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Information (in) Chains: Information Transmission through Production Chains

1.1
Introduction

The production of a final good in modern economies usually involves multiple stages of processing. Although the notion that pricing decisions are tied to the interdependence of firms at different stages of production has been presented at least since Means (1935), how information flows along these stages are far less understood. In order to fill this gap, we study the transmission of information in a model with two features that are typical of modern economies: (a) a vertical input-output structure and (b) dispersed information. The goal of this paper is to understand how dispersed information affects not only the weight on public information but also the precision of information along the chain.

The backbone of our model is a variant of the beauty contest game of Morris and Shin (2002). We introduce an input-output structure considering a payoff function that yields the same (log-linear) optimal flexible pricing decision as the model of monopolistically competitive goods supply with production chains in Huang and Liu (2001). In this sense, we are considering *vertical* complementarities instead of *horizontal* complementarities.[1] As in Morris and Shin (2002), information is dispersed and firms observe noisy private and public signals about the underlying fundamental.

If we consider that all firms in all stages observe the same exogenous public signal, our model of vertical complementarities converges to Morris and Shin (2002) and Angeletos and Pavan (2007) models of horizontal complementarities. In this case, when actions are strategic complements, agents wish to coordinate their actions, and because public information is a relatively better predictor of others' actions, agents find it optimal to rely more on public information relative to a situation in which actions are strategically independent.

[1] We bring this terminology from Matsuyama (1995).

The result above is obtained with an exogenous information structure and presumes that the precision of public information remains invariant along the chain. We argue that this is unlikely to be the case when public information is endogenous through prices or other macroeconomic indicators. We extend our model to consider that firms observe input prices with noise, which endogenize the precision of information that is public within a stage but not across stages (semi-public signal).

In contrast to the case with exogenous public information, our main result is that agents may find optimal to rely less on semi-public information along the chain. The rationale of this result is that equilibrium input prices play the same role as financial prices in Grossman and Stiglitz (1976) and Angeletos and Werning (2006), aggregating disperse private information, while avoiding perfect revelation due to unobservable semi-public information of the previous stage. Since the semi-public signal of a stage n includes unobservable information of all previous stages, the precision of the semi-public signal decreases along the chain. As a result, the weight on public information also decreases if the share of goods produced at stage $(n-1)$ in the production of stage n is not high enough. Finally, we show that, while information precision remains unchanged with exogenous public signals (*information chains*), it may decrease along the chain with endogenous semi–public signals (*information in chains*). This result is also a direct consequence of the semi-public signal precision's decrease.

The plan for the rest of the paper is as follows. We relate our approach with the pertinent literatures in the next section and introduce our basic model in Section 1.3. The core of the paper is Section 1.4 where, after presenting our exogenous public information model as a benchmark, we incorporate an endogenous semi-public signal and study how the information revealed by input prices affects the precision of as well as the weight on public information. Section 1.5 concludes. The Appendix contains proofs omitted in the main text.

1.2 Related Literature

The literature of production chains as a propagation mechanism has a long tradition. Means (1935), as pointed out by Basu (1995), showed that, in the Great Depression, simple goods, such as agricultural products, declined heavily in price, while their quantity was almost unchanged. Complex manufactured goods, on the other hand, showed the opposite pattern, with small price changes and consequently huge declines in the quantity of sales. Blanchard (1984) shows that a simple reduced-form model incorporating a vertical production chain with prices staggered across different stages of processing can generate patterns of price changes similar to those noted by Means (1935). Gordon (1990) considers the input–output table as an essential component in the description of price stickiness.

Recent studies confirm Means's observation on the patterns of price changes at different stages of production. None of these works, however, consider the impact of dispersed information on prices. For example, Clark (1999) studies a broad range of data sets and finds that prices at early stages of production respond more to a monetary policy shock than do prices at subsequent stages of production. Recently, Huang and Liu (2001) present a dynamic stochastic general equilibrium model embedded with a vertical input-output structure, with staggered price contracts at each stage of production. Working through the input-output relations and the timing of firms' pricing decisions, the model generates persistent fluctuations in aggregate output and the observed patterns of price dynamics following a monetary shock.

In recent years there has been a growing interest in models that feature heterogeneous information about aggregate economic conditions and a moderate degree of complementarity in actions. These models, however, consider a horizontal roundabout input–output structure within a single stage of production. Examples of this kind of models were used to capture applications such as the effects of monetary or fiscal policy, as in Woodford (2002), Lorenzoni (2009), and Angeletos and Pavan (2009), and the welfare effects of public information dissemination, as in Morris and Shin (2002), Hellwig (2005), and Angeletos and Pavan (2007).

The role of endogenous public information has also been considered with interest. An especially important source of endogenous public information is prices, and papers by Tarashev (2003), Angeletos and Werning (2006), Hellwig et al. (2006), and Morris and Shin (2006) have pursued various methods of combining endogenous public information with coordination games. These papers, however, focus on financial prices rather than input prices. Closely related to our work is Gorodnichenko (2008), who combine menu costs with the aggregate price level in the previous period serving as an endogenous public signal to generate rigidity in price setting even when there is no real rigidity. Although the model of Gorodnichenko (2008) also features information aggregation, it focuses on how firms make state-dependent decisions on both pricing and acquisition of information across periods. We focus on how the weight on public information and information precision changes along the chain within a period.

Our model is also related to the literature on herding and information cascades that consider how firms, *having observed the actions of those ahead of him*, follow the behavior of the preceding firm without regard to his own information.[2] Instead, our model deals with firms deciding prices *simultaneously based on an aggregate signal of its suppliers.*

1.3 The Model

Consider a monopolistic competition model with production chains as in Huang and Liu (2001).[3] In the model economy, the production of a final consumption good requires N stages of processing, from crude material to intermediate goods, then to more advanced goods, and so on. At each stage, there is a specific continuum of monopolistically competitive firms indexed in the interval $[0, 1]$ producing differentiated goods. The production at stage 1 requires only homogeneous labor services provided by a representative household, and the production at stage $n \in \{2, ..., N\}$ uses both labor and goods produced at stage $n - 1$. To keep the model simple, we consider a quadratic profit function that yields the same optimal price decision as the firms of Huang and Liu (2001)'s model. The resulting model is as a variant of the imperfect common knowledge models of Woodford (2002) and Morris and Shin (2002). All variables are in logs and all distributions are Normal. As a result, prices under imperfect information equal the expected price under complete information.

1.3.1 Actions and Payoffs

A firm $j \in [0, 1]$ in stage $n \in \{1, 2, ..., N\}$ sets price $p_n(j)$ to maximize the profit function

$$\Pi_n\left(p_n\left(j\right), p_n^*\right) \equiv \Pi_n^* - \left(p_n\left(j\right) - p_n^*\right)^2,$$

where $\Pi_n^* \in \mathbb{R}$ is maximum profit. The target price p_n^* is given by

$$p_n^* \equiv rP_{n-1} + (1 - r)\,\theta,$$

where $P_n \equiv \int p_n(j)\,dj$ is the stage-n aggregate price, $r \in (0, 1)$ is the share of goods produced at stage $n - 1$ in the production of firm j of stage n, θ is exogenous nominal aggregate demand for final (N^{th} stage) goods. For simplicity, we set $P_0 = \theta$. The model differs from Woodford (2002) as

[2]The seminal papers are Banerjee (1992) and Bikhchandani et al. (1992).
[3]See appendix for details.

it considers the influence of stage-$(n-1)$ price level, P_{n-1}, instead of the single stage economy price level P on profits. Following the terminology in Matsuyama (1995), we are considering *vertical* complementarities instead of *horizontal* complementarities.

1.3.2 Timing and Information

Before firms move, nature draws θ from an improper uniform prior over $\mathbb{R}$, as in Morris and Shin (2002). Instead of observing the fundamentals, each firm $j \in [0,1]$ of stage $n \in \{1,..,N\}$ receives two noisy signals about θ: (i) a private signal $x_n(j)$ and (ii) a public signal y or a semi-public signal y_n. Finally, firms simultaneously set prices based on the information they received.

We consider that each firm receives an exogenous unbiased private noisy signal $x_n(j)$ about the fundamental

$$x_n(j) = \theta + \xi_n(j),$$

where the idiosyncratic noise terms $\xi_n(j)$ are normally distributed with zero mean and variance α^{-1}, independent of θ and from one another.

We also consider that each firm receives a public or a semi-public signal about θ. As our benchmark case, we first consider that each firm receives an exogenous public noisy signal y

$$y = \theta + \epsilon,$$

where the common noise term ϵ is normally distributed, independent of θ and all $\xi_n(j)' s$, with mean zero and variance β^{-1}. After, in order to investigate the role of endogenous information, we consider instead that stage-n firms observe stage-$(n-1)$ aggregate price with noise, our endogenous *semi-public signal* y_n

$$y_n = P_{n-1} + \epsilon_n,$$

where the stage noise term ϵ_n is also normally distributed, independent of θ and all $\xi_n(j)' s$, with mean zero and variance β^{-1}.

The structure of the signals, fundamental θ plus an error term, as well as the distributions of the errors are common knowledge. Because input prices depend on the underlying fundamental θ, the equilibrium stage price levels will convey information that is valuable in the coordination game.

1.3.3
Equilibrium

All firms j of every stage n choose prices simultaneously in order to maximize profits. We first consider equilibrium under complete information. After, we consider the incomplete information case.

Complete Information

The optimal pricing decision when θ is common knowledge yields the same price for all firms of stage $n \in \{1, 2, ..., N\}$

$$p_n^* = P_n = rP_{n-1} + (1 - r)\,\theta,$$

or equivalently

$$P_n = r^n P_0 + (1 - r^n)\,\theta.$$

Because we assume $P_0 = \theta$, there is a unique optimal price for all stages that equal the fundamental

$$P^* = \theta.$$

As a result, if θ is common knowledge, the equilibrium entails $p_n(j) = \theta$ for all $j \in [0, 1]$ and $n \in \{1, 2..., N\}$, so that information is completely transfer from one stage to another.

Incomplete Information

Consider now that information is dispersed. Before agents move, nature draws the fundamental θ from a uniform distribution over $\mathbb{R}$ and each firm $j \in [0, 1]$ of stage $n \in \{1, 2, ..., N\}$ receives noisy signals about θ. As a result, the optimal pricing decision of firm j of stage n is

$$p_n(j) = E\left[rP_{n-1} + (1 - r)\,\theta\,|\,x_n(j)\,, y_n\right], \tag{1-1}$$

where $y_n = y$ for all n if all firms in all stages observe a public instead of a semi-pubic signal.

1.4
Results

Now, we consider the optimal price as a function of the noisy signals received by the firms. We first consider the case of an exogenous public signal. After, we analyze the impact of endogenous semi-public information.

1.4.1
The Model with Exogenous Public Signal

The private posterior for a firm j of stage n that receives signals $x_n(j)$ and y then becomes

$$\theta \,|\, x_n(j), y \sim N\left((1-\lambda)\, x_n(j) + \lambda y, (\alpha+\beta)^{-1}\right),$$

where the Bayesian weight on public information is

$$\lambda \equiv \frac{\beta}{\alpha+\beta} \in (0,1).$$

The optimal price is analogous to the equilibrium price of Morris and Shin (2002) or the equilibrium use of information of Angeletos and Pavan (2007)

$$p_n(j) = (1-\lambda_n)\, x_n(j) + \lambda_n y, \tag{1-2}$$

with the only difference that the weight on public signal is a function of the firm's stage n

$$\lambda_n \equiv \lambda \left(\frac{1-\Gamma^n}{1-\Gamma}\right). \tag{1-3}$$

where

$$\Gamma \equiv r(1-\lambda) \in (0,1). \tag{1-4}$$

In order to clarify the characteristics of our vertical complementarity model with exogenous information, we compare it with the horizontal complementarity model with exogenous information of Angeletos and Pavan (2007). The major difference of our model is that it offers a dynamic instead of a static behavior for the weight on the public signal y. First, we can show that λ_n increases monotonically with the stage n. After, we consider how the weight on public information λ_n evolves when the number of stages n in the economy increases.

Theorem 1 *The weight on public information increases along the chain. For chains long enough, vertical complementarity converges to horizontal complementarity.*

$$\lambda_n > \lambda_{n-1}, \text{ for all } n, \tag{1-5}$$

$$\lim_{n\to\infty} \lambda_n = \lambda + \left(\frac{\lambda r(1-\lambda)}{1-r(1-\lambda)}\right). \tag{1-6}$$

As is evident from equation (1-3), the sensitivity of the equilibrium to private and public information depends not only on the relative precision of the two, captured by the Bayesian weight λ, but also on the depth of the chain until that stage, captured by $\left(\frac{1-\Gamma^n}{1-\Gamma}\right)$. When $n = 1$, the weight λ_n

is simply the Bayesian weight. If $n > 1$, the public signal y contains not only information about the fundamental θ but also reveals information used for firms from previous stages. As a result, the equilibrium value of public information increases along the chain.

A correlate question is what happens to the weigh on public information λ_n when the number of stages increases arbitrarily. The right-end side expression of (1-6) is exactly the weight on public information in Angeletos and Pavan (2007). As a direct consequence of this result, an observed strategic complementary in pricing decisions could really means a complex-multiple stages economic structure. If vertical complementarity instead of horizontal complementarity is the major factor regarding prices, however, is out of the scope of this paper.

Our model with exogenous information is also similar to horizontal complementarity models in that incomplete information generates *price dispersion* (variation in the cross section of the population) and *non-fundamental volatility* (variation in aggregate activity around the complete-information level).

Theorem 2 *Price dispersion, σ_n^2, decreases while non-fundamental volatility, ϱ_n^2, increases through the chain.*

$$\begin{aligned} \sigma_n^2 &\equiv Var\left[p_n\left(j\right) - P_n\right] \\ &= \left(1-\lambda_n\right)^2 \alpha^{-1} \Rightarrow \sigma_n^2 < \sigma_{n-1}^2, \end{aligned} \tag{1-7}$$

$$\begin{aligned} \varrho_n^2 &\equiv Var\left[\theta - P_n\right] \\ &= \left(\lambda_n\right)^2 \beta^{-1} \Rightarrow \varrho_n^2 > \varrho_{n-1}^2. \end{aligned} \tag{1-8}$$

Once again, our model offers a dynamic evolution of both dispersion and volatility. On one hand, dispersion decreases along the chain because firms relies more on public information. On the other hand, the increasing relevance of public information amplifies non-fundamental volatility.

We conclude the comparative exercise of our model decomposing the information structure into its *accuracy* (the precision of the agents' forecasts about the fundamental θ) and its *commonality* (the correlation of forecast errors across agents).

Theorem 3 (Information Chains) *Information precision, $V_n^{-1}(j)$, and* *commonality, $\chi_n(i,j)$, remain unchanged through the chain.*

$$V_n^{-1}(j) \equiv Var\left[\upsilon_n(j)\right]^{-1} = \alpha + \beta, \tag{1-9}$$

$$\chi_n(i,j) \equiv Corr\left[\upsilon_n(i), \upsilon_n(j)\right] = \lambda, \tag{1-10}$$

where $\upsilon_n(j) \equiv \theta - E\left[\theta \,|\, x_n(j), y\right]$.

Although encompasses an input-output structure, our model yields the same information precision and commonality than horizontal complementarity models when public information is exogenous. The intuition behind this result is that firms rely more on public information along the chain because a growing number of firms at previous stages are using the public sign, but not because the signs are more informative about the fundamental θ.

As the results above suggest, our model of vertical complementarities converges to Morris and Shin (2002) and Angeletos and Pavan (2007) models of horizontal complementarities with exogenous public information. The inclusion of an input-output structure *per se* is not enough to change the main conclusion that firms find it optimal to rely more on public information when pricing decisions are strategic complements. Now, we investigate the robustness of this result when the public signal is specific of the stage (semi-public) and endogenous.

1.4.2
Input Prices: Endogenous Semi-public Signals

The results above presume that the precision of public information remains invariant along the chain. We argue that this is unlikely to be the case when public information is endogenous through prices or other macroeconomic indicators. To investigate the role of prices, we consider that stage-n firms observe stage-$(n-1)$ prices with noise, our endogenous semi-public signal y_n, instead of an exogenous public signal y.

Because of the linearity of the best-response condition (1-1) and the Gaussian specification of the information structure, the equilibrium prices are convex combinations of the signals

$$\hat{p}_n(j) = (1-\hat{\lambda}_n)x_n(j) + \hat{\lambda}_n y_n,$$

where $\hat{\lambda}_n$ is a weight to be determined.[4]

Given this function, we infer that the aggregate price level must satisfy

$$\hat{P}_n = (1 - \hat{\lambda}_n)\theta + \hat{\lambda}_n y_n.$$

The firms of stage n don't observe the price level $\hat{P}_{n-1}$ or the semi-public signal y_{n-1} of the previous stage. However, they know that $\hat{P}_{n-1}$ is a convex combination of the fundamental θ and the signal y_{n-1}. As a result, we can rewrite the semi-public signal y_n as

$$\begin{aligned} y_n &= \hat{P}_{n-1} + \epsilon_n \\ &= (1 - \hat{\lambda}_{n-1})\theta + \hat{\lambda}_{n-1} y_{n-1} + \epsilon_n \\ &= \theta + \hat{\lambda}_{n-1}\left(y_{n-1} - \theta\right) + \epsilon_n \\ &= \theta + \hat{\epsilon}_n, \end{aligned}$$

which is an unbiased signal of θ, where the modified stage-n error $\hat{\epsilon}_n$ is

$$\hat{\epsilon}_n = \epsilon_n + \hat{\lambda}_{n-1}\hat{\epsilon}_{n-1}, \quad \hat{\epsilon}_n \sim N(0, \hat{\beta}_n^{-1}), \tag{1-11}$$

where $\hat{\beta}_n$ is a precision to be determined for $n \geq 2$ and $\hat{\epsilon}_1 = \epsilon_1 \sim N(0, \beta^{-1})$.

The optimal price decision for stage-n firms is then

$$\hat{p}_n\left(j\right) = E\left[\theta + r\hat{\lambda}_{n-1}\hat{\epsilon}_{n-1} \mid x_n\left(j\right), y_n\right]$$

and the posterior joint distribution of $(\theta, \hat{\epsilon}_{n-1})$ given the signals $\left(x_n\left(j\right), y_n\right)$ yields

$$\begin{aligned} E\left[\theta \,|\, x_n\left(j\right), y_n\right] &= \left(1 - \delta_n\right) x_n\left(j\right) + \delta_n y_n, \\ E\left[\hat{\epsilon}_{n-1} \,|\, x_n\left(j\right), y_n\right] &= \gamma_n\left[y_n - x_n\left(j\right)\right], \end{aligned}$$

where

$$\begin{aligned} \delta_n &\equiv \frac{\beta\hat{\beta}_{n-1}}{\alpha(\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2) + \beta\hat{\beta}_{n-1}}, \\ \gamma_n &\equiv \frac{\alpha\beta\hat{\lambda}_{n-1}}{\alpha(\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2) + \beta\hat{\beta}_{n-1}}. \end{aligned}$$

[4]This conjecture can be verified following the same argument as in Morris and Shin (2002).

As a result, the weight on public information has the following expression

$$\hat{\lambda}_n = \frac{\beta(\hat{\beta}_{n-1} + r\alpha\hat{\lambda}_{n-1}^2)}{\alpha(\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2) + \beta\hat{\beta}_{n-1}}.$$

Note that $\hat{\lambda}_n \in (0,1)$ since we can rewrite this expression as

$$\hat{\lambda}_n = \frac{b_n}{a_n + b_n},$$

where

$$a_n \equiv \alpha(\hat{\beta}_{n-1} + (1-r)\,\beta\hat{\lambda}_{n-1}^2) > 0,$$
$$b_n \equiv \beta(\hat{\beta}_{n-1} + r\alpha\hat{\lambda}_{n-1}^2) > 0.$$

From equation (1-11), we observe that precision $\hat{\beta}_n$ is recursively defined as

$$\hat{\beta}_n = \frac{\beta\hat{\beta}_{n-1}}{\hat{\beta}_{n-1} + \beta\hat{\lambda}_{n-1}^2} > 0. \tag{1-12}$$

We can use the last equation to obtain expressions for δ_n and $\hat{\lambda}_n$, as functions of the primitive parameters α, β, and r together with stage-n modified precision $\hat{\beta}_n$

$$\delta_n \equiv \frac{\hat{\beta}_n}{\alpha + \hat{\beta}_n}, \quad \hat{\lambda}_n = \frac{(\beta - r\alpha)\,\hat{\beta}_n + r\alpha\beta}{\beta(\alpha + \hat{\beta}_n)}. \tag{1-13}$$

In order to fully characterize the equilibrium, we need to calculate the precisions $\{\hat{\beta}_n\}$. Given $\hat{\beta}_1 = \beta$ and $\hat{\lambda}_1 = \lambda$, we observe that the precision of the semi-public signal decreases from stage 1 to 2

$$\hat{\beta}_2 = \left[\frac{(\alpha+\beta)^2}{(\alpha+\beta)^2 + \beta^2}\right]\beta < \beta.$$

However, the evolution of $\hat{\beta}_n$ for $n > 2$ is not so simple to deal with analytically. After simulating the model under standard values for the *exogenous* parameters α, β, and r, we obtain that *endogenous* semi-public information precision $\hat{\beta}_n$ decreases monotonically to a strict positive limit $\hat{\beta}^*$[5]

$$\beta \equiv \hat{\beta}_1 > \hat{\beta}_2 > ... > \hat{\beta}^* > 0.$$

We are now able to analyze how the endogenous information structure affects the results of the model. A direct consequence of the fall of public

[5]See appendix for details.

precision is that the Bayesian weight on public information δ_n decreases monotonically when stage n increases.

Theorem 4 *The Bayesian weight on public information, δ_n, decreases along the chain and, if the chain is long enough, eventually reaches a lower bound δ^*.*

$$\delta_1 > \delta_2 > ... > \delta^* = \frac{\hat{\beta}^*}{\alpha + \hat{\beta}^*} > 0. \tag{1-14}$$

This result is in contrast with the exogenous public information case, where the Bayesian weight on public information remains constant along the chain. The semi-public signal y_n reveals information about the fundamental θ that is embedded in stage-$(n-1)$ price level $\hat{P}_{n-1}$. This indirect rather than direct information increases the noise in the public signal. Thus, result (1-14) follows from the accumulation of noise in stage-n modified error that comes from all previous stages.

Note that the same is not necessarily true for the weight $\hat{\lambda}_n$ on public information in price setting.

Theorem 5 *The weight on public information in price setting, $\hat{\lambda}_n$, decreases along the chain if and only if $r < \lambda$.*

$$\frac{\partial \hat{\lambda}_n}{\partial \hat{\beta}_n} = \frac{\alpha \left[\beta - r\left(\alpha + \beta\right)\right]}{\beta(\alpha + \hat{\beta}_n)^2} > 0 \Longleftrightarrow r < \lambda. \tag{1-15}$$

In contrast with the exogenous public information case, the weight on public information may decrease along the chain. The decrease in precision along the chain translates into a decrease on the relative importance of public information if the share of goods produced at stage $(n-1)$ in the production of stage n is not high enough.

Endogenous information also affects the behavior of price dispersion and non-fundamental volatility.

Theorem 6 *Non-fundamental volatility, $\hat{\varrho}_n^2$, increases through the chain*

$$\begin{aligned} \hat{\varrho}_n^2 &\equiv V\left[\theta - \hat{P}_n\right] \\ &= (\hat{\lambda}_n)^2 \hat{\beta}_n^{-1} \Rightarrow \hat{\varrho}_n^2 > \hat{\varrho}_{n-1}^2, \end{aligned} \tag{1-16}$$

while price dispersion, $\hat{\sigma}_n^2$, increases if and only if $r < \lambda$

$$\begin{aligned} \hat{\sigma}_n^2 &\equiv V\left[\hat{p}_n\left(j\right) - \hat{P}_n\right] \\ &= (1 - \hat{\lambda}_n)^2 \alpha^{-1} \Rightarrow \hat{\sigma}_n^2 > \hat{\sigma}_{n-1}^2 \Longleftrightarrow r < \lambda. \end{aligned} \tag{1-17}$$

Now, in contrast with the exogenous public information case, dispersion may increase along the chain because firms rely less on more imprecise public information when $r < \lambda$. This result is a direct consequence of the decrease of the weight on public information $\hat{\lambda}_n$ due to the decrease of endogenous public precision $\hat{\beta}_n$.

As in the exogenous public information case, however, volatility also increases. This apparent contradictory result is due to the combination of two distinct effects of endogenous information precision $\hat{\beta}_n$. First, $\hat{\beta}_n$ has a *indirect* impact through the weigh on public information $\hat{\lambda}_n$. Note that $\hat{\lambda}_n$ has a proportional impact on volatility. When $\hat{\lambda}_n$ increases (as in the exogenous public information case), volatility also increases. Alternatively, if $\hat{\lambda}_n$ decreases (as in the endogenous public information case if $r < \lambda$), volatility also decreases. Second, $\hat{\beta}_n$ has a *direct* and inversely proportional impact on volatility. In the exogenous case, this precision is invariant along the chain. In the endogenous case, however, volatility increases while public precision decreases. This second and direct effect of endogenous information precision $\hat{\beta}_n$ supplants the indirect effect on $\hat{\lambda}_n$, which establishes the result.

Finally, consider the impact of endogenous semi-public information on precision and commonality.

Theorem 7 (Information in Chains) *Information precision, $\hat{V}_n^{-1}(j)$, and commonality, $\hat{\chi}_n(i,j)$, decreases along the chain.*

$$\begin{aligned} \hat{V}_n^{-1}(j) &\equiv Var\left[\upsilon_n(j)\right]^{-1} \\ &= \alpha + \hat{\beta}_n \Rightarrow \hat{V}_n^{-1}(j) < \hat{V}_{n-1}^{-1}(j), \end{aligned} \tag{1-18}$$

$$\begin{aligned} \hat{\chi}_n(i,j) &\equiv Corr\left[\upsilon_n(i), \upsilon_n(j)\right] \\ &= \delta_n \Rightarrow \hat{\chi}_n(i,j) < \hat{\chi}_{n-1}(i,j), \end{aligned} \tag{1-19}$$

where $\upsilon_n(j) \equiv \theta - E\left[\theta \,|\, x_n(j), y_n\right].$

Our final result summarizes the major impacts of the introduction of an endogenous semi-public signal. Although the two measures remain unchanged with the inclusion of an input-output structure, the same is not true when we change the information structure. Precision decreases along the chain because the public signal accumulates noise or, alternatively, precision $\hat{\beta}_n$ decreases. But, when $\hat{\beta}_n$ decreases, firms rely less on public information to obtain θ, which means that δ_n decreases. As a result, commonality also decreases.

1.5 Discussion

We show how endogenous public information affects the weight on public information and the precision of information in a production chain. We consider that firms observe input prices with noise, which endogenize the precision of semi-public information.

Our main result is that agents may find optimal to rely less on stage-specific public information along the chain when semi-public signals are endogenous. This result is in contrast with the existing literature, which states that, when actions are strategic complements, agents wish to coordinate their actions, and because public information is a relatively better predictor of others' actions, agents find it optimal to rely more on public information relative to a situation in which actions are strategically independent.

An important implication of our main result regards information precision along the chain and highlights the importance of the information structure. We show that, while information precision remains unchanged with exogenous public signals (*information chains*), it decreases along the chain when signals are semi-public and endogenous (*information in chains*). This result is also a direct consequence of the semi-public signal precision's decrease.

There are various possible extensions, variations, and applications of our model. From the theoretical side, we don't consider any measure of social welfare in our model. A direct candidate is the so called "efficient use of information", proposed by Angeletos and Pavan (2007). This benchmark is the strategy that maximizes ex-ante welfare taking as given the dispersion of information in the population. It can be represented as the solution to a planner's problem, where the planner can perfectly control how an agent's action depends on his own information, but cannot transfer information from one agent to another. In contrast with standard efficiency concepts based on Mirrlees (1971) or Holmstrom and Myerson (1983) that assume costless communication and focus on incentive constraints, the efficient use of information takes decentralization of information as the main constraint, sharing with Hayek (1945) and Radner (1962) the idea that information is dispersed and can not be centralized in a planner.

The simple framework of our model can also help the understanding of diverse phenomena concerning pricing behavior. The sluggish response of consumer prices to monetary policy changes and to exchange rate devaluations are important applications.[6] How does prices are set if firms in an input-output

[6]For works in this line relating information and exchange rates see Bacchetta and van Wincoop (2006) and Areosa (2010).

structure observe not only our endogenous semi-public signal, the previous stage price index, but also a public signal, the nominal interest rate? And what if they observe the (public) exchange rate? Our model can be easily modified to deal with these questions as well as to its policy implications.