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A Sticky-Dispersed Information Phillips Curve: A Model with Partial and Delayed Information

3.1
Introduction

Over the last years, there has been a renewed interest in the idea pioneered by Phelps (1968) and Lucas (1972) that prices fail to respond quickly to nominal shocks due to the fact agents are imperfectly informed about those shocks. As an example, Mankiw and Reis (2002) suggest that, perhaps due to acquisition and re-optimization costs, information (rather than prices) is *sticky*, i.e., new information is disseminated slowly in the economy rather than being fully revealed to the agents. As a result, although prices are always changing, pricing decisions are not always based on current information, and, consequently, do not respond instantaneously to nominal shocks.

In contrast to models that assume that information is sticky, there is large literature that assumes that agents have access to timely but heterogenous information about fundamentals. As a result, in the *dispersed-information* models of Morris and Shin (2002), Angeletos and Pavan (2007) and others, prices reflect the interaction among differently informed agents and their heterogenous beliefs about the state and about what *others* know about the state.

In this paper, we study how individual firms set prices when information is both sticky *and* dispersed, and analyze the resulting dynamics for aggregate prices and inflation rates. In our model, the firms' optimal price is a convex combination of the current state of the economy and the aggregate price level. Moreover, as in Mankiw and Reis (2002), only a fraction of firms update their information set at each period. Those who update receive two sources of information: the first piece is the value of all previous periods states, while the second piece is a noisy, idiosyncratic, private signal about the current state of the economy. Since noisy signals are idiosyncratic, the firms that update their information set will have heterogenous information about the state (as in Morris and Shin (2002) and Angeletos and Pavan (2007)). Hence, in our model, *heterogenous* information disseminates slowly in the economy.

As individual prices depend on the current state and the aggregate price level, firms that update their information sets must not only form beliefs about the current state but also form beliefs about the other firms' beliefs about current the state, and so on and so forth. Hence, the pricing decisions by firms induce an incomplete information game among them, and we prove that there exists a unique equilibrium of such game. This allows us to unequivocally speak about the sticky-dispersed-information (SDI) aggregate price level and Phillips curve. The SDI aggregate price level we derive depends on all the prices firms have set in the past. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the far past. This is a *direct* effect of sticky information. Second, even firms that have just adjusted their information set will be, at least partly, backward-looking. This happens because of an *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices

From aggregate prices, we are able to derive the SDI Phillips curve. It is immediate that, since current aggregate prices depend on all prices set by firms in the past, the current inflation rate will also depend on inflation rates that prevailed in the past. Therefore, in spite of the fact that firms are forward looking in our model, the Phillips curve that results from their interaction displays a non-trivial dependence on inflation rates that prevailed in the past. This is an implication of the stickiness of information in our model and was already present in Mankiw and Reis (2002).

In our model, however, on top of being sticky, information is also *disperse*. The effect of dispersion is captured by the positive weight given to the state from periods $t-j$, $j > m$, by a firm that has its information set updated in $t-m$. As the private signal the firm observes is noisy, it is always optimal to place some weight on past states to forecast the current state. Hence, in comparison to the economy described in Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with disperse information.

Our model nests as special cases the complete information model, the dispersed information model and the sticky information model. To better understand the roles played by information stickiness and dispersed information, we also decompose our SDI Phillips curve into function of the three benchmark inflation rates that can be obtained as limiting cases of our model: (i) complete-information inflation, (ii) dispersed-information inflation, and (iii) sticky-information inflation.

We study the individual contribution to the SDI Phillips curve of each of the main parameters of our model: (i) Degree of strategic complementarity, (ii) Degree of informational stickiness, (iii) Public information precision, and (iv) Private information precision.. First, we analyze the impact of current and past complete-information inflation rates on current SDI inflation. Second, we consider the inflation response to shocks. Finally, we compare the variance of SDI inflation with the variances of complete-information inflation, dispersed-information inflation, and sticky-information inflation.

On top of the effects discussed above, the introduction of dispersed information in an otherwise standard sticky-information model sheds light on two different issues. First, dispersion in an sticky- information setting generates price and inflation inertia irrespective of assumptions regarding the firms' capacity to predict equilibrium outcomes. Indeed, although they may not have their information sets up to date, the firms in our model correctly predict the equilibrium behavior of their opponents. In spite of correctly predicting the *strategies* (i.e., contingent plans) adopted by the opponents in equilibrium, a firm cannot infer what is the actual price set by them (i.e., the action taken), since it cannot observe its opponents' private signals. Hence, a firm that hasn't updated its information set cannot infer the current state from the behavior of its opponents. This is in contrast to Mankiw and Reis (2002) who, in order to obtain price and information inertia in a model with sticky but non-dispersed information, (implicitly) assume that agents cannot condition on equilibrium behavior from the opponents. In fact, in their main experiment, there is a (single) nominal shock that only a fraction of the firms observe. Trivially, the prices set by those firms (as well as aggregate prices) will reflect such change in the fundamental. Hence, a firm that hasn't observed the shock but can predict the equilibrium behavior of the opponents will be able to infer the fundamental from such behavior.[1] It follows that *all* firms will adjust prices in response.

The second issue relates to policy. In a world in which information is dispersed, for a benevolent central banker who has (imperfect) information about the state, the optimal communication policy is far from trivial. On the one hand, any information disclosed by the central banker about the state will have the benefit of allowing the agents to count on an additional piece of information about the state when deciding on their prices. This benefit is particularly relevant when information is sticky for a fraction of firms is setting prices based on outdated information about the current state. On the other hand, since the information disclosed by the central banker is a public signal, agents will place too much weight on any information disclosed by

[1]The argument here is similar to the one in Rational Expectations Equilibrium models *à la* Grossman (1981).

the central banker as this is a public signal (e.g., Morris and Shin (2002), Angeletos and Pavan (2007). We believe the model we put forth in this paper is a suitable framework to study optimal communication policy by central banks when information is heterogenous and sticky.[2]

Related Literature. This work follows a growing number of papers that sheds new insights into the long-tradition literature of price setting under imperfect information that dates back to Phelps (1968) and Lucas (1972). This paper makes no attempt to survey this literature. The reader is referred to Mankiw and Reis (2002) for the most recent survey of aggregate supply under imperfect information and Veldkamp (2009) for an extensive coverage of the topics regarding information choice in macroeconomics and finance. As already mentioned, this paper will, however, follow two distinct lines of research regarding informational frictions. From one hand, information in our model is sticky, following Mankiw and Reis (2002) and related work.[3] From the other hand, we follow Woodford (2002), Morris and Shin (2002), and subsequent work, and also considers that information is dispersed.[4]

The works that most ressembles ours are Angeletos and La'O (2009)(1) and Mankiw and Reis (2010)(82). Mankiw and Reis (2010), while offering the most recent survey of this literature, compare a partial (dispersed) information model with a delayed (sticky) information model and derive their common implications.[5] Angeletos and La'O (2009) also considers dispersed information, but merges it with sticky prices *à la* Calvo (1983). In doing so, the authors highlighs the role of higher-order believes in the formalization of their model.

This paper, instead, contributes to this literature by explicit formalizing the solution of a model where information is *both* sticky and dispersed as a function of higher-order beliefs, offering the first integrated approach to analyze the interactions of these two of the most debated forms of informational frictions.

[2]In a companion paper, Areosa et al. (2010a), we incorporate a policy signal in our SDI model to analyze the impact of central bank communication on price setting and their implications on welfare.

[3]See, for example, Carroll (2003), Mankiw et al. (2004), Dupor and Tsuruga (2005), Mankiw and Reis (2006, 2007, 2010), Carvalho and Schwartzman (2008), Crucini et al. (2008), and Curtin (2009).

[4]Examples are Bacchetta and van Wincoop (2006), Hellwig (2008), Angeletos and Pavan (2007), Angeletos and La'O (2009), Hellwig and Veldkamp (2009), Hellwig and Venkateswaran (2009), Lorenzoni (2009, 2010), and Woodford (2009).

[5]The theories of "rational inattention" proposed by Sims (2003, 2009) and "inattentiveness" proposed by Reis (2006a, 2006b) have been used to justify models of dispersed information and sticky information.

Organization. The paper is organized as follows. In section 3.2, the set-up of the model is described. In section 3.3, we derive the unique equilibrium of the pricing game played by the firms, and derive the implied aggregate price and inflation rate. In section 3.4, we compare our SDI Phillips curve with three benchmarks: the complete information, the sticky-information and the dispersed information Phillips curves. Section 3.5 calibrates our SDI Phillips curve for different values of the main parameters of the model. Section 3.6 draws the concluding remarks. All derivations that are not in the text can be found in the Appendix.

3.2
The Model

The model is a variation of Mankiw and Reis (2010) sticky information model.[6] There is a continuum of firms, indexed by $z \in [0,1]$, that set prices at every period $t \in \{1,2,...\}$. Although prices can be re-set at no cost at each each $t \in \{1,2,...\}$, information regarding the state of the economy is made available to the firms infrequently. At period t, only a fraction λ of firms is selected to update their information sets about the current state. For simplicity, the probability of being selected to adjust information sets is the same across firms and independent of history.

We depart from this standard sticky-information model by allowing information to be *heterogeneous* and *dispersed*: a firm that updates its information set receives public information regarding the past states of the economy as well as a *private* signal about the current state.

Pricing Decisions: Every period t, each firm z chooses its price $p_t(z)$. We can derive from a model of monopolistic competition in the spirit of Blanchard and Kiyotaki (1987) that the (log-linear) price decision that solves a firm's profit maximization problem, p_t^*, is the same for all firms and given by

$$p_t^* = rP_t + (1-r)\,\theta_t, \qquad (3\text{-}1)$$

where $P_t \equiv \int_0^1 p_t(z)\,dz$ is the aggregate price level and θ_t is the nominal aggregate demand, the current state of the economy.

Information: Every firm z knows that the state θ_t follows a random walk

$$\theta_t = \theta_{t-1} + \epsilon_t, \qquad (3\text{-}2)$$

[6]Subsequent refinements of the sticky information models can be found in Mankiw and Reis (2006, 2007, 2010) and Reis (2006, 2006b, 2009).

with $\epsilon_t \sim N(0, \alpha^{-1})$. If firm z is selected to update its information set in period t, it observes all *previous* periods realizations of the state, $\{\theta_{t-j}, j \geq 1\}$. Moreover, it obtains a noisy private signal about the current state. Denoting such signal by $x_t(z)$, we follow the literature and assume:

$$x_t(z) = \theta_t + \xi_t(z), \tag{3-3}$$

where $\xi_t(z) \sim N(0, \beta^{-1})$, β is the precision of $x_t(z)$, and the error term $\xi_t(z)$ is independent of ϵ_t for all z, t.

As a result, if one defines

$$\Theta_{t-j} = \{\theta_{t-k}\}_{k=j}^{\infty}, \tag{3-4}$$

at period t, the information set of a firm z that was selected to update its information j periods ago is

$$\Im_{t-j}(z) = \{x_{t-j}(z), \Theta_{t-j-1}\}. \tag{3-5}$$

3.3 Equilibrium

Using (3-1), the best response for a firm z that was selected to update its information j periods ago is the forecast of p_t^* given its information set $\Im_{t-j}(z)$:

$$p_{j,t}(z) = E[p_t^* \mid \Im_{t-j}(z)]. \tag{3-6}$$

Denoting by Λ_{t-j} the set of firms that last updated its information set at period $t-j$, we can express the aggregate price level P_t as

$$\begin{aligned} P_t &= \int_{\cup_{j=0}^{\infty}\Lambda_{t-j}} p_t(z)\, dz \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E[p_t^* \mid \Im_{t-j}(z)]\, dz. \end{aligned} \tag{3-7}$$

Since the optimal price p_t^* is, according to (3-1), a convex combination of the state θ_t and the aggregate price level P_t, firm z needs to forecast the state of the economy *and* the pricing behavior of the other firms in the economy. The pricing behavior of each of these firms, in turn, depends on their own forecast of the other firms' aggregate behavior. It follows that firm z must not only forecast the state of the economy but also, to predict the behavior of the other firms in the economy, must make forecasts of these firms' forecasts about the state, forecasts about the forecasts of these firms forecasts about the state, and so on and so forth. In other words, higher-order beliefs will play a key role in the derivation of an equilibrium in our model.

Indeed, if one defines the average k-th order belief about the current state recursively as follows:

$$\bar{E}^k[\theta_t] = \begin{cases} \theta_t, & : k = 0, \\ \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[\bar{E}^{k-1}[\theta_t] \mid \Im_{t-j}(z)\right] dz, & : k \geq 1, \end{cases} \tag{3-8}$$

we can express the equilibrium aggregate price level as

$$P_t = (1-r) \sum_{k=1}^{\infty} r^{k-1} \bar{E}^k [\theta_t]. \tag{3-9}$$

3.3.1 Computing the Equilibrium

In this section, we derive the unique equilibrium of the pricing game played by the firms. Following Morris and Shin (2002), we do this in two steps. We first derive an equilibrium for which the aggregate price level is a linear function of fundamentals. We then establish, using (3-9), that this linear equilibrium is the unique equilibrium of our game.

Expectations

In the Appendix, we show that, given the distribution of the private signals and the process $\{\theta_t\}$ implied by (3-2), a firm z that updated its information set in period $t-j$ makes use of the variables $x_{t-j}(z) = \theta_{t-j} + \xi_{t-j}(z)$ and $\theta_{t-j-1} = \theta_{t-j} - \epsilon_{t-j}$, to form the following belief about the current state θ_{t-j}:

$$\theta_{t-j} \mid \Im_{t-j}(z) \sim N\left((1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1}, (\alpha+\beta)^{-1}\right), \tag{3-10}$$

where

$$\delta \equiv \frac{\alpha}{\alpha+\beta} \in (0,1). \tag{3-11}$$

Hence, a firm that updated its information set in $t-j$ expects the current state to be a convex combination of the private signal $x_{t-j}(z)$ and a (semi) public signal θ_{t-j-1} – the only relevant piece of information that comes from learning all previous states $\{\theta_{t-j-k}\}_{k\geq 1}$.[7] The relative weights given to $x_{t-j}(z)$ and θ_{t-j-1} when the firm computes the expected value of state θ_{t-j} depend on the precisions of such signals.

Using (3-2), one has that, for $m \leq j$,

$$\theta_{t-m} = \theta_{t-j} + \sum_{k=0}^{j-m-1} \epsilon_{t-m-k}. \tag{3-12}$$

Thus, the expectation of a firm z that last updated its information set at $t-j$ about θ is

[7] θ_{t-j-1} is the only piece of information in $\Theta_{t-j} = \{\theta_{t-j-k}\}_{k=1}^{\infty}$ the firm needs to use because the state's process is Markovian.

$$E\left[\theta_{t-m} \mid \Im_{t-j}(z)\right] = \begin{cases} E\left[\theta_{t-j} \mid \Im_{t-j}(z)\right] = (1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1} & : m \leq j, \\ \theta_{t-m} & : m > j. \end{cases} \tag{3-13}$$

In words, a firm that last updated its information set in period $t-j$ expects that all future values of the fundamental θ will be the same as the expected value of the fundamental at the period $t-j$. Moreover, since at the moment it adjusts its information set the firm observes all previous states, the firm will know for sure the value of θ_{t-m} for $m > j$.

Linear Equilibrium

To derive the linear equilibrium, we adopt a standard guess and verify approach. We assume that the (equilibrium) aggregate price level is linear and then show that the implied best responses for the individual firms indeed lead to linear aggregate prices.

Toward that, assume that

$$P_t = \sum_{j=0}^{\infty} c_j \theta_{t-j}. \tag{3-14}$$

for some constants c_j, $j \geq 0$.

In such case, the optimal price for a firm that last updated information at $t-m$ is

$$\begin{aligned} p_t &= E\left[(1-r)\,\theta_t + rP_t \mid \Im_{t-m}\right] \\ &= (1-r)\, E\left[\theta_t \mid \Im_{t-m}\right] + r\sum_{j=0}^{\infty} c_j E\left[\theta_{t-j} \mid \Im_{t-m}\right] \\ &= (1-r)\, E\left[\theta_t \mid \Im_{t-m}\right] + r\sum_{j=0}^{m} c_j E\left[\theta_{t-j} \mid \Im_{t-m}\right] + r\sum_{j=m+1}^{\infty} c_j E\left[\theta_{t-j} \mid \Im_{t-m}\right] \\ &= \left[1 - r\left(1 - C_m\right)\right]\left[(1-\delta)\, x_{t-m} + \delta\theta_{t-m-1}\right] + r\sum_{j=m+1}^{\infty} c_j \theta_{t-j} \\ &= (1-\delta)\left[1 - r\left(1 - C_m\right)\right] x_{t-m} + \delta\left[1 - r\left(1 - C_{m+1}\right)\right]\theta_{t-m-1} + r\sum_{j=m+2}^{\infty} c_j \theta_{t-j}, \end{aligned}$$

where

$$C_m \equiv \sum_{j=0}^{m} c_j.$$

Aggregating such individual prices and using (3-7), we get

$$\begin{aligned} P_t &= \sum_{m=0}^{\infty} \int_{\Lambda_{t-m}} \left[1 - r\left(1 - C_m\right)\right]\left[(1-\delta)\, x_{t-m} + \delta\theta_{t-m-1}\right] + r\sum_{j=m+1}^{\infty} c_j \theta_{t-j} dz \\ &= \lambda\sum_{m=0}^{\infty} (1-\lambda)^m \left\{\left[1 - r\left(1 - C_m\right)\right]\left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right] + r\sum_{j=m+1}^{\infty} c_j \theta_{t-j}\right\} \\ &= \lambda\sum_{m=0}^{\infty} (1-\lambda)^m \left\{\left[1 - r\left(1 - C_m\right)\right]\left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right]\right\} \\ &+ r\sum_{m=0}^{\infty} c_m \left[1 - (1-\lambda)^m\right]\theta_{t-m}. \end{aligned}$$

Note that the above equality can be re-written as

$$(1-r)\,P_t = \lambda\,(1-\delta)\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-r\,(1-C_m)\right]\theta_{t-m}$$
$$+\,\lambda\delta\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-r\,(1-C_m)\right]\theta_{t-m-1}$$
$$-\,r\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m c_m\theta_{t-m},$$

so that the implied aggregate price will be linear in the values of the fundamental, as assumed in (3-14). Matching coefficients, we obtain

$$c_k \equiv \begin{cases} \frac{\lambda(1-\delta)(1-r)}{1-r\lambda(1-\delta)} & \text{if } k=0, \\ \frac{\lambda(1-r)\phi(1-\lambda)^{k-1}}{\left[1-r\left[1-\phi(1-\lambda)^{k-1}\right]\right]\left[1-r\left[1-\phi(1-\lambda)^{k}\right]\right]} & \text{if } k\geq 1, \end{cases} \tag{3-15}$$

where

$$\phi \equiv 1-\lambda\,(1-\delta)\,,$$
$$C_\infty \equiv \lim_{m\to\infty}\textstyle\sum_{j=0}^{m}c_j = 1.$$

We have then shown:

Proposition 16 (Linear Equilibrium) *There exists an equilibrium in which the aggregate price level in period* t*,* P_t*, are linear in the states* $\{\theta_{t-j}\}_{j=0}^{\infty}$.

To compare our equilibrium aggregate price level with the one obtained within a standard sticky-price framework, we can also write P_t as a function of the shocks ϵ:

$$P_t = \textstyle\sum_{k=0}^{\infty}\Phi_k\epsilon_{t-k}, \tag{3-16}$$

where the coefficients

$$\Phi_k \equiv \frac{(1-r)\left[1-\phi\,(1-\lambda)^k\right]}{1-r\left[1-\phi\,(1-\lambda)^k\right]},$$

converts to the same coefficients obtained by Mankiw and Reis (2010) in a sticky-price model if we ignore dispersion and set $\delta = 0$ ($\phi = 1-\lambda$). If we derive (3-16) with respect to δ:

$$\frac{\partial P_t}{\partial\delta} = \textstyle\sum_{k=0}^{\infty}\frac{\partial\Phi_k}{\partial\delta}\epsilon_{t-k},$$

where

$$\frac{\partial\Phi_k}{\partial\delta} \equiv -\frac{(1-r)\,(1-\lambda)^k\,\lambda}{\left[1-r\left[1-\phi\,(1-\lambda)^k\right]\right]^2} < 0,$$

we obtain that dispersion *decreases* the sensibility of prices to fundamental's shocks.

Uniqueness of Equilibrium: Beliefs

As shown in (3-9), an alternative way to describe the aggregate price level in period t is through a weighed average of all (average) higher-order beliefs about the state θ_t. In this section, we derive such beliefs and establish that the implied aggregate price level will be identical to the one derived in Proposition 16. This will establish that the linear equilibrium is unique.

First-Order Beliefs: Using (3-13), we are able to compute (3-8) for the case $k = 1$.

$$\bar{E}^1 [\theta_t] = \lambda \textstyle\sum_{j=0}^{\infty} (1-\lambda)^j [(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}] . \tag{3-17}$$

Higher-Order Beliefs: In the Appendix, we use (3-17) and the recursion (3-8) to derive the following useful result:

Lemma 17 (Higher-Order Beliefs) *The average k-th order forecast of the state is given by*

$$\bar{E}^k [\theta_t] = \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m [\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}] , \tag{3-18}$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ recursively defined for $k \geq 1$

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} [1-(1-\lambda)^m]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta\left[1-(1-\lambda)^m\right]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

and the initial weights are $(\kappa_{1,k}, \delta_{1,k}) \equiv (1-\delta, \delta)$.

Plugging (3-18) into the expression for the aggregate price level P_t, (3-9), we get, after a few manipulations, the following expression for the aggregate price level:

$$P_t = \textstyle\sum_{m=0}^{\infty} K_m [(1-\Delta_m)\theta_{t-m} + \Delta_m\theta_{t-m-1}] , \tag{3-19}$$

where the weights K_m and Δ_m are

$$K_m \equiv \frac{(1-r)\lambda(1-\lambda)^m}{(1-r[1-(1-\lambda)^m])\left(1-r\left[1-(1-\lambda)^{m+1}\right]\right)},$$
$$\Delta_m \equiv \frac{\delta[1-r[1-(1-\lambda)^m]]}{1-r\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right]+\delta[1-(1-\lambda)^m]\right]}.$$

Comparing the coefficients above with the $\{c_j\}_{j=0}^{\infty}$ defined in (3-15), for

$$\begin{aligned} &c_0 \text{ with } K_0 (1 - \Delta_0), \\ &c_k \text{ with } K_{m-1}\Delta_{m-1} + K_m (1 - \Delta_m), \ m \geq 1, \end{aligned}$$

one sees that the aggregate price level implied by (3-19) is exactly the same as the one derived in Proposition 16.

Having shown that the equilibrium is unique, we can unequivocally speak about *the* Philips curve of our economy. Denoting the inflation rate by π_t, by taking first differences of equation (3-19), we can write our **sticky-dispersed information Phillips curve** as

$$\pi_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1 - \Delta_m)(\theta_{t-m} - \theta_{t-m-1}) + \Delta_m (\theta_{t-m-1} - \theta_{t-m-2})\right]. \quad (3\text{-}20)$$

We summarize all the discussion above in the following result:

Proposition 18 (SDI Phillips Curve) *In an economy in which information is sticky and dispersed, and the state follows (3-2), there is a unique equilibrium in the pricing game played by the firms. In such equilibrium, the aggregate price level is given by*

$$P_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1 - \Delta_m)\theta_{t-m} + \Delta_m \theta_{t-m-1}\right], \quad (3\text{-}21)$$

and the SDI Phillips curve is given by

$$\pi_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1 - \Delta_m)(\theta_{t-m} - \theta_{t-m-1}) + \Delta_m (\theta_{t-m-1} - \theta_{t-m-2})\right], \quad (3\text{-}22)$$

where

$$K_m \equiv \frac{(1-r)\lambda(1-\lambda)^m}{(1 - r[1-(1-\lambda)^m])\left(1 - r\left[1-(1-\lambda)^{m+1}\right]\right)}, \quad (3\text{-}23)$$

$$\Delta_m \equiv \frac{\delta[1 - r[1-(1-\lambda)^m]]}{1 - r\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta[1-(1-\lambda)^m]\right]}. \quad (3\text{-}24)$$

Note that the current aggregate price level P_t depends on all the prices firms have set in the past. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the far past. This is a *direct* effect of sticky information. Second, even firms that have just adjusted their information set will be, at least partly, backward-looking. This happens because of an *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices. The direct and strategic effects of sticky information are captured by the terms K_m.

It is immediate that, since current aggregate prices depend on all prices set by firms in the past, the current inflation rate will also depend on inflation rates that prevailed in the past. Therefore, in spite of the fact that firms are forward looking in our model, the Philips curve that results from their interaction displays a non-trivial dependence on inflation rates that prevailed in the past. This is an implication of the stickiness of information in our model and was already present in Mankiw and Reis (2002).

In our model, however, on top of being sticky, information is also *disperse*. The effect of dispersion is captured by the positive weight given to the state in period θ_{t-m-1} by a firm that has its information set updated in $t-m$. As the private signal the firm observes is noisy, it is always optimal to place some weight on past states to forecast the current state. Hence, in comparison to an economy *à la* Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with disperse information.

Also, and perhaps more importantly, the introduction of dispersion in an sticky information model allows us to generate price and inflation inertia irrespective of assumptions regarding the firms' capacity to predict equilibrium outcomes. Indeed, although they may not have their information sets up to date, the firms in our model correctly predict the equilibrium behavior of their opponents. In spite of correctly predicting the *strategies* (i.e., contingent plans) adopted by the opponents in equilibrium, a firm cannot infer what is the actual price set by them (i.e., the action taken), since it cannot observe its opponents' private signals. Hence, a firm that hasn't updated its information set cannot infer the current state from the behavior of its opponents.

This is in contrast to Mankiw and Reis (2002) who, in order to obtain price and information inertia in a model with sticky but non-dispersed information, (implicitly) assume that agents cannot condition on equilibrium behavior from the opponents. In fact, in their main experiment, there is a (single) nominal shock that only a fraction of the firms observe. Trivially, the prices set by those firms (as well as aggregate prices) will reflect such change in the fundamental. Hence, a firm that hasn't observed the shock but can predict the equilibrium behavior of the opponents will be able to infer the fundamental from such behavior.[8] It follows that *all* firms will adjust prices in response.

[8]The argument here is similar to the one in Rational Expectations Equilibrium models *à la* Grossman (1981).

3.4
Benchmarks for the SDI Phillips Curve

Our model nests the dispersed information model ($\lambda = 1$) and the sticky information model ($\beta^{-1} \to 0$) as special cases. In order to understand the properties of the SDI Phillips curve, in what follows, we compare it to those two benchmarks as well as to the inflation rate implied by the complete information case.

3.4.1
Benchmark 1: Complete-Information Inflation

Under complete information, the price of any firm z is

$$p_t(z) = p_t^* \equiv rP_t + (1-r)\,\theta_t.$$

Since firms are identical, they all set the same price. As a result

$$P_t = rP_t + (1-r)\,\theta_t \Rightarrow P_t = \theta_t.$$

Hence, if θ is common knowledge, the equilibrium entails an inflation rate $\pi_{C,t}$ – that we call the *complete-information inflation* – that is equal to the variation in the state:

$$\pi_{C,t} = \theta_t - \theta_{t-1}. \tag{3-25}$$

3.4.2
Benchmark 2: Dispersed-Information Inflation

If stickiness vanishes ($\lambda = 1$), our result converges to the ones obtained by Morris and Shin (2002) and Angeletos and Pavan (2007). Denoting the inflation rate for the economy without stickiness by $\pi_{D,t}$ (the dispersed information inflation), we have:

$$\pi_{D,t} = (1-\Delta)\,\pi_{C,t} + \Delta\pi_{C,t-1}, \tag{3-26}$$

so that the inflation rate in period t is a convex combination of the complete information inflations of period t and $t-1$, with the weight on period $t-1$ complete information inflation given by

$$\Delta = c_1 \equiv \frac{\delta}{1-r(1-\delta)}, \tag{3-27}$$

$1-\Delta = c_0$, and $c_k = 0$, $\forall\ k > 1$.[9]

[9] Alternatively, as in Morris and Shin (2002), we can say that inflation in t is a convex combination of the "state/fundamental", $\pi_{C,t}$, and the "public signal", $\pi_{C,t-1}$.

When compared to the full information case, the inflation rate that prevails under dispersed information displays more inertia. Moreover, note that

$$E\left[\pi_{D,t} \mid \Im_t(z)\right] = (1-\Delta) E\left[\pi_{C,t} \mid \Im_t(z)\right] + \Delta \pi_{C,t-1}.$$

Hence, when information is dispersed, the forecast error

$$\pi_{D,t} - E\left[\pi_{D,t} \mid \Im_t(z)\right] = (1-\Delta)\left[\pi_{C,t} - E\left[\pi_{C,t} \mid \Im_t(z)\right]\right]$$

is proportional to the forecast error of the complete information inflation $\pi_{C,t}$.

3.4.3
Benchmark 3: Sticky-Information Inflation

The other polar case occurs when information is sticky but not dispersed ($\delta = 0$). In such case, the Phillips curve we obtain resembles the one in Mankiw and Reis (2002). Denoting the sticky information inflation by $\pi_{S,t}$, we have

$$\pi_{S,t} = \textstyle\sum_{m=0}^{\infty} K_m \pi_{C,t-m}, \tag{3-28}$$

where inflation is also a function of current and past complete-information inflation, but with the weights K_m in (3-23) replacing the coefficients c_m defined in (3-15). Note that, for $m = 0$

$$c_0 \equiv \frac{(1-r)\lambda(1-\delta)}{1-r\lambda(1-\delta)} < \frac{(1-r)\lambda}{1-r\lambda} \equiv K_0$$

because

$$\frac{\partial c_0}{\partial \delta} \equiv \frac{-(1-r)\lambda}{\left[1-r\lambda(1-\delta)\right]^2} < 0.$$

3.4.4
Benchmark Contributions to SDI Inflation

We can rewrite our SDI Phillips curve as a combination of the inflation rates that prevail under the three benchmarks cases discussed above. First, note that the SDI inflation π is a function of complete information inflations π_C of current and previous periods. Indeed, using (3-14) or (3-21), we obtain

$$\begin{aligned} \pi_t &= \textstyle\sum_{j=0}^{\infty} c_j \pi_{C,t-j} \\ &= \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m)\pi_{C,t-m} + \Delta_m \pi_{C,t-m-1}\right]. \end{aligned} \tag{3-29}$$

Using (3-28) and (3-29), we can also relate the SDI inflation to the sticky-information inflation π_S as follows:

$$\pi_t = \pi_{S,t} - \sum_{m=0}^{\infty} K_m \Delta_m \left(\pi_{C,t-m} - \pi_{C,t-m-1}\right).$$

Finally, if we combine this last equation with (3-26), we obtain a decomposition of SDI inflation that includes all the proposed benchmarks

$$\pi_t = \pi_{S,t} + \sum_{m=0}^{\infty} K_m \left(\frac{\Delta_m}{\Delta}\right) \left[\pi_{D,t-m} - \pi_{C,t-m}\right]. \quad (3\text{-}30)$$

Thus, compared to the case in which information is sticky, inflation under sticky *and* dispersed information will be higher if, and only if, dispersed information inflation, $\pi_{D,t-m}$, is on "average" higher than complete information inflation $\pi_{C,t-m}$.

3.5 Inflation Behavior under SDI

Having derived the SDI Phillips curve, we now examine how it behaves in response to changes in the main parameters of the model. Making use of the fact that we can write the SDI inflation as a weighted average of all past complete information inflation rates, we start in Figure 3.1 by analyzing the impact of period $t - k$ complete information inflation $\pi_{C,t-k}$ on SDI current inflation π_t. Afterwards, we consider the inflation response to shocks in Figure 3.2. Finally, in Figure 3.3, we consider the behavior of SDI's inflation variance as well as the variances of the three benchmarks considerd in Section 3.4: complete-information inflation, dispersed-information inflation, and sticky-information inflation.

To isolate effects, we perform each of the above exercises for different values of the key parameters of the model as listed in Table 3.1: (a) Strategic complementarity r, (b) Information stickiness λ, (c) Public information precision α, and (d) Private information precision β.

3.5.1 Calibration

The model's structural parameters are r, λ α, and β. The baseline values we use for r and λ (see Table 3.1) are standard and based on Mankiw and Reis (2002). A value of $\lambda = 0.25$ can be interpreted as implying that, on average, firms adjust their information set (and therefore their prices) once a year. This is compatible with the most recent microeconomic evidence on

Table 3.1: Baseline calibration

Parameter	Description	Range	Benchmark Value
r	Degree of strategic complementarity	$[0, 1]$	0.90
λ	Degree of informational stickiness	$[0, 1]$	0.25
α	Public information precision	$[0, 1]$	0.50
β	Private information precision	$[0, 1]$	0.50

price-setting.[10] The higher the value of r, the more important becomes the aggregate price level (and therefore the strategic interaction component) for (of) the firms's optimal price. We set $\alpha = \beta = 0.5$ as our benchmark value to keep the baseline calibration as neutral as possible regarding the importance of public versus private information precision.

To better understand the impact of each individual parameter on the SDI Philips curve, in what follows, we always keep three of the four key parameters fixed at their benchmark values and vary the fourth one.

3.5.2 Impact of Complete Information Inflation

We first consider the impact of period $t-k$ complete information inflation $\pi_{C,t-k}$ on the current SDI inflation π_t. Using equation (3-29), one can readily see that such impact is fully captured by the coefficients $c_j's$ in Equation (3-15). We plot the results in Figure 3.1, where each panel shows the effect of changes in one of the four parameters of the model.

Consider Panel (a) of Figure 3.1. The weight on the current complete information inflation is higher the lower the degree of strategic complementarity, r. As the degree of strategic complementarity rises, the incentive for firms to align prices increases. As a result, even informed firms will attach a higher weight on past information. This leads to a higher impact of *past* complete information on *current* SDI inflation.

Panel (b) of Figure 3.1 captures the role of informational stickiness on the impact of past full information inflation rates on current SDI inflation. It can be seen that higher values of λ (i.e., smaller degrees of information stickiness) are related to lower weights on past complete information inflation. As the degree of information stickiness increases, however, the share of SDI inflation that comes from the past is higher, since firms have incentives to align prices

[10]See, for example, Klenow and Malin (2010).

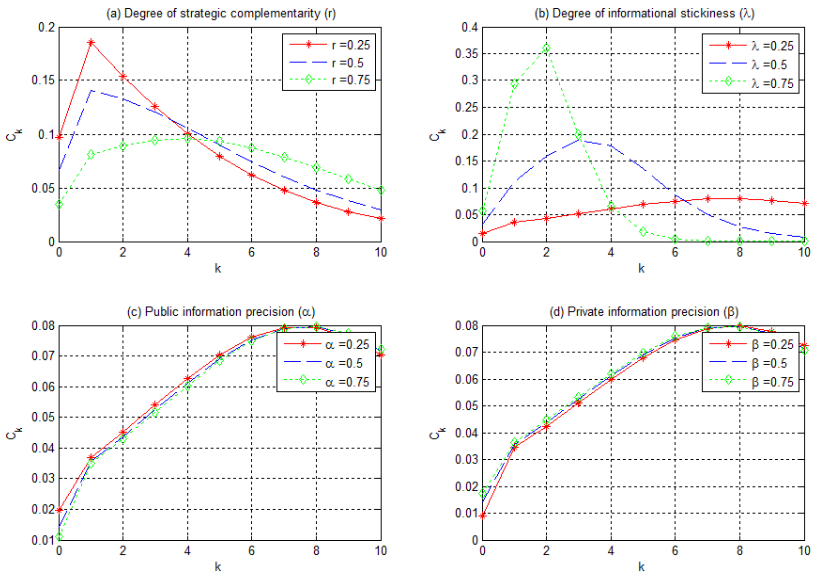

Figure 3.1: Coefficients $c_j's$ for different values of the parameters $(r, \lambda, \alpha, \beta)$.

and, the lower λ, the larger the faction of price setters that are stuck with past information about the state.

The impact of information dispersion on SDI inflation is shown in Panels (c) and (d) of Figure 3.1. Firms attach more weight on a given piece of information the more precise it is. Consider an increase in public information precision α or a decrease in private information precision β. As a result, as $\delta \equiv \alpha/(\alpha+\beta)$ increases, firms attach more weight to the past since, the larger δ, the more (relatively to their private information) the firms can be confident about past fundamentals being a good source of information about the current fundamental.

3.5.3
Impulse Response Functions

Figure 3.2 shows the impulse responses of current SDI inflation, π_t, to a shock in the fundamental process $\{\epsilon_t\}$ in (3-2).

From Panel (a) of Figure 3.2, we observe that, as r increases, inflation becomes more inertial. When $r = 0$, the firms' desired prices respond only to the value of the fundamental, θ. In such case, inflation responds quickly to the shock. By contrast, when $0 < r < 1$, firms also care about the overall price level and, therefore, need to consider what information other firms have. In the SDI model, as well as in the sticky-information model, this strategic complementarity in prices is a source of inflation inertia.

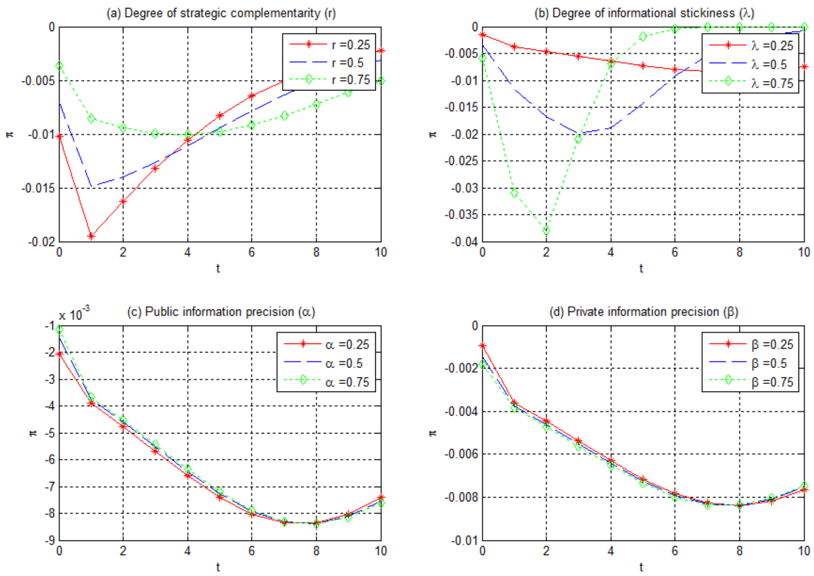

Figure 3.2: Responses of π_t to a shock in the fundametal process ϵ_t for different values of $(r, \lambda, \alpha, \beta)$.

Panel (b) of Figure 3.2 considers the impact of information stickiness on inflation dynamics. For higher values of λ (smaller degree of information stickiness), inflation not only responds more quickly to a shock in the fundamental but also returns to its pre-shock levels at a faster rate.

Finally, Panels (c) and (d) of Figure 3.2 shows the impact of information dispersion on SDI inflation. Once again, remember that $\delta \equiv \alpha/(\alpha + \beta)$ increases with higher values of public information precision α or lower values of private information precision β. Higher values for δ imply that previous values of θ are relatively more precise signals of the state than the firm's private information. As a result, for large δ, even firms that update their information sets *at* the moment of the shock respond less to such new piece of information.

Also, for a given δ, an additional strategic effect leads the firms to place a larger weight on past information about the state. Indeed, a firm that wishes to align its price to other firms' prices relies more heavily on public information because it is a better predictor of other firms' prices than private information. This effect has been already pointed out by authors such as Morris and Shin (2002), Angeletos and Pavan (2007), and others in related contexts.

3.5.4 Inflation Variance

We now analyze the variance of inflation under SDI. Using equation (3-25), we obtain complete-information inflation variance

$$Var\left[\pi_{C,t}\right] = \alpha^{-1}.$$

From equations (3-26) and (3-28), we obtain the variances of dispersed-information inflation and sticky-information inflation

$$Var\left[\pi_{D,t}\right] = \left[(1-\Delta)^2 + \Delta^2\right] Var\left[\pi_{C,t}\right],$$
$$Var\left[\pi_{S,t}\right] = \kappa Var\left[\pi_{C,t}\right],$$

where Δ, defined in (3-27), is a function of (r, α, β) while

$$\kappa \equiv \sum_{j=0}^{\infty} K_j^2$$

is a function of (r, λ), as can be seen by the definition of K_j in (3-23).

Finally, from equation (3-29), we obtain the variance of SDI inflation

$$Var\left[\pi_t\right] = \Omega Var\left[\pi_{C,t}\right],$$

where Ω, which is a function of the parameters $(r, \lambda, \alpha, \beta)$, is given by

$$\Omega \equiv \sum_{j=0}^{\infty} c_j^2 \in (0, 1),$$

where the $c_j's$ are defined in (3-15).

Hence, the variance of the SDI inflation, $Var\left[\pi_t\right]$, is proportional to the variance of complete information inflation, $Var\left[\pi_{C,t}\right]$. A bit more surprising is the fact that the informational frictions we consider in the model reduce the variance of inflation when compared to the complete information benchmark. The reason is as follows. As discussed throughout the paper, the combination of sticky and dispersed information with strategic interdependence in price setting leads to inflationary inertia, which, in turn, reduces the variance of inflation under SDI.

Figure 3.3 plots $Var\left[\pi_t\right]$ as well as $Var\left[\pi_{C,t}\right]$, $Var\left[\pi_{D,t}\right]$, and $Var\left[\pi_{S,t}\right]$ as a function of $(r, \lambda, \alpha, \beta)$.[11]

As can be seen from Figure 3.3, the variances of complete-information inflation $Var\left[\pi_{C,t}\right]$ and dispersed-information inflation $Var\left[\pi_{D,t}\right]$ are always

[11] We use $\Omega_{\bar{k}} \equiv \sum_{j=0}^{\bar{k}} c_j^2$ and $\bar{\kappa} \equiv \sum_{j=0}^{\bar{k}} K_j^2$ rather than Ω and κ for computational reasons.

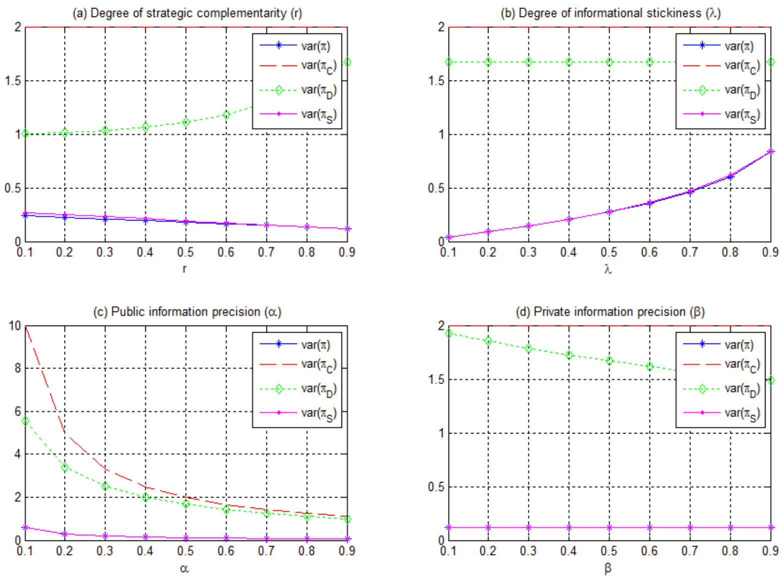

Figure 3.3: Variances of SDI inflation π_t, complete-information inflation $\pi_{C,t}$, dispersed-information inflation $\pi_{D,t}$, and sticky-information inflation $\pi_{S,t}$ as a function of $(r, \lambda, \alpha, \beta)$.

higher than SDI inflation $Var\left[\pi_t\right]$ and sticky-information inflation $Var\left[\pi_{S,t}\right]$. Besides, $Var\left[\pi_t\right]$ and $Var\left[\pi_{S,t}\right]$ are extremely similar and are only quantitatively affected by the degree of informational stickiness λ. For higher values of λ (smaller degree of information stickiness), $Var\left[\pi_t\right]$ and $Var\left[\pi_{S,t}\right]$ increase.

As the signals become more precise, more similar are the information sets of the firms. As a result, dispersed-information inflation $Var\left[\pi_{D,t}\right]$ decreases considerably as information precisions α and β increase. $Var\left[\pi_{D,t}\right]$ is also affected by the degree os strategic complementarity r. As r increases, more weight is given by a firm to its forecast about the forecast of the others, increasing $Var\left[\pi_{D,t}\right]$.

3.6 Conclusion

Costs to acquire and process information make its diffusion through the economy slow: i.e., information is sticky. Likewise, heterogeneity in the sources and interpretation of new information is likely to make relevant information about the economy dispersed across agents. In this paper, we have considered the impact of sticky and dispersed information on individual price setting decisions, and the resulting effect on the aggregate price level and the inflation rate.

Compared to a setting in which information is solely sticky as in Mankiw and Reis (2002), sticky *and* dispersed information always leads to non-trivial effects on prices regardless of assumptions about the agents' capability to predict equilibrium behavior by their opponents. Moreover, the effects of information on aggregate prices and inflation rates will be more pronounced: aggregate prices and inflation rates will be more inertial than their sticky information counterparts.

There are several interesting dimensions in which our model of price setting under SDI can be extended. Perhaps the most important one is to explore the policy implications of dispersed information. In a world in which information is dispersed, a benevolent central banker's optimal communication policy is far from trivial. On the one hand, any information disclosed by the central banker about the state will have the benefit of allowing the agents to count on an additional piece of information about the state when deciding on their prices. On the other hand, from a social perspective, agents will place too much weight on any information disclosed by the central banker as this is a public signal (e.g., Morris and Shin (2002) and Angeletos and Pavan (2007)). One can remedy this latter effect by setting a tax that corrects the incentives the agents have to "coordinate" on such public signal.

Our derivation of the equilibrium played by firms and the prevailing Phillips curve when information is sticky and dispersed is a necessary first step toward answering the policy questions suggested above. In fact, in a companion paper, Areosa et al. (2010a), we incorporate a policy signal in our SDI model to analyze the impact of central bank communication on price setting and their implications on welfare. In another paper, Areosa et al. (2010b), we consider the case when the public *signal* is also a policy *instrument* affecting fundamental dynamics.