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Collateralized Assets and Asymmetric Information

4.1
Introduction

In a two-period incomplete markets economy with nominal assets, Cornet & de Boisdeffre (2002) contribute to the theory of asymmetric information extending the classical non-arbitrage asset pricing procedure. They propose a decentralized mechanism in which agents anticipate asset prices and make, before the trade of commodities and assets, a refinement of their signals by precluding arbitrage opportunities.

In this context, a vector of asset prices is implementable as equilibrium only if the pooling information, obtained after the exclusion of arbitrage opportunities, is non-empty (see de Boisdeffre (2007)). In particular, there are financial structures for which only asset prices that fully reveal information are equilibria.

To illustrate this point, consider a two-period economy with two states of nature in the second period, $\{u, d\}$. There is only one commodity and only one asset, an Arrow security contingent to $s = u$. There are two types of agents, $\{A, B\}$. Individuals of type A are uninformed about the realization of the uncertainty, while individuals of type B know that the state $s = d$ will occur with certainty. Then, applying the refinement mechanism of Cornet & de Boisdeffre (2002), only when the asset has zero price the pooling information (obtained by the elimination of arbitrage opportunities) will be non-empty. Furthermore, in the unique equilibrium, there is no trade and uninformed agents become fully informed.

Note that when an agent anticipates an asset price, she believes that it is a non-arbitrage price. Thus, the individual never anticipates asset prices that give unbounded gains today without any risk tomorrow. For this reason, in the preceding example, type B individuals will never anticipate a positive price for the asset.

Moreover, the existence of a financial position that gives unbounded gains tomorrow without any cost today will be interpreted by any agent as a signal that some states of nature will not occur. This will induce agents to refine their private information. For instance, in our example, uninformed individuals will become fully informed when they anticipate a zero price.

A natural question arises: why will uninformed individuals anticipate a zero price (becoming fully informed)? For Cornet & de Boisdeffre (2002) this happens because, even without rational expectations, agents will coincide in their forecasted prices. In fact, given that credit markets are frictionless, informed individual always will anticipate a zero price.

On the other hand, when borrowers can default, financial markets need to implement mechanisms to protect lenders of excessive losses. Usually, these mechanisms will set limits on the amount of debt and, therefore, may also preclude (endogenously) the unbounded gains associated to an arbitrage opportunity. For instance, in the previous example, if we burden borrowers to constitute collateral requirements that will be seized in case of default, then a positive price for the Arrow security may emerge. In fact, the financial frictions induced by the collateral constraints will prevent type B agents from obtaining unbounded gains when the security has a positive price. Therefore, new equilibria will appear when default is allowed.

In this paper, we extend the model of Cornet & de Boisdeffre (2002) to allow for default and collateral, as in Geanakoplos & Zame (2002). We prove that equilibrium always exists, with no need to update information through a predefined (centralized or decentralized) mechanism. In this sense, the set of common non-arbitrage prices will increase when default is allowed.

Essentially, Cornet & de Boisdeffre (2002) point out that in the absence of default, a non-arbitrage price (common to every agent) may no longer exist, thus agents may need to update information to preclude arbitrage opportunities. When default is allowed and assets are collateralized, the existence of a margin between the market value of the collateral and the asset price will bound the amount of wealth that more informed borrowers extract from less informed lenders. Thus, endogenously, markets only allow for limited arbitrage opportunities and physical-financial trade becomes possible. Alternatively, from the perspective of an environment that allows agents to update information before commodities and assets can be traded, we focus on the second stage. Thus, we assure the existence of equilibrium independently of either the nature of the mechanism that was used to update information or the final distribution of information.

4.2
Collateralized Assets in an Economy with Asymmetric Information

4.2.1
Model

We consider a discrete time economy with two periods, $t \in \{0, 1\}$. There is no uncertainty at $t = 0$ and we denote by $s = 0$ the unique state of nature at this date. At $t = 1$, there is a finite set S of states of nature that can be reached. Let $\Sigma = \{0\} \bigcup S$.

There is a finite set of commodities, L, that can be traded at each $s \in \Sigma$. Commodities may suffer depreciation contingent on the state of nature. Thus, if one unit of good $l \in L$ is consumed in $t = 0$, an amount $Y_s(l, l')$ of the good $l' \in L$ is obtained at $s \in S$. Given $s \in S$, the matrix $Y_s = (Y_s(l, l'))_{(l,l') \in L \times L}$ has non-negative entries. Note that we allow for perishable and perfect durable goods in our economy.

As in Geanakoplos & Zame (2002), the financial sector is characterized by a finite set J of collateralized assets that can be negotiated at $t = 0$ and that make promises contingent on the states $s \in S$. More formally, each $j \in J$ is characterized by a process $(A(s, j);\, s \in S) \in \mathbb{R}_+^{L \times S}$ of state contingent real promises, and by its physical unitary collateral requirements, $C_j \in \mathbb{R}_+^L$. Collateral guarantees are always held by the borrowers.

Following Cornet & de Boisdeffre (2002), there is a finite number of agents, $h \in H$, that have a private information $S_h \subset S$ about the states of nature that will occur in $t = 1$. This private information is correct in the sense that the true state will belong to S_h. Therefore, the state of nature that actually will occur in $t = 1$ will belong into the *pooled information set*: $\underline{S} := \bigcap_h S_h$.

Each $h \in H$ may trade commodity and assets at $t = 0$, and make plans for consumption at each $s \in S_h$. Let $\Sigma_h = \{0\} \cup S_h$. We assume that consumer $h \in H$ is also characterized by his endowments, $w^h = (w_s^h; s \in \Sigma_h) \in \mathbb{R}_+^{\Sigma_h \times L}$, and by his preferences over consumption, that are represented by a function $U^h : \mathbb{R}_+^{\Sigma_h \times L} \to \mathbb{R}_+$.

The consumption allocation of $h \in H$, which includes the physical collateral requirements, is denoted by $x^h = (x_s^h;\, s \in \Sigma_h)$. We denote by θ_j^h (resp. φ_j^h) the quantity of asset j that agent h buys (resp. short-sells). Let $\theta^h = (\theta_j^h;\, j \in J)$ and $\varphi^h = (\varphi_j^h;\, j \in J)$.

We assume that each $h \in H$ observes commodity prices at $t = 0$, $p_0 \in \mathbb{R}_+^L$. Moreover, future (state contingent) commodity prices will be anticipated by h, denominated $p_s^h \in \mathbb{R}_+^L$ for each $s \in S_h$. Let $p^h = (p_s^h; s \in S_h)$. Also, each agent will form expectations about the unitary asset price, $q^h \in \mathbb{R}_+^J$.

Individuals trade assets and demand commodities after the anticipation of prices. As the only penalty in case of default is the seizure of the collateral guarantees, borrowers will pay the minimum between the depreciated value of the collateral and the market value of the original debt. Thus, as any agent h believes that her forecasted prices are correct, she will anticipate that the (unitary) nominal payment made by $j \in J$ at the state of nature $s \in S_h$ is given by $D_{s,j}(p^h) := \min\{p^h_s A(s,j), p^h_s Y_s C_j\}$.

Therefore, given prices (p_0, p^h, q^h), each agent $h \in H$ chooses an allocation $(x^h, \theta^h, \varphi^h)$ in his budget set $B^h(p_0, p^h, q^h) \subset \mathbb{E}^h := \mathbb{R}_+^{\Sigma_h \times L} \times \mathbb{R}_+^J \times \mathbb{R}_+^J$, which is given by the collection of vectors $(x, \theta, \varphi) = ((x_s)_{s\in\Sigma_h}, (\theta_j, \varphi_j)_{j\in J}) \in \mathbb{E}^h$ such that, $x_0 \geq \sum_{j\in J} C_j \varphi_j$ and

$$\begin{aligned} p_0 x_0 + q^h(\theta - \varphi) &\leq p_0 w_0^h; \\ p_s^h(x_s - w_s^h) &\leq p_s^h Y_s x_0 + \sum_{j\in J} D_{s,j}(p^h)\,(\theta_j - \varphi_j), \quad \forall s \in S_h. \end{aligned}$$

Note that the restriction $x_0 \geq \sum_{j\in J} C_j \varphi_j$ assures that first period consumption is compatible with financial promises, as the physical collateral required by the market is effectively constituted and held by the borrowers.

Definition. An equilibrium for our economy is given by a vector of prices $(\overline{p}_0, (\overline{p}^h, \overline{q}^h)_{h\in H})$ jointly with individual allocations $(\overline{x}^h, \overline{\theta}^h, \overline{\varphi}^h)_{h\in H} \in \prod_{h\in H} \mathbb{E}^h$, such that,

(a) For each $h \in H$, $(\overline{x}^h, \overline{\theta}^h, \overline{\varphi}^h) \in \operatorname{argmax}_{(x,\theta,\varphi)\in B^h(\overline{p}_0, \overline{p}^h, \overline{q}^h)} U^h(x)$.

(b) Physical and asset markets clear at states of nature that may occur, i.e.,

$$\sum_{h\in H} (\overline{x}_0^h, \overline{\theta}^h) = \sum_{h\in H} (w_0^h, \overline{\varphi}^h); \qquad \sum_{h\in H} \overline{x}_s^h = \sum_{h\in H} w_s^h + Y_s \sum_{h\in H} w_0^h, \;\; \forall s \in \underline{S}.$$

(c) For each $(h, h') \in H \times H$, $\overline{q}^h = \overline{q}^{h'}$. Moreover, at each $s \in \underline{S}$, $\overline{p}_s^h = \overline{p}_s^{h'}$, for all $(h, h') \in H \times H$.

Note that, as in de Boisdeffre (2007), we assume that in equilibrium forecasted prices coincide in the states of nature that may occur, although agents do not know the characteristics of the other individuals and information is private.[1] However, individuals do not need to coincide on the expected

[1] Without imposing rational expectations hypotheses, in a contemporaneous working paper, Daher, Martins-da-Rocha, Páscoa & Vailakis (2006) analyze a temporary equilibrium model with collateralized asset, in which agents are allowed to have beliefs about the characteristics of the others. In this context, the perfect foresight behaviour described in our model may appear, for some readers, as more natural.

commodity prices at states of nature $s \notin \underline{S}$.

4.2.2 Equilibrium Existence

As we point out earlier, the collateralization of financial contracts assures a natural mechanism to protect lenders in case of default and will induce endogenous bounds on short-sales. Thus, as in Geanakoplos & Zame (2002), equilibrium will always exist, even when agents do not make any refinement of their private information.

THEOREM. Assume that,

A. For each $h \in H$, $(w_0^h, (w_s^h + Y_s w_0^h)_{s \in S_h}) \gg 0$.

B. For each $j \in J$, $C_j \neq 0$.

C. For each $h \in H$, $U^h : \mathbb{R}_+^{\Sigma_h \times L} \to \mathbb{R}_+$ is continuous, strictly increasing and quasi-concave.

D. For each $j \in J$, there exists $s \in \bigcup_{h \in H} S_h$ such that $(A(s,j), Y_s C_j) \gg 0$.

Then, given any vector of commodity prices forecasts $(\hat{p}_s^h,\ h \in H,\ s \in S_h \setminus \underline{S}) \gg 0$, there exists an equilibrium $\left[(\overline{p}_0, (\overline{p}^h, \overline{q}^h)_{h \in H});\ (\overline{x}^h, \overline{\theta}^h, \overline{\varphi}^h)_{h \in H}\right]$ for our economy in which $\overline{p}_s^h = \hat{p}_s^h$ for any $h \in H$ and $s \in S_h \setminus \underline{S}$.

PROOF. Let $\underline{\Sigma} = \{0\} \cup \underline{S}$ and, for each $h \in H$, define $\eta^h = (\hat{p}_s^h)_{s \in (S_h \setminus \underline{S})}$. As in Geanakoplos & Zame (2002), collateral constraints jointly with feasibility conditions (item (b) of equilibrium definition) will assure that, under Assumption B, equilibrium individual allocations, if there exists, are uniformly bounded at the states $s \in \underline{\Sigma}$, independently of the price level. Now, as $\eta^h \gg 0$, budget restrictions will imply that consumption allocations of agent h at nodes $s \in S_h \setminus \underline{S}$ are also uniformly bounded.

Thus, for each $h \in H$ there exists a non-empty, compact and convex set $K^h \subset \mathbb{E}^h$ such that, to find an equilibrium we can restrict, without loss of generality, individual h to choose budget feasible allocations in K^h. Moreover, sets K^h are constructed in such way that feasible allocations are in the interior of $\prod_{h \in H} K^h$. Let

$$\mathbb{P} = \{(p_0, (\mu_s)_{s \in \underline{S}}, q) \in \mathbb{R}_+^L \times \mathbb{R}_+^{L \times \underline{S}} \times \mathbb{R}_+^J :\ \|(p_0, q)\|_1 = 1;\ \|\mu_s\|_1 = 1,\ \forall s \in \underline{S}\},$$

where $\|\cdot\|_1$ denotes the norm of the sum.

We will find an equilibrium for our economy as a fixed point of a set-valued mapping. To attempt this objective we define, for each $h \in H$, a correspondence $\Psi^h : \mathbb{P} \twoheadrightarrow K^h$ by

$$\Psi^h(p_0, (\mu_s)_{s\in\underline{S}}, q) = Argmax_{(x,\theta,\varphi)\in B^h(p_0,p^h,q)\cap K^h} U^h(x),$$

where $p^h = ((\mu_s)_{s\in\underline{S}}, \eta^h)$.

Moreover, if $\Delta_+^{L+J} := \{z \in \mathbb{R}_+^L \times \mathbb{R}_+^J : \|z\|_1 = 1\}$ and $\Delta_+^L := \{z \in \mathbb{R}_+^L : \|z\|_1 = 1\}$, let $\Psi_0 : \prod_{h\in H} K^h \twoheadrightarrow \Delta_+^{L+J}$ be the correspondence,

$$\Psi_0((x^h, \theta^h, \varphi^h)_{h\in H}) = Argmax_{(p_0,q)\in\Delta_+^{L+J}} \; p_0 \sum_{h\in H}(x_0^h - w_0^h) + q \sum_{h\in H}(\theta^h - \varphi^h).$$

and, for each $s \in \underline{S}$, define $\Psi_s : \prod_{h\in H} K^h \twoheadrightarrow \Delta_+^L$ by,

$$\Psi_s((x^h, \theta^h, \varphi^h)_{h\in H}) = Argmax_{\mu_s\in\Delta_+^L} \; \mu_s \sum_{h\in H}(x_s^h - w_s^h - Y_s w_0^h).$$

Now, it is not difficult to see that, under Assumptions A-C and as a consequence of Berge Maximum Theorem (see Aliprantis & Border (1999), Theorem 16.31) each one of the correspondences above is upper hemicontinuous and has non-empty, convex and compact values. Therefore, as the set $\mathbb{P} \times \prod_{h\in H} K^h$ is non-empty, convex and compact, it follows from Kakutani Fixed Point Theorem (see Aliprantis & Border (1999), Corollary 16.51) that there exists a fixed point, $[(\overline{p}_0, (\overline{\mu}_s)_{s\in\underline{S}}, \overline{q}); (\overline{x}^h, \overline{\theta}^h, \overline{\varphi}^h)_{h\in H}]$, for the set-valued mapping $\Omega : \mathbb{P} \times \prod_{h\in H} K^h \twoheadrightarrow \mathbb{P} \times \prod_{h\in H} K^h$ defined by,

$$\Omega((p_0, (\mu_s)_{s\in\underline{S}}, q); (x^h, \theta^h, \varphi^h)_{h\in H}) =$$

$$\prod_{s\in\underline{\Sigma}} \Psi_s((x^h, \theta^h, \varphi^h)_{h\in H}) \times \prod_{h\in H} \Psi^h(p_0, (\mu_s)_{s\in\underline{S}}, q).$$

Let, for each $h \in H$, $\overline{p}^h := ((\overline{\mu}_s)_{s\in\underline{S}}, \eta^h)$. Under Assumption C and D, analogous arguments to those made in Araujo, Páscoa & Torres-Martínez (2002, Lemma 2) will guarantee that

$$[(\overline{p}_0, (\overline{p}^h)_{h\in H}, \overline{q}); (\overline{x}^h, \overline{\theta}^h, \overline{\varphi}^h)_{h\in H}]$$

is an equilibrium for our economy. Finally, by construction, we have that, for each $h \in H$, equilibrium prices $\overline{p}_s^h$ coincide with forecasted prices $\hat{p}_s^h$, for any $s \in S_h \setminus \underline{S}$. □