$$

Idiosyncratic risk arises due to the presence of Calvo's price setting. We will assume that a fraction $1-\theta_w$ of households are allowed to change their prices. The remaining θ_w can only partially index their nominal wages by past inflation. Indexation is controlled by $\chi_w \in [0,1]$. Let nominal wage at period t be denoted by W_t, hence the real wage in the next period is:

$$W_{j,t+1} = \Pi_t^{\chi_w} W_{j,t} \rightarrow \frac{W_{j,t+1}}{p_{t+1}} = \Pi_t^{\chi_w} \frac{W_{j,t}}{p_t} \frac{p_t}{p_{t+1}}$$
$$\rightarrow w_{j,t+1} = \frac{\Pi_t^{\chi_w}}{\Pi_t} w_{j,t}$$

therefore, after τ periods:

$$w_{j,t+\tau} = \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} w_{j,t}$$

The relevant part of the household's optimization problem is:

$$\max_{w_{j,t}} \quad E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left\{ -\psi \frac{l_{j,t+\tau}^{1+\gamma}}{1+\gamma} + \lambda_{j,t} \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} w_{j,t} l_{j,t+\tau} \right\}$$

s.t

$$l_{j,t+\tau} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_{j,t}}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^d$$

Using the restriction in the value function:

$$\max_{w_{j,t}} \quad E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left\{ \lambda_{j,t} \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_{j,t}}{w_{t+\tau}} \right)^{1-\eta} w_{t+\tau} l_{t+\tau}^d - \psi \frac{\left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_{j,t}}{w_{t+\tau}} \right)^{-\eta(1+\gamma)}}{1+\gamma} (l_{t+\tau}^d)^{1+\gamma} \right\}$$

All houses, which are allowed to change wages, set the same wage because complete markets allow them to hedge the risk of the timing of wage change. Therefore, in the first order condition we can drop the j index from wages and the lagrange multiplier:

$$\frac{\eta-1}{\eta} w_t^* E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^d$$
$$= E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \psi \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_t^*}{w_{t+\tau}} \right)^{-\eta(1+\gamma)} (l_{t+\tau}^d)^{1+\gamma}$$

Define the above equation as:

$$f_t = \frac{\eta-1}{\eta} w_t^* E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^d$$
$$f_t = E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \psi \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_t^*}{w_{t+\tau}} \right)^{-\eta(1+\gamma)} (l_{t+\tau}^d)^{1+\gamma}$$

The above equations can be expressed recursively as:

$$f_t = \frac{\eta-1}{\eta} (w_t^*)^{1-\eta} \lambda_t w_t^\eta l_t^d + \beta\theta_w E_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}} \right)^{1-\eta} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta-1} f_{t+1}$$
$$f_t = \psi \left(\frac{w_t}{w_t^*} \right)^{\eta(1+\gamma)} (l_t^d)^{1+\gamma} + \beta\theta_w E_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}} \right)^{-\eta(1+\gamma)} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta(1+\gamma)} f_{t+1}$$

The real wage index evolves as:

$$w_t^{1-\eta} = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{1-\eta} w_{t-1}^{1-\eta} + (1-\theta_w) w_t^{*1-\eta}$$

3.2 Firms

The supply side of this economy is characterized by a firm which buys intermediate goods from a continuum of monopolistic firms and aggregate them into a composite good which is sold in the final good market to households. Hence, the supply side has two problems: the final good producer problem and the intermediate good producers problem.

Final good producer

Final good y_t^d is produced by a competitive firm which uses a CES technology to aggregate the different goods produced by intermediate firms:

$$y_t^d = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3-8}$$

where ε is the elasticity of substitution. Final good firm seeks to maximize profits by choosing y_{it}:

$$v = \max_{y_{it}} p_t y_t^d - \int_0^1 p_{it} y_{it} di$$
$$\text{s.t} \quad y_t^d = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where p_{jt} and p_t are time t prices of the intermediate good i and the consumption good. The first order condition of this problem implies a demand for good i that is decreasing in its relative price and increasing in aggregate demand y_t^d.

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t^d \tag{3-9}$$

Price index p_t is obtained by noting that profits of the aggregating firm are zero, using (3-9) in v:

$$p_t = \left[\int_0^1 p_{it}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{3-10}$$

Intermediate good producers

Intermediate good $i \in (0, 1)$ is produced by a price-setting monopolist with a Cobb-Douglas technology:

$$y_{i,t} = \begin{cases} A_t k_{i,t}^{\alpha} (l_{i,t}^d)^{1-\alpha} - \phi z_t & \text{if} \quad A_t k_{i,t}^{\alpha} (l_{i,t}^d)^{1-\alpha} \geq \phi z_t \\ 0 & \text{otherwise} \end{cases} \tag{3-11}$$

Each firm i produces output using capital services $k_{i,t}$ and labor services $l_{i,t}^d$ supplied by the aggregating labor firm. ϕ denotes fixed costs of production and $z_t = A_t^{\frac{1}{1-\alpha}}$ implies that profits are equal to zero in the steady state. For this reason, we assume that A_t follows the following process:

$$A_t = A_{t-1} e^{\Lambda_A + z_{A,t}}$$

where $z_{A,t} = \sigma_A \varepsilon_{A,t}$ and $\varepsilon_{A,t} \sim N(0, 1)$. Therefore, z_t follows:

$$z_t = z_{t-1} e^{\Lambda_z + z_{z,t}}$$

where $z_{z,t} = \frac{z_{A,t}}{1-\alpha}$ and $\Lambda_z = \frac{\Lambda_A}{1-\alpha}$.

The i-th firm faces two different problems. It has to decide how much labor and capital services it will demand and it has to set the price that it will charge to the final good firm for its differentiated good.

Factor markets: The i-th firm demands capital and labor services in perfectly competitive factor markets, optimal demand of factors are obtained from the following problem:

$$\min_{k_{i,t}, l^d_{i,t}} r^k_t k_{i,t-1} + w_t l^d_{i,t}$$

$$\text{s.t} \quad y_{i,t} = A_t k^{\alpha}_{i,t-1} (l^d_{i,t})^{1-\alpha} - \phi z_t$$

where r^k_t and w_t are real capital return and real wage rate, respectively. The solution of this problem implies a capital-labor ratio equal across firms:

$$\frac{k_{i,t-1}}{l^d_{i,t}} = \frac{\alpha}{1-\alpha} \frac{w_t}{r^k_t}$$

Real cost is:

$$RC_t = \left(\frac{1}{1-\alpha}\right) w_t l^d_{i,t}$$

Real marginal cost:

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{w_t^{1-\alpha} (r^k_t)^{\alpha}}{A_t} \tag{3-12}$$

Real marginal cost does not depend on the index i since all firms face the same technology and they do not have price power in factor markets.

Price setting: We assume that firms set prices according to Calvo (1983). We will use a variation of the indexation scheme presented in Christiano et al. (2005). Assume that $1 - \theta_p$ of the firms are permitted to reoptimize. The remaining θ_p firms that are not permitted to set new prices index their prices using past inflation with an indexation parameter $\chi \in [0, 1]$ (the scheme is the same used by households).

The problem of the firms is defined as follows:

$$\max_{p_{i,t}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\prod_{s=1}^{\tau} \Pi^{\chi}_{t+s-1} \frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{i,t+\tau}$$

s.t.

$$y_{i,t+\tau} = \left(\prod_{s=1}^{\tau} \Pi^{\chi}_{t+s-1} \frac{p_{it}}{p_{t+\tau}} \right)^{-\varepsilon} y^d_{t+\tau}$$

Substituting the constraint in the value function:

$$\max_{p_{i,t}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\left(\prod_{s=1}^{\tau} \frac{\Pi^{\chi}_{t+s-1}}{\Pi_{t+s}} \frac{p_{it}}{p_t} \right)^{1-\varepsilon} - \left(\prod_{s=1}^{\tau} \frac{\Pi^{\chi}_{t+s-1}}{\Pi_{t+s}} \frac{p_{it}}{p_t} \right)^{-\varepsilon} mc_{t+\tau} \right) y^d_{t+\tau}$$

Lets define p^*_t as the value of p_{it} set by a firm that can reoptimize at time t. p^*_t does not depend on i since all firms that are able to reoptimize choose the same price. See Christiano et al. (2005) for additional details. The first order condition is:

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \lambda_{t+\tau} \left\{ \left((1-\varepsilon) \left(\prod_{s=1}^{\tau} \frac{\Pi^{\chi}_{t+s-1}}{\Pi_{t+s}} \right)^{1-\varepsilon} \frac{p^*_t}{p_t} + \varepsilon \left(\prod_{s=1}^{\tau} \frac{\Pi^{\chi}_{t+s-1}}{\Pi_{t+s}} \right)^{-\varepsilon} mc_{t+\tau} \right) y^d_{t+\tau} \right\} = 0$$

As in the household wage problem, we can define the first order condition recursively:

$$g^1_t = E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\Pi^{\chi}_{t+s-1}}{\Pi_{t+s}} \right)^{-\varepsilon} mc_{t+\tau} y^d_{t+\tau}$$

$$g^2_t = E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\Pi^{\chi}_{t+s-1}}{\Pi_{t+s}} \right)^{1-\varepsilon} \frac{p^*_t}{p_t} y^d_{t+\tau}$$

which can be written as:

$$g^1_t = \lambda_t mc_t y^d_t + \beta\theta_p E_t \left(\frac{\Pi^{\chi}_t}{\Pi_{t+1}} \right)^{-\varepsilon} g^1_{t+1} \tag{3-13}$$

$$g^2_t = \lambda_t \Pi^*_t y^d_t + \beta\theta_p E_t \left(\frac{\Pi^{\chi}_t}{\Pi_{t+1}} \right)^{1-\varepsilon} \frac{\Pi^*_t}{\Pi^*_{t+1}} g^2_{t+1} \tag{3-14}$$

where $\Pi^*_t = \frac{p^*_t}{p_t}$ and $\varepsilon g^1_t = (\varepsilon - 1) g^2_t$.

In addition, price dynamics is defined by the following relation:

$$1 = \theta_p \left(\frac{\Pi^{\chi}_{t-1}}{\Pi_t} \right)^{1-\varepsilon} + (1-\theta_p)(\Pi^*_t)^{1-\varepsilon} \tag{3-15}$$

3.3 Fiscal and monetary policy and aggregate constraint

Conditional on a specific regime which is captured by the non observable variable S_t, we assume that government purchases follow:

$$G_t = G_{t-1}^{\rho_g(S_t)} e^{\sigma_g(S_t)\varepsilon_{g,t}} \tag{3-16}$$

S_t is a state variable which follows a Markov process with a transition matrix $[P_{ij}]$ for $i, j = 1, 2$. We will identified each regime of this process with periods of big wars: World War II and Korean War; and periods of peace or small wars (low government military spending as a percentage of GDP).

Monetary policy is assumed to follow a Taylor rule with interest rate smoothing:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{\frac{y_t^d}{y_{t-1}^d}}{\Lambda y_t^d}\right)^{\gamma_y}\right) e^{\sigma_R \varepsilon_{R,t}} \tag{3-17}$$

where $\varepsilon_{R,t} \sim N(0,1)$. Transfer are such that deficit is equal to zero every period:

$$G_t + T_t = \frac{\int_0^1 m_{j,t} dj}{p_t} - \frac{\int_0^1 m_{j,t-1} dj}{p_t} + \frac{\int_0^1 b_{j,t+1} dj}{p_t} - R_{t-1} \frac{\int_0^1 b_{j,t} dj}{p_t}$$

Household's aggregate budget constraint becomes

$$c_t + i_t + G_t = w_t l_t^d + (r_t u_t - a[u_t]) k_{t-1} + F_t \tag{3-18}$$

Aggregate resource constraint Aggregate demand is given by:

$$y_t^d = c_t + i_t + G_t + a[u_t] k_{t-1} \tag{3-19}$$

We can use this expression in (3-9) to obtain the demand for each intermediate good:

$$y_{it} = (c_t + i_t + G_t + a[u_t] k_{t-1}) \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon}$$

Note that y_{it} is defined by the production function in (3-11), then:

$$A_t k_{i,t-1}^{\alpha} (l_{it}^d)^{1-\alpha} - \phi z_t = (c_t + i_t + G_t + a[u_t] k_{t-1}) \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon}$$

Since the capital-labor ratio is the same across firms and $\int_0^1 k_{i,t-1} di = u_t k_{t-1}$:

$$\frac{k_{i,t-1}}{l_{i,t}^d} = \frac{u_t k_{t-1}}{l_t^d}$$

This implies the following condition for the production function:

$$A_t k_{i,t-1}^{\alpha}(l_{i,t}^d)^{1-\alpha} = A_t \left(\frac{k_{i,t-1}}{l_{i,t}^d}\right)^{\alpha} l_{i,t}^d = A_t \left(\frac{u_t k_{t-1}}{l_t^d}\right)^{\alpha} l_{i,t}^d$$

After integrating out the above expression:

$$\begin{aligned}\int_0^1 A_t \left(\frac{u_t k_{t-1}}{l_t^d}\right)^{\alpha} l_{i,t}^d di &= A_t \left(\frac{u_t k_{t-1}}{l_t^d}\right)^{\alpha} \int_0^1 l_{i,t}^d di \\ &= A_t \left(u_t k_{t-1}\right)^{\alpha} (l_t^d)^{1-\alpha}\end{aligned}$$

All the above framework is useful to obtain the price distortion on the aggregate resource constraint:

$$A_t(u_t k_{t-1})^{\alpha}(l_t^d)^{1-\alpha} - \phi z_t = (c_t + i_t + G_t + a[u_t]k_{t-1}) \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$$

Denote $\nu_t^p = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$. The dynamics of this new term is determined by the Calvo's assumption:

$$\nu_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} \nu_{t-1}^p + (1-\theta_p)\Pi_t^{*-\varepsilon}$$

Therefore, the resource constraint becomes:

$$\frac{A_t(u_t k_{t-1})^{\alpha}(l_t^d)^{1-\alpha} - \phi z_t}{\nu_t^p} = c_t + i_t + G_t + a[u_t]k_{t-1}$$

Finally, we need to find an expression for aggregate labor demand $l_t = \int_0^1 l_{j,t} dj$ as function of l_t^d. Labor demand of labor variety j is:

$$l_{j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\eta} l_t^d$$

Integrating out

$$l_t = \int_0^1 l_{j,t} dj = \int_0^1 \left(\frac{w_{j,t}}{w_t}\right)^{-\eta} dj l_t^d$$

Define $\nu_t^w = \int_0^1 \left(\frac{w_{j,t}}{w_t}\right)^{-\eta} dj$, then:

$$l_t = \nu_t^w l_t^d$$

The new term has the following dynamics:

$$\nu_t^w = \theta_w \left(\frac{w_{t-1}}{w_t} \frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{-\eta} \nu_{t-1}^w + (1-\theta_w)(\Pi_t^{w*})^{-\eta}$$

3.4
Stationary Equilibrium

Equilibrium is characterized by an allocation of quantities and prices that satisfy the households' optimality conditions, the final and intermediate firms' optmality conditions, the monetary policy rule, the government process, and markets clearing conditions.

This model presents technology growth which implies that variables are growing in steady state. For this reason, we need to express the model in stationary terms. To accomplish this goal, we deflate all variables by z_t; hence, stationary variables are defined in the following way:

$$\begin{array}{lll} \tilde{c}_t = \frac{c_t}{z_t} & \tilde{\lambda}_t = \lambda_t z_t & \tilde{i}_t = \frac{i_t}{z_t} \\ \tilde{w}_t = \frac{w_t}{z_t} & \tilde{w}_t^* = \frac{w_t^*}{z_t} & \tilde{k}_t = \frac{k_t}{z_t} \\ \tilde{g}_t = \frac{G_t}{z_t} & \tilde{y}_t^d = \frac{y_t^d}{z_t} & \end{array}$$

Therefore, a stationary symmetric equilibrium of this economy is:

- A contingent path of variables that satisfy household's first order conditions.
- A contingent path of variables that satisfy firms' first order conditions.
- Price and wage dynamics.
- Government rules and market clearing conditions.

Household's first order conditions:

$$(\tilde{c}_t - \Phi\tilde{c}_{t-1}\frac{z_{t-1}}{z_t})^{-1} - \Phi\beta E_t(\tilde{c}_{t+1}\frac{z_{t+1}}{z_t} - \Phi\tilde{c}_t)^{-1} = \tilde{\lambda}_t$$

$$\tilde{\lambda}_t = \beta E_t\left\{\tilde{\lambda}_{t+1}\frac{z_t}{z_{t+1}}\frac{R_t}{\Pi_{t+1}}\right\}$$

$$r_t = a'[u_t]$$

$$q_t = \beta E_t\left\{\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\frac{z_t}{z_{t+1}}((1-\delta)q_{t+1} + r_{t+1}u_{t+1} - a[u_{t+1}])\right\}$$

$$1 = q_t\left(1 - S\left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\frac{z_t}{z_{t-1}}\right) - S'\left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\frac{z_t}{z_{t-1}}\right)\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\frac{z_t}{z_{t-1}}\right) + \beta E_t q_{t+1}\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}\frac{z_t}{z_{t+1}}S'\left(\frac{\tilde{i}_{t+1}}{\tilde{i}_t}\frac{z_{t+1}}{z_t}\right)\left(\frac{\tilde{i}_{t+1}}{\tilde{i}_t}\frac{z_{t+1}}{z_t}\right)^2$$

$$f_t = \frac{\eta-1}{\eta}(\tilde{w}_t^*)^{1-\eta}\tilde{\lambda}_t\tilde{w}_t^{\eta}l_t^d + \beta\theta_w E_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta}\left(\frac{\tilde{w}_{t+1}^*}{\tilde{w}_t^*}\frac{z_{t+1}}{z_t}\right)^{\eta-1} f_{t+1}$$

$$f_t = \psi\left(\Pi_t^{w*}\right)^{-\eta(1+\gamma)}(l_t^d)^{1+\gamma} + \beta\theta_w E_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta(1+\gamma)}\left(\frac{\tilde{w}_{t+1}^*}{\tilde{w}_t^*}\frac{z_{t+1}}{z_t}\right)^{\eta(1+\gamma)} f_{t+1}$$

Firms' first order conditions:

$$\frac{u_t\tilde{k}_{t-1}}{l_t^d} = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t}{r_t}\frac{z_t}{z_{t-1}}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}(\tilde{w}_t)^{1-\alpha}r_t^{\alpha}$$

$$g_t^1 = \tilde{\lambda}_t mc_t\tilde{y}_t^d + \beta\theta_p E_t\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon} g_{t+1}^1$$

$$g_t^2 = \tilde{\lambda}_t\Pi_t^*\tilde{y}_t^d + \beta\theta_p E_t\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon}\frac{\Pi_t^*}{\Pi_{t+1}^*}g_{t+1}^2$$

$$\varepsilon g_t^1 = (\varepsilon-1)g_t^2$$

Wage and price dynamics:

$$1 = \theta_p\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{1-\varepsilon} + (1-\theta_p)(\Pi_t^*)^{1-\varepsilon}$$

$$1 = \theta_w\left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta}\left(\frac{\tilde{w}_{t-1}}{\tilde{w}_t}\frac{z_{t-1}}{z_t}\right)^{1-\eta} + (1-\theta_w)(\Pi_t^{w*})^{1-\eta}$$

Government relations:

$$\frac{\tilde{g}_t}{g} = \left(\frac{\tilde{g}_{t-1}}{g}\right)^{\rho_g(S_t)} e^{\sigma_g(S_t)\varepsilon_{g,t}}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{\frac{\tilde{y}_t^d}{\tilde{y}_{t-1}^d}\frac{z_t}{z_{t-1}}}{\Lambda y_t^d}\right)^{\gamma_y}\right) e^{\sigma_R \varepsilon_{R,t}}$$

Market clearing conditions:

$$\tilde{y}_t^d = \tilde{c}_t + \tilde{i}_t + \tilde{g}_t + a[u_t]\tilde{k}_{t-1}\frac{z_{t-1}}{z_t}$$

$$\tilde{y}_t^d = \frac{\frac{A_t}{A_{t-1}}\frac{z_{t-1}}{z_t}(u_t\tilde{k}_{t-1})^\alpha (l_t^d)^{1-\alpha} - \phi}{\nu_t^p}$$

$$l_t = v_t^w l_t^d$$

$$\nu_t^w = \theta_w \left(\frac{\tilde{w}_{t-1}}{\tilde{w}_t}\frac{z_{t-1}}{z_t}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} \nu_{t-1}^w + (1-\theta_w)(\Pi_t^{w*})^{-\eta}$$

$$\nu_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} \nu_{t-1}^p + (1-\theta_p)\Pi_t^{*-\varepsilon}$$

$$\tilde{k}_t\frac{z_t}{z_{t-1}} - (1-\delta)\tilde{k}_{t-1} - \left(1 - S\left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\frac{z_t}{z_{t-1}}\right)\right)\tilde{i}_t\frac{z_t}{z_{t-1}} = 0$$

3.5 Model Solution

We use an approximation method to solve our benchmark economy. We suppose that there are two economies which differ in the persistence of the government process, its variance, and the value of the steady state government process. We perform a first and second order taylor expansion of the policies functions. We present this method in the Appendix C.

To solve our Markov model we follow Farmer et. al. (2011). The objective of their method is to find the model equilibria which can be of two types: minimal state variable (MSV) equilibria and non-fundamental equilibria. Assume the following general form of a model:

$$A(s_t)_{n\times n}x_t = B(s_t)_{n\times n}x_{t-1} + \mathit{\Psi}(s_t)_{n\times k}\varepsilon_t + \mathit{\Pi}(s_t)_{n\times l}\eta_t \tag{3-20}$$

where $x_t \sim n \times 1$, $\varepsilon_t \sim k \times 1$, and $\eta_t \sim l \times 1$. x_t is a vector of endogenous and predetermined (exogenous) variables, ε_t is an iid vector of stationary shocks, and η_t is a vector of expectational errors. s_t follows a h-regime Markov process, where $s_t \in \{1, \cdots, h\}$ with transition matrix given by:

$$p_{ij} = Pr(s_t = i|s_{t-1} = j)$$

In general, $x'_t = [y'_t \ z'_t \ E_t y'_{t+1}]$ and $y_t = E_{t-1}y_t + \eta_t$ is added to the system to make it understand how expectations operate in the model. y_t is the endogenous component and z_t is the exogenous component consisting of lagged and exogenous variables.

The *Theorem 1* of Farmer et. al. (2011) states that if $\{x_t, \eta_t\}_{t=1}^{\infty}$ is a Minimal State Variable (MSV) solution of the system in (3-20), then:

$$x_t = V(s_t)F_1(s_t)x_{t-1} + V(s_t)G_1(s_t)\varepsilon_t \tag{3-21a}$$

$$\eta_t = -\left(F_2(s_t)x_{t-1} + G_2(s_t)\varepsilon_t\right) \tag{3-21b}$$

definitions of V, F, and G can be found in Farmer's paper in equations (7)-(9). The Farmer's method requires initial conditions to obtain the MSV solution and it does not rule out indeterminacy. To see if the equilibrium is unique, Farmer et. al. suggest to randomly select a bunch of initial conditions and check it when converge to the true solution. If there is only one equilibrium, then it is the MSV solution. In addition, the solution has to be stationary which occurs if and only if all the eigenvalues of

$$(P \otimes I_{n^2})\,\text{diag}\left(V_1F_{11} \otimes V_1F_{11}, \cdots, V_hF_{1h} \otimes V_hF_{1h}\right)$$

are all inside the unit circle. In a related paper Cho (2014) uses the forward method for solving rational expectation models and provide conditions for determinacy and indeterminacy of solutions mean square stable. When the model is determined, both approaches give the same result.

It is important to stress that in our model, shocks and the regime process are exogenous determined, i.e. the state of the economy does not affect the behavior of the distributions of these two processes. However, one can argue that the distribution of shocks as well as the probabilities of regime switches may depend on the state of the economy, this approach is suggested in Barthélemy and Marx (2013).

Barthélemy and Marx (2013) prove that the existence and the uniqueness of a bounded solution for state dependent model rely on the existence of a unique solution for a simplified, linear model with exogenous regime switching, hence for small shocks the properties of models with dependent shocks and regimes mimics that of the models with exogenous behavior (like our model). They also show that when shocks are small, state-dependence does not alter the determinacy conditions up to a first order expansion. In addition, they explain that in models with state-dependent transition probabilities of switching across regimes, the state-dependence does not matters at first order approximation only when the steady state do not change across regimes (as in our model), it is important only at second order expansions.

Maybe the most critical point raised by this paper is related to the determinacy conditions for bounded solutions. As we mention above, to obtain the MSV solution, one need to use an iterative algorithm to search for all possible solutions. However, Farmer et. al. (2011) do not provide a theoretical argument for determinacy. The approach suggested by Cho (2014) which uses Mean Square Stability concepts following Costa et al. (2005) also does not provide a theoretical background. Therefore, up to this moment, according to Barthélemy and Marx (2013) "*there is not theoretical argument ensuring the consistency of this concept of stability with the perturbation approach for non linear Markov switching*".

3.6 Calibration

Our calibration is based on three sources: (i) Bianchi (2012), (ii) Christiano et. al. (2005), (iii) Férnandez-Villaverde (2006), and (iv) Zubairy (2009). The following table shows the model's parameters:

TABLE 3.1: Model Parameters

Parameter	Calibrated value	Source
Consumer side		
Φ	0.96	Zubairy (2009)
β	$1.03^{-0.25}$	Christiano et. al. (2005)
ε	10	Fernández-Villaverde (2006)
η	10	Fernández-Villaverde (2006)
Firm, technology, and economy		
δ	0.025	Christiano et. al. (2005)
α	0.36	Christiano et. al. (2005)
κ	9.64	Bianchi (2012)
γ_2	0.0001	Fernández-Villaverde (2006)
Π	1.01	Christiano et. al. (2005)
$s_g = \frac{G}{Y}$	0.20	Assumption for the benchmark model
Stickness		
θ_p	0.82	Fernández-Villaverde (2006)
θ_w	0.68	Fernández-Villaverde (2006)
χ_p	0.63	Fernández-Villaverde (2006)
χ_w	0.62	Fernández-Villaverde (2006)
Monetary Policy		
γ_R	0.80	Zubairy (2009)
γ_π	1.50	Zubairy (2009)
γ_y	0.10	Zubairy (2009)

The parameters of the consumer side are obtained from Fernández-Villaverde (2006) and Christiano et. al. (2005). In the case of the discount parameter, it is a common value in the literature and implies a value of 0.9926 for β. The habit persistence parameter is obtained from Fernández-Villaverde (2006). We use this source for two reasons: (i) it is similar to the value presented in other papers, for example Bianchi (2012), (ii) it reproduces multiplier similar to the ones in empirical literature.

In the second group of parameters we use δ and α from Christiano et. al. (2005). These values are quarterly specified and imply a rate of depreciation of 10 percent and a steady state share of capital income of around 36 percent. The parameters that define cost adjustment κ and the one implied by the capital utilization rate γ_2 are obtained from Bianchi (2012). Note that the parameter κ is greater than the one in Christiano et. al. (2005) (which is 2.48). We follow Bianchi because we define the function $S(\cdot)$ as in his paper. This value is similar to the one in Hall (2009) and Fernandez-Villaverde (2006).

For the stickness parameters we follow Fernandez-Villaverde (2006) since in our model we use price and wage rigidities and partial indexation. Finally, Taylor rule parameters are the standard ones, we use a smoothing parameter of 0.8. This avoids interest rate to respond strongly to movements in inflation and GDP. In addition, for inflation rate, the parameter is set at 1.5. This guarantees uniqueness of the solution. For values less than 1 indeterminacy arises. Finally, the parameter for the response of interest rate to GDP is set at 0.1.

4
Benchmark model

In this section we present the effect of a government spending shock over the benchmark model presented above. To solve the model, as explained in the section 3.5, we use a first order Taylor approximation of the equilibrium conditions. Then, we use a one standard deviation perturbation on the government process to shock the model and obtain impact multipliers. We also perform a second order approximation to gauge the effect of the variance of the government process. Nevertheless, since we are using a log-preference specification, the effect of the government variance is negligible as can be observed in the results presented in the Appendix D.

In this section we also investigate the role of a different government steady state value. It is a common observation that during the World War II and the Korean War, government spending was higher than. We explore this observation by considering a higher government expenditure to GDP ratio in steady state.

4.1
First order approximation

We explore the effect of government spending and assess how this model behaves during the war and peace episodes. As we mentioned before, we identify a peace episode when government's persistence is high and its variance is low. On the other side, we identify war episodes by a lower government's persistence parameter and a higher variance.

We treat each specification separately as if they were two distinct economies (when we introduce the Markov process in section 5, we will use these estimates in a single economy environment). We will compare the fiscal multipliers obtained in each economy and see how they differ at impact and over the horizon. The multiplier definition that we are using is the following:

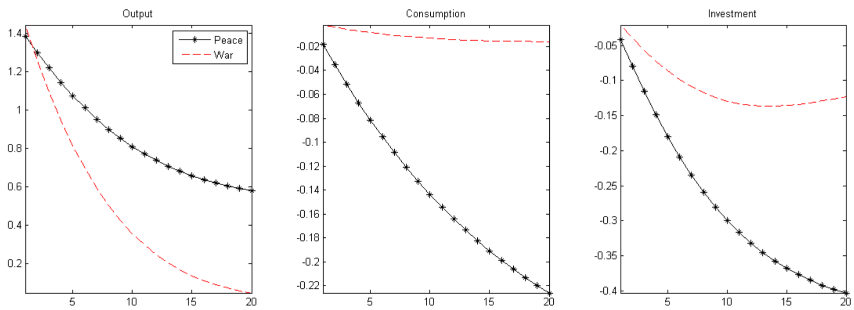
$$\text{Multiplier at horizon k} = \frac{y_{t+k} - y}{g_t - g}$$

In Figure 4.1 we present the fiscal multipliers for GDP, consumption, and investment. The black line presents the peace multiplier and the red one the war multiplier.

First, note that GDP war multiplier is higher at impact than the peace multiplier. The mechanism behind this observation is the negative wealth effect of government purchases in this kind of models. When the fiscal authority intervenes in the economy, it makes households become poorer and, hence, consumption and investment decrease. This wealth effect suggests that fiscal intervention is not appropriate in these economies. As we explain in the literature review, there are some shortcuts to overcome this characteristic of DSGE models; for instance, assume deep habits in the household's preferences or impose a rule of thumb behavior in a fractions of households. Since we are interested in reporting multiplier differences when we assume a changing government spending process, we do not care about the effect of the negative wealth channel.

The first thing to note about our multipliers is that there is an important difference between war and peace episodes as time passes. In the Figure 4.1, one can observe that multiplier difference is of order 4 ten quarters after the fiscal shock. This difference is the result of the higher persistent process that governs peace episodes. Note that this is a first order approximation, hence the variance of the process has no role for the multipliers. In the Appendix D we explore the role of the variance solving the model using a second order approximation. However, given our preference specification (log-preferences) this effect is almost negligible. To assess the impact of the government's variance we will need to use a non-separable utility function (something that we left for future research).

Figure 4.1: Government spending multiplier



Note: We present the impact multiplier at different horizons. The first figure shows the output multiplier. The second one the consumption multiplier, and the last one the investment multiplier.

Consumption and investment show a more pronounce decrease in peace episodes. This, again, is the consequence of the negative wealth effect over of government purchases over consumption and the crowding out over investment.

On the light of these multipliers, we can observe that there is an important difference between peace and war multipliers in these three key variables. In the case of GDP, the impact multiplier is almost 1.4 in both cases, then after 3 quarters, peace multiplier becomes greater. The multiplier's bias that we found is more pronounced after quarter 5 and then it continues growing. This is an important result since it suggests that there are differences between these multipliers after a short time and the effectiveness of fiscal intervention in peace episodes overcomes the strength of the multiplier in war times in this horizon.

4.1.1 Government steady state problem

One important observation of Figures 2.1 and 2.2 is that government spending behaves different in war and peace times. This suggests that the steady state value of this variable is changing between these two episodes. To asses the implications of this observation, we assume that during peace times the steady state value of the government process is as in the benchmark calibration; and in the war episodes, the steady state ratio GDP - government spending is 0.3 (ratio's mean for the World War II and Korean War episodes).

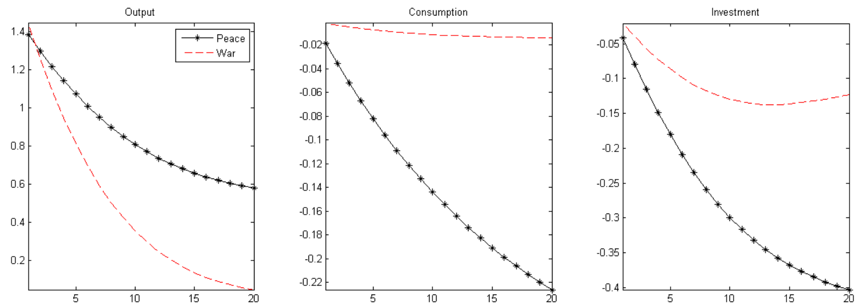
The reason for which we perform this analysis is twofold. First, to differentiate both economies we need a distinct persistent parameter and government steady state since in the benchmark economy we do not control for this parameter.

In addition, we assume a different steady state value since in the Markov model we cannot use a particular government value for each episode. The numerical methods we are using to approximate the Markov model do not handle different steady states. Bianchi (2013) mentions this problem in the algorithm of Farmer et. al. (2011) but he does not provide a justification for which we are not allowed to use two steady states. It will be the case that the solution of the linear model behaves well in a vicinity of the steady state; hence, it is not clear in the presence of a regime dependent steady state, from which point is the approximation taken. For this reason, we incorporate this observation in this section to try to shed light on the importance of different government spending in steady state.

The Figure 4.2 presents our results. First note that output multiplier for the peace time is the same as in the benchmark economy. The only one that is changing is the war multiplier which corresponds with a government expenditure to GDP ratio of 0.3. The behavior of this series is almost the same suggesting that the importance of a steady state changing across regimes is negligible. This is a very important result since it suggests that the value of the multiplier is regime dependent even though we control for a different steady state, something that goes in line with our hypothesis. The multiplier is above 1.4 at impact in both cases and it decreases as time passes.

The same observation is obtained for investment, in this case the crowding out effect of government purchases is still present once we allow for the presence regime dependent steady state. In the case of consumption, the effect of the steady state is more important than in the other variables. This is due to the importance of the negative wealth effect which makes consumption multiplier under a higher GDP - government ratio more negative.

FIGURE 4.2: GOVERNMENT SPENDING MULTIPLIER WITH DIFFERENT STEADY-STATES



Note: We present the impact multiplier at different horizons under the assumption of different steady-states. The first figure shows the output multiplier. The second one the consumption multiplier, and the last one the investment multiplier.

5 Markov DSGE model

In this section we present fiscal multipliers obtained in the model for GDP, consumption, and investment. We use the definition of the impact multiplier and different horizon multipliers in Spilimbergo et. al. (2009) and the definition of present value multiplier in Mountford and Uhlig (2009).

Fiscal multiplier is defined as the ratio of a change in GDP to a change in government purchases with respect to their baseline value. In our model this baseline value is the steady state of each variable. In this paper we use the following multipliers:

$$\text{Impact multiplier} = \frac{y_t - y}{g_t - g} = \frac{\hat{y}_t}{\hat{g}_t}\frac{y}{g} \tag{5-1}$$

and

$$\text{Multiplier at horizon k} = \frac{y_{t+k} - y}{g_t - g} = \frac{\hat{y}_{t+k}}{\hat{g}_t}\frac{y}{g} \tag{5-2}$$

where $\hat{y}_t$ denotes GDP log deviation, i.e. $\hat{y}_t = \frac{y_t - y}{y}$. This same applies for government process and for consumption and investment.

These two definitions are standard in the literature. Mountford and Uhlig (2009) propose another definition for multipliers. The idea behind this approach is to note that magnitudes in different periods are not equal and hence they cannot be compared directly. For this reason, these authors suggest to bring future magnitudes to present value and compare them. This multiplier is defined as follow:

$$\text{Present Value Multiplier at lag k} = \frac{\sum_{j=0}^{k} R^{-j}\hat{y}_j}{\sum_{j=0}^{k} R^{-j}\hat{g}_j}\frac{y}{g} \tag{5-3}$$

where R is the average of the gross interest rate in the sample. We will present model multipliers using these three definitions.

5.1 Results

5.1.1 Benchmark economy

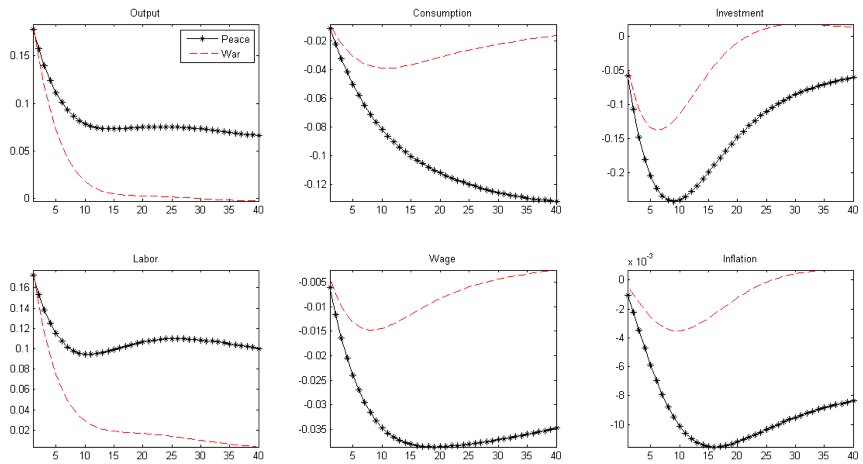
Our benchmark economy is defined in section 3. We show multipliers of government purchases for GDP, consumption, and investment. We use as government process the Markov switching model described in section 3, i.e. the model in (2-1).

In Figure 5.1 we present impulse response functions to a 1 percent shock in government process using (2-1). All the magnitudes are expressed in percent deviation from the steady state.

The first thing to note from this figure is that the response for output and labor is positive. This is in line with the evidence of RBC and New Keynesian models which predict positive response for output and labor when government's consumption rises. In addition, these results also match the response of output and labor presented in the empirical section (chapter 2) of this paper. For both estimated VARs, the responses of GDP and hours are positive.

Consumption and investment fall after the government shock. This result is standard in the RBC literature, see Baxter and King (1993) due to the negative wealth effect. In the case of investment, New Keynesian models also predict a decline in investment due to the crowding out effect of government purchases. Consumption is a controversial variable for neoclassical and keynesian literature. Neoclassical literature argues that the negative wealth effect of government purchases make labor to increase, wages to go down, and consumption to fall. On the other hand, as Galí et. al. (2007) show that the negative wealth effect is mitigated when the model is modified to include *rule of thumb* consumers and non competitive labor market. In their model consumption rises as well as wages. However this model has very strong assumptions, for example that the half of model consumers are subject to credit constraints (rule of thumb). When the parameter that governs the proportion of rule of thumb consumers is less than 0.3 (empirically supported), then even with non competitive labor market, consumption falls.

FIGURE 5.1: IMPULSE RESPONSE TO A GOVERNMENT SPENDING SHOCK



The response of wages is not so strong (see that all variables are presented in percent deviations), this is the case since we are assuming wage stickness which reduces the volatility in wages response.

Finally, the difference in both curves (for each variable) is assigned to the persistence of the shock (the shock is divided by the standard deviation in each regime to make them comparable). In peace periods we estimate a higher persistence component, hence the negative wealth effect is greater implying a higher increase in labor and GDP and a higher decrease in consumption and investment in peace periods.

Given these impulse response functions, we can compute fiscal multipliers for both regimes. Figure 5.2 shows multipliers at different horizons (see definition in (5-2)). First, note that the multiplier on GDP at impact is higher in the case of war episodes. Impact multiplier is 0.92 in war periods and 0.89 in peace periods (see Table 5.1). After the second quarter, peace multipliers are greater than war multipliers and this is the case for the rest of the sample. This difference is maintained and stabilized after quarter 15 with a difference of almost 4 times between both magnitudes.

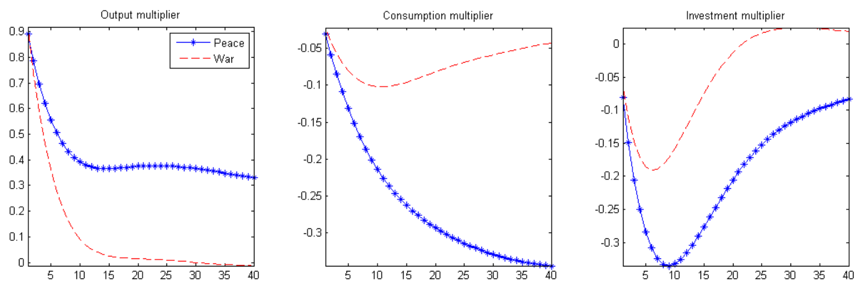
This difference is the first evidence of the bias of the multipliers. This dynamic model, which exhibits a regime dependent government rule, identifies peace episodes with a greater fiscal effect on the GDP. As we mention, this difference is small in the first part of the sample, however it becomes greater as time passes. Our motivation is to report differences between fiscal multipliers depending on the state of the economy, hence a difference of 4 between these multipliers is a very promising result for our investigation. This result also suggest that multipliers obtained using war episodes as exogenous variation may be misleading in reporting the real effect of government purchases on the GDP when the economy is in peace.

Consumption multiplier also presents this difference. Impact multiplier in both cases is almost zero at impact, however the effect of government purchases is more negative for peace episodes due to the higher negative wealth effect during the sample. For this reason, the multiplier for consumption is more negative in the case of peace episodes than in war.

Investment multiplier also is more negative in the case of peace episodes reaching the lowest level 10 quarters after the shock. The difference between these two regimes is due to the more severe crowding out of government purchases over investment in peace episodes. Note that investment multiplier in war episodes becomes positive 23 quarters after the shock.

Note the difference between multipliers obtained from the benchmark model and this Markov model. These differences can be attributed to the increase in uncertainty in the Markov model which generates a more negative response in consumption in both states showing a possible precautionary motive.

FIGURE 5.2: MULTIPLIERS TO A GOVERNMENT SPENDING SHOCK



Following Mountford and Uhlig (2009), we also estimate present value multipliers for these three variables in Table 5.1. As in the case of multipliers in Figure 5.2, multipliers for GDP are higher in the case of war episodes in the first part of the sample, then peace multiplier become higher and 20 quarters after the shock it is almost 4 times higher to that of war episodes. peace multiplier after 20 quarters is 0.4290 and in the case of war episodes it is 0.1337.

Consumption multiplier, however, is more negative in the first part of the sample for peace episodes and then it becomes greater than war multipliers. This is the result of two forces; first, interest rates are higher during war episodes, hence the factor which bring magnitudes to present value is higher for war episodes; second, the response of consumption in war episodes is higher than the response of government (i.e. the numerator in (5-3) is higher for war than for peace) making multipliers to be more negative in war episodes for larger samples. Once again we can report a significant difference between multipliers of both regimes.

Investment multiplier in peace episodes, is more negative at impact, in the first part of the sample, and at 20 quarters after the shock. In the case of 4 and 8 quarters war multipliers are more negative than the peace ones.

Table 5.1: PRESENT VALUE MULTIPLIERS

	Impact	2 qtrs	4 qtrs	8 qtrs	10 qtrs	20 qtrs
Output						
Peace	0.8896	0.7913	0.6332	0.4545	0.4172	0.4290
War	0.9151	0.8286	0.6576	0.3657	0.2602	0.1337
Consumption						
Peace	-0.0300	-0.0587	-0.1114	-0.1957	-0.2283	-0.3355
War	-0.0221	-0.0461	-0.0994	-0.2223	-0.2910	-0.7503
Investment						
Peace	-0.0805	-0.1499	-0.2555	-0.3498	-0.3545	-0.2355
War	-0.0628	-0.1253	-0.2430	-0.4120	-0.4488	-0.1159

5.1.2 Ramey revisited: the importance of news

In section 2 we explain the implications of Ramey (2011). The same point was raised by Alexopoulos (2011). This is a timing assumption, in our benchmark calibration we assume that government shock in period t becomes known to agent only in t. However, it is well known that agents have information about future shocks. To use this observation in our model, we follow Christiano et. al. (2014) who assume that the perturbation component of equation (2-1) has two terms: (i) an unanticipated term and (ii) an anticipated or news term:

$$g_t = \rho_g(S_t)g_{t-1} + \xi_t^0 + \xi_{t-1}^1$$

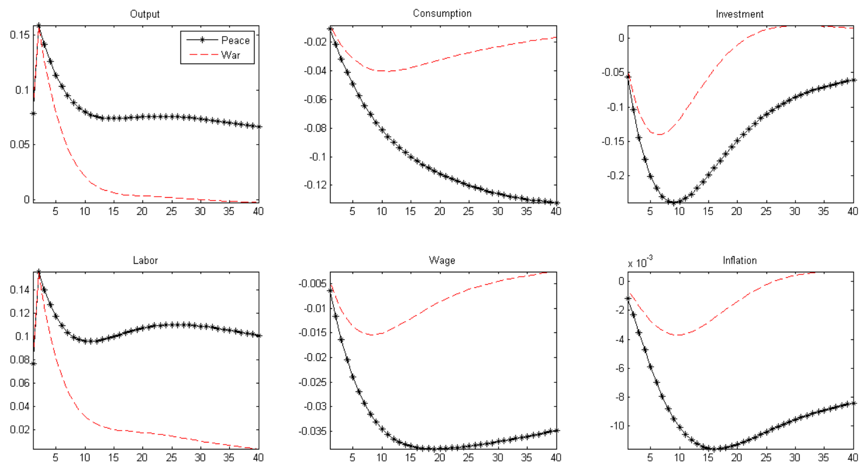
We use this specification and introduce two shocks in this economy. We assume that the economy is subject to an unanticipated shock ξ_t^0 of 0.5 percent and that agents have news about future government purchases ξ_t^1 also of 0.5 percent.

In Figure 5.3 we present impulse response functions to a 0.5 percent shock in ξ_t^0 and 0.5 percent shock in ξ_t^1. All the magnitudes are expressed in percentage deviation from the steady state.

Note that GDP and labor have a positive response which peaks in the second period, due to the news effect. The difference between the response of output and labor in peace and war periods remains when the model includes anticipated shocks.

The effect over consumption and investment is the same as in the benchmark calibration since agents internalize today the effect of future purchases. In this case the difference between the episodes remains and for consumption the negative wealth effect is important.

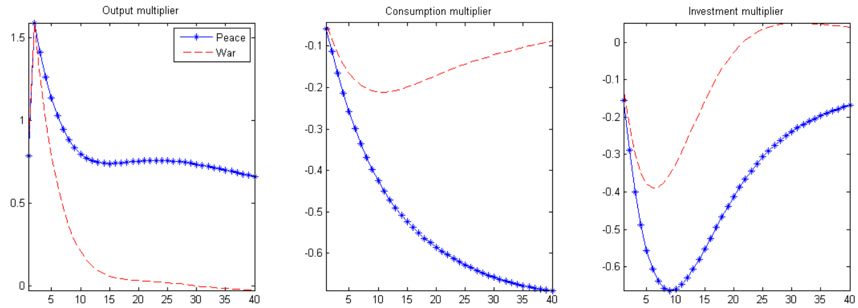
FIGURE 5.3: IMPULSE RESPONSE TO A GOVERNMENT SPENDING SHOCK



We compute fiscal multipliers in Figure 5.4. See that GDP multiplier peaks in the second quarter with a value above 1.5 in both cases (this is due to the increase in GDP in the second period). Impact multiplier is lower than in the benchmark calibration. For peace episodes it is 0.78 and in war periods it is 0.83 (see Table 5.2). As in our benchmark, we observe an increasing difference between multipliers in war and peace after quarter 5.

Consumption multiplier also presents this difference. War consumption multiplier is higher than peace multiplier since the negative wealth effect remains stronger for peace process. The addition of news does not changes the results obtained in the benchmark calibration. Investment multiplier is more negative in the case of peace episodes. In war episodes, this multiplier becomes positive after quarter 25.

FIGURE 5.4: MULTIPLIERS TO A GOVERNMENT SPENDING SHOCK



Present value multipliers are in Table 5.2. Multipliers for GDP are higher in the case of war episodes in the first part of the sample, then peace multiplier become higher. Present value multiplier of 20 quarters are almost 4 times higher in peace than in war periods, with values of 0.43 and 0.15, respectively.

Consumption multiplier is more negative in the first part of the sample for peace episodes. However, 20 quarters after the shock consumption multiplier of peace episodes is higher (less negative) than war multipliers. Investment multiplier in peace episodes, is more negative at impact, in the first part of the sample, and 20 quarters after the shock. In the case of 8 and 10 quarters war multipliers are more negative than the peace ones.

TABLE 5.2: Present Value multipliers

	Impact	2 qtrs	4 qtrs	8 qtrs	10 qtrs	20 qtrs
Output						
Peace	0.7861	0.7979	0.6423	0.4630	0.4243	0.4308
War	0.8306	0.8372	0.6717	0.3853	0.2803	0.1487
Consumption						
Peace	-0.0585	-0.0572	-0.1091	-0.1931	-0.2259	-0.3338
War	-0.0444	-0.0441	-0.0958	-0.2162	-0.2837	-0.7356
Investment						
Peace	-0.1554	-0.1449	-0.2486	-0.3439	-0.3499	-0.2354
War	-0.1250	-0.1187	-0.2325	-0.3986	-0.4360	-0.1158

6
Conclusion

In this paper we use a calibrated DSGE model to assess whether multipliers are state-dependent. In particular, our objective was to document differences in output multipliers during in peace periods *vis-à-vis* war episodes. For this purpose we follow two approaches. First we use a DSGE model and solve it using a first and second order approach. We obtain multipliers that are different depending on the state of the economy. This observations holds if we assume that the steady state of the model is changing. In addition, the importance of the precautionary motive is negligible in part because of the households' preference specification.

Moreover, we estimate a Markov switching model for a simple government purchases rule and identify peace episodes with a highly persistent coefficient and a small variance. War episodes were less lasting but more volatile. We solve the model taking into account the changes in regimes and calculate multipliers for output, consumption, and investment.

Our results suggest that the war multiplier is greater at impact, but it becomes smaller after a while. The peace multiplier becomes higher quickly. The difference between both multipliers is of order four in our benchmark calibration. We observe the same result if we analyze present value multipliers (the literature suggests that they are more adequate).

We also include news in the model to assess the important observation that the information set of market participants depends critically on the news that they have about the future state of the economy. We include this observation in the model and we obtain a multiplier that is different in magnitude, but similar in the reported differences between war and peace multipliers.

7
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A
Steady state

In the steady state, all variables are constant. Both technology variables z_t and A_t growth at the following rate:

$$\tilde{z} = \frac{z_t}{z_{t-1}} = e^{\Lambda_z}$$
$$\tilde{A} = \frac{A_t}{A_{t-1}} = e^{\Lambda_A}$$

In addition, all variables growth at the same path than technology process, i.e. $\Lambda_c = \Lambda_i = \Lambda_w = \Lambda_{w^*} = \Lambda_{y_t^d} = \Lambda_z$. Moreover, we assume that $u = 1$.

Household's steady state conditions:

$$\frac{1}{\tilde{c} - \Phi\frac{1}{\tilde{z}}\tilde{c}} - \Phi\beta\frac{1}{\tilde{c}\tilde{z} - \Phi\tilde{c}} = \tilde{\lambda}$$
$$1 = \beta\frac{1}{\tilde{z}}\frac{R}{\Pi}$$
$$r = a'[1]$$
$$q = \beta\frac{1}{\tilde{z}}((1-\delta)q + r - a[1])$$
$$1 = q\left(1 - S\left(\tilde{z}\right) - S'\left(\tilde{z}\right)\tilde{z}\right) + \beta q\frac{1}{\tilde{z}}S'\left(\tilde{z}\right)\left(\tilde{z}\right)^2$$
$$f = \frac{\eta-1}{\eta}(\tilde{w}^*)^{1-\eta}\tilde{\lambda}\tilde{w}^{\eta}l^d + \beta\theta_w\left(\frac{\Pi^{\chi_w}}{\Pi}\right)^{1-\eta}(\tilde{z})^{\eta-1}f$$
$$f = \psi\left(\Pi^{w*}\right)^{-\eta(1+\gamma)}\left(l^d\right)^{1+\gamma} + \beta\theta_w\left(\frac{\Pi^{\chi_w}}{\Pi}\right)^{-\eta(1+\gamma)}\tilde{z}^{\eta(1+\gamma)}f$$

Firms' first order conditions:

$$\frac{u\tilde{k}}{l^d} = \frac{\alpha}{1-\alpha}\frac{\tilde{w}}{r}\tilde{z}$$
$$mc = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}(\tilde{w})^{1-\alpha}r^{\alpha}$$
$$g^1 = \tilde{\lambda}mc\tilde{y}^d + \beta\theta_p\left(\frac{\Pi^{\chi}}{\Pi}\right)^{-\varepsilon}g^1$$
$$g^2 = \tilde{\lambda}\Pi^*\tilde{y}^d + \beta\theta_p\left(\frac{\Pi^{\chi}}{\Pi}\right)^{1-\varepsilon}g^2$$
$$\varepsilon g^1 = (\varepsilon-1)g^2$$

Wage and price dynamics:

$$1 = \theta_p \left(\frac{\Pi^{\chi}}{\Pi}\right)^{1-\varepsilon} + (1-\theta_p)(\Pi^*)^{1-\varepsilon}$$
$$1 = \theta_w \left(\frac{\Pi^{\chi_w}}{\Pi}\right)^{1-\eta} \tilde{z}^{\eta-1} + (1-\theta_w)(\Pi^{w*})^{1-\eta}$$

Market clearing conditions:

$$\tilde{y}^d = \tilde{c} + \tilde{i} + \tilde{g} + a[1]\tilde{k}\frac{1}{\tilde{z}}$$
$$\tilde{y}^d = \frac{\frac{\tilde{A}}{\tilde{z}}(u\tilde{k})^{\alpha}(l^d)^{1-\alpha} - \phi}{\nu^p}$$
$$l = v^w l^d$$
$$\nu^w = \theta_w \left(\frac{1}{\tilde{z}}\frac{\Pi^{\chi_w}}{\Pi}\right)^{-\eta} \nu^w + (1-\theta_w)(\Pi^{w*})^{-\eta}$$
$$\nu^p = \theta_p \left(\frac{\Pi^{\chi}}{\Pi}\right)^{-\varepsilon} \nu^p + (1-\theta_p)\Pi^{*-\varepsilon}$$
$$\tilde{k}\tilde{z} - (1-\delta)\tilde{k} - (1 - S(\tilde{z}))\,\tilde{i}\tilde{z} = 0$$

We need to express the above system recursively to obtain each variable in terms of the deep model parameters. In addition, we need to assume specifications for the functions $a[\cdot]$ and $S[\cdot]$:

$$a[u] = \gamma_1(u-1) + \frac{\gamma_2}{2}(u-1)^2$$
$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - \Lambda_i\right)^2$$

These two functions imply that in the steady state: (i) $a[1] = 0$, (ii) $a'[1] = r = \gamma_1$, and (iii) $S[\Lambda_i] = S'[\Lambda_i] = 0$.

To obtain the steady state, first notice that the return over capital impose restrictions on the value of γ_1:

$$r = \frac{1 - \frac{\beta}{\tilde{z}}(1-\delta)}{\frac{\beta}{\tilde{z}}} = \gamma_1$$

In addition, optimal relative prices are given by:

$$\Pi^* = \left(\frac{1 - \theta_p \Pi^{-(1-\varepsilon)(1-\chi)}}{1-\theta_p}\right)^{\frac{1}{1-\varepsilon}}$$

Marginal cost depends on inflation and optimal relative prices:

$$mc = \frac{\varepsilon - 1}{\varepsilon} \frac{1 - \beta\theta_p \Pi^{(1-\chi)\varepsilon}}{1 - \beta\theta_p \Pi^{-(1-\chi)(1-\varepsilon)}} \Pi^*$$

Optimal relative wage:

$$\Pi^{w*} = \left(\frac{1 - \theta_w \Pi^{-(1-\chi_w)(1-\eta)} \tilde{z}^{-(1-\eta)}}{1 - \theta_w} \right)^{\frac{1}{1-\eta}}$$

Wage and optimal wage evolve according to the following relations:

$$\tilde{w} = (1-\alpha) \left(mc \left(\frac{\alpha}{r} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$
$$\tilde{w}^* = \tilde{w} \Pi^{w*}$$

We use the above equations and the wage household decision equations to obtain a function that relates $\tilde{\lambda}$ and l_t^d:

$$\frac{1 - \beta\theta_w \tilde{z}^{\eta(1+\gamma)} \Pi^{\eta(1-\chi_w)(1+\gamma)}}{1 - \beta\theta_w \tilde{z}^{(\eta-1)} \Pi^{-(1-\eta)(1-\chi_w)}} = \frac{\psi (\Pi^{w*})^{-\eta\gamma} (l_t^d)^{\gamma}}{\frac{\eta-1}{\eta} w^* \tilde{\lambda}}$$

We derive another expression that relates $\tilde{\lambda}$ and l_t^d to solve the steady state. Price and wage dispersion are given by:

$$\nu^p = \frac{1 - \theta_p}{1 - \theta_p \Pi^{(1-\chi)\varepsilon}} \Pi^{*-\varepsilon}$$
$$\nu^w = \frac{1 - \theta_w}{1 - \theta_w \Pi^{(1-\chi_w)\eta} \tilde{z}^{\eta}} (\Pi^{*w})^{-\eta}$$

In addition, production function and capital in steady state are given by:

$$\tilde{y}^d = \frac{\frac{\tilde{A}}{\tilde{z}} (\tilde{k})^{\alpha} (l^d)^{1-\alpha} - \phi}{v^p}$$
$$\tilde{k} = \frac{\tilde{z}}{\tilde{z} - (1-\delta)} \tilde{i}$$

Using the resource constraint:

$$\tilde{c} + \frac{\tilde{z} - (1-\delta)}{\tilde{z}} \tilde{k} + \tilde{g} = \frac{\frac{\tilde{A}}{\tilde{z}} (\tilde{k})^{\alpha} (l^d)^{1-\alpha} - \phi}{v^p}$$

Using the fact that

$$\frac{\tilde{k}}{l^d} = \frac{\alpha}{1-\alpha} \frac{\tilde{w}}{r} \tilde{z} = \Gamma_0 \rightarrow \tilde{k} = \Gamma_0 l_t^d$$

then:

$$\begin{aligned}\tilde{c} &= \frac{\frac{\tilde{A}}{\tilde{z}}(\Gamma_0^\alpha(l^d) - \phi}{v^p} - \frac{\tilde{z}-(1-\delta)}{\tilde{z}}\Gamma_0 l^d \\ &= \left(\frac{\frac{\tilde{A}}{\tilde{z}}\Gamma_0^\alpha}{v^p} - \frac{\tilde{z}-(1-\delta)}{\tilde{z}}\Gamma_0\right) l^d - \frac{1}{v^p}\phi\end{aligned}$$

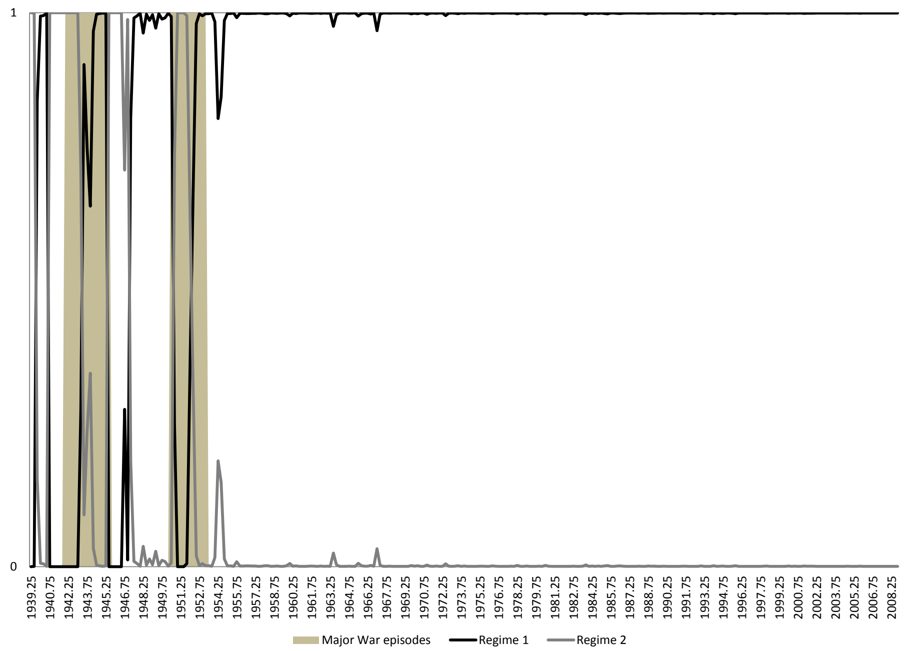
Finally, the second relation is given by:

$$(1-\Phi\beta)\tilde{z}\left(1-\frac{h}{\tilde{z}}\right)^{-1}\left(\left(\frac{\frac{\tilde{A}}{\tilde{z}}\Gamma_0^\alpha}{v^p} - \frac{\tilde{z}-(1-\delta)}{\tilde{z}}\Gamma_0\right) l^d - \frac{1}{v^p}\phi\right)^{-1} = \tilde{\lambda}$$

After obtaining l_t^d we can solve for the other variables.

B
Empirical Evidence, smoothed probabilities

Figure B.1: Smoothed probabilities for the Markov Estimation



Note: The figure presents the Markov estimation of the government process presented in the Empirical Evidence section. As suggested by the shaded area, the algorithm identifies the two major war events in the US history.

C
Solution method

C.1
First Order approximation method

To develop the first order approximation of this model we use the next variation of a first order Taylor expansion. Let's assume that an equation of this model could be represented by:

$$f(X_t, Z_t) = g(Y_t)$$

with a steady state given by:

$$f(X_{ss}, Z_{ss}) = g(Y_{ss})$$

Note that we can express the first equation as:

$$f(e^{X_t}, e^{Z_t}) = g(e^{Y_t})$$

Now, the first order approximation of each part equation around the steady state is:

$$f(e^{X_t}, e^{Z_t}) \approx f(X_{ss}, Z_{ss}) + \left[\begin{array}{cc} \frac{\partial f(e^{X_t}, e^{Z_t})}{\partial \log X_t} & \frac{\partial f(e^{X_t}, e^{Z_t})}{\partial \log Z_t} \end{array} \right]_{t=ss} \left[\begin{array}{c} \log X_t - \log X_{ss} \\ \log Z_t - \log Z_{ss} \end{array} \right]$$

Which is the same as:

$$f(X_t, Z_t) \approx f(X_{ss}, Z_{ss}) + f_1(X_{ss}, Z_{ss}) X_{ss} x_t + f_2(X_{ss}, Z_{ss}) Z_{ss} z_t$$

Doing the same calculation with the right part of the equation and after some algebra, we get:

$$f_1(X_{ss}, Z_{ss}) X_{ss} x_t + f_2(X_{ss}, Z_{ss}) Z_{ss} z_t = g'(Y_{ss}) Y_{ss} y_t$$

C.2
Second Order approximation method

In this section, we explain the algorithm developed in Schmitt-Grohé and Uribe (2004) to find second order approximations of policy functions. Assume a general model as follows:

$$F(x,\sigma) \equiv E_t f(g(h(x,\sigma)+\eta\sigma\epsilon'), g(x,\sigma), h(x,\sigma)+\eta\sigma\epsilon', x) = 0 \tag{C-1}$$

Now, second order Taylor approximation of policy functions $g(x,\sigma)$ and $h(x,\sigma)$ around the steady state $[x_t,\sigma] = [\bar{x},0]$ are given by:

$$\begin{aligned}
[g(x,\sigma)]^i = {} & [g(\bar{x},0)]^i + [g_x(\bar{x},0)]^i_a[(x-\bar{x})]_a + [g_x(\bar{x},0)]^i[\sigma] \\
& + \frac{1}{2}[g_{xx}(\bar{x},0)]^i_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b \\
& + \frac{1}{2}[g_{x\sigma}(\bar{x},0)]^i_a[(x-\bar{x})]_a[\sigma] \\
& + \frac{1}{2}[g_{\sigma x}(\bar{x},0)]^i_a[(x-\bar{x})]_a[\sigma] \\
& + \frac{1}{2}[g_{\sigma\sigma}(\bar{x},0)]^i[\sigma]^2
\end{aligned} \tag{C-2}$$

and

$$\begin{aligned}
[h(x,\sigma)]^j = {} & [h(\bar{x},0)]^j + [h_x(\bar{x},0)]^j_a[(x-\bar{x})]_a + [h_x(\bar{x},0)]^j[\sigma] \\
& + \frac{1}{2}[h_{xx}(\bar{x},0)]^j_{ab}[(x-\bar{x})]_a[(x-\bar{x})]_b \\
& + \frac{1}{2}[h_{x\sigma}(\bar{x},0)]^j_a[(x-\bar{x})]_a[\sigma] \\
& + \frac{1}{2}[h_{\sigma x}(\bar{x},0)]^j_a[(x-\bar{x})]_a[\sigma] \\
& + \frac{1}{2}[h_{\sigma\sigma}(\bar{x},0)]^j[\sigma]^2
\end{aligned} \tag{C-3}$$

In these two expressions, the unknowns are g_{xx}, $g_{x\sigma}$, $g_{\sigma x}$, $g_{\sigma\sigma}$, h_{xx}, $h_{x\sigma}$, $h_{\sigma x}$, and $h_{\sigma\sigma}$. To find them one has to note that since equation (C-1) is equal to zero, all its derivative are zero as well, then this system will give the solution to each of the needed expressions. For deep explanations and the proof of the next result one can refer to the paper. A theoretical result of the paper of Schmitt-Grohé and Uribe is that $h_{\sigma x} = 0$ and $g_{\sigma x} = 0$. With this, it is clear that neither of the policy functions will depend of the variance of the processes at least by a constant $[g_{\sigma\sigma}(\bar{x},0)][\sigma]^2$ and $[h_{\sigma\sigma}(\bar{x},0)][\sigma]^2$, respectively.

As we mentioned before, for this method we need to define the steady state values of each of the variables. The steady state value of the variables of the model are given in the Appendix A.

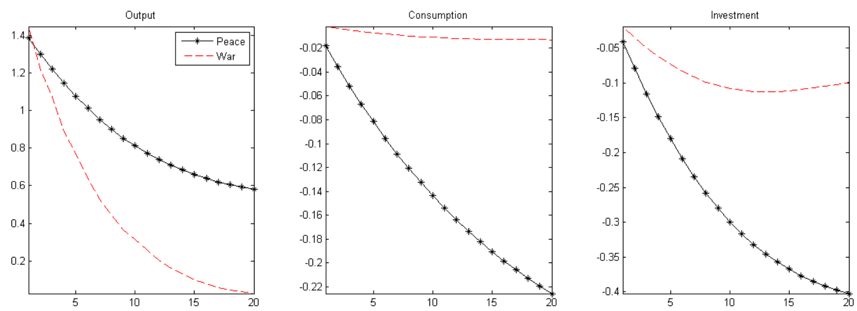
D
Second order approximation, results

In this appendix we show government spending multipliers when we solve the model using a second order approach. Our objective here is to formally analyze the role of government spending variance in the different multipliers. In addition, we allow government spending steady state to change across regimes.

The results are very similar to the ones presented in chapter 4. This suggests two important results: (i) the choice of preference function (log - preferences) mutes the possible effect of government spending variance (see Figure D.1) and hence other type of utility function is needed to asses how volatility affects multipliers; (ii) the effect of government spending steady state is negligible suggesting that this correction to war episodes do not capture the difference that we observe in peace and war regimes (see Figure D.2), therefore we can attribute this difference to the changing spending process.

D.1
Second order approximation

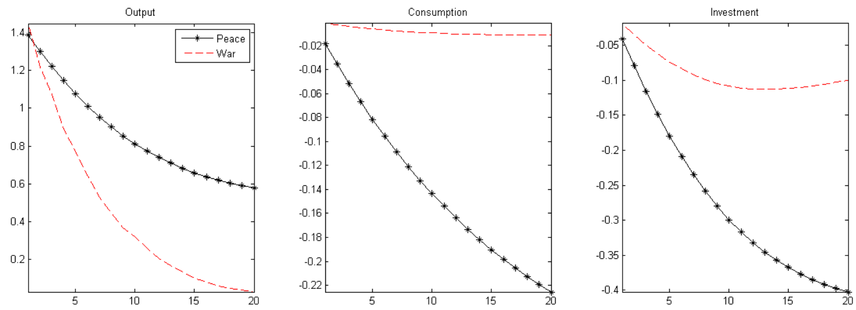
FIGURE D.1: GOVERNMENT SPENDING MULTIPLIER, SECOND ORDER APPROACH



Note: We present the impact multiplier at different horizons using a second order approximation solution. The first figure shows the output multiplier. The second one the consumption multiplier, and the last one the investment multiplier.

D.1.1
The effect of government variance

FIGURE D.2: GOVERNMENT SPENDING MULTIPLIER WITH DIFFERENT STEADY-STATES, SECOND ORDER APPROACH



Note: We present the impact multiplier at different horizons using a second order approximation solution under the assumption of different steady-states. The first figure shows the output multiplier. The second one the consumption multiplier, and the last one the investment multiplier.