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5 Appendixes

5.1 Appendix of Chapter 1

5.1.1 The Model of Huang and Liu (2001)

In the model economy, the production of a final consumption good requires N stages of processing. The production of each good at stage 1 requires labor services only, with a constant-returns-to-scale technology given by $Y_1(j) = H_1(j)$, where $H_1(j)$ is the labor input and $Y_1(j)$ is the output. The production of each good at stage $n \in \{2, \dots, N\}$ uses labor and all goods produced at the previous stage as inputs according to the following technology

$$Y_n(j) = \left[\int_0^1 Y_{n-1}(j, z)^{\frac{1}{\mu}} dz \right]^{r\mu} H_n(j)^{1-r},$$

where $Y_n(j)$ is the output of a stage- n firm of type j , $Y_{n-1}(j, z)$ is the input supplied to j by a stage- $(n-1)$ firm of type z , $\mu > 1$ is function of the elasticity of substitution between such goods, $H_n(j)$ is the labor input used by j , and $r \in (0, 1)$ is the share of composite of stage- $(n-1)$ goods in j 's production. Firms behave as imperfect competitors in their output markets and are price-takers in their input markets. Given the constant-returns-to-scale technologies, the unit cost is also the marginal cost and is firm-independent. The optimal price decision under flexible prices is just a mark-up over marginal costs

$$P_n(j) = \mu [\bar{r} (P_{n-1})^r (W)^{1-r}], \quad (5-1)$$

where $\bar{r} \equiv r^{-r} (1-r)^{-(1-r)}$, $P_{n-1} \equiv \left[\int_0^1 P_{n-1}(z)^{\frac{1}{1-\mu}} dz \right]^{1-\mu}$ is a price index for goods produced at stage $n-1$, and W is nominal wage per hour.

The representative household is infinitely lived and maximizes expected life utility. The following equation describes the labor supply decision of the household

$$\frac{W}{P_N} = C, \quad (5-2)$$

Table 5.1: Parameter Values

Parameter	Description	Interval
α	Private information precision	[0.10, 50.0]
β	Public or Semi-public information precision	[0.10, 50.0]
r	Share of goods produced at stage $(n - 1)$ in the production of stage n	[0.10, 0.90]

where consumption C is a Dixit and Stiglitz (1977) composite of the final-stage goods

$$C = \left[\int_0^1 Y_N(z)^{\frac{1}{\mu}} dz \right]^{\mu} \equiv Y_N, \quad (5-3)$$

where $Y_N(z)$ is a type z good produced at stage N . We combine equations (5-1), (5-2), and (5-3) to obtain

$$\frac{P_n(j)}{P_N} = \mu \left(\frac{P_{n-1}}{P_N} \right)^r (Y_N)^{1-r},$$

which log-linearized yields $p_n^* \equiv rP_{n-1} + (1-r)\theta$, where $\theta \equiv Y_N + P_N$ is nominal aggregate demand.

5.1.2 Simulation

We simulate the model in order to understand the evolution of semi-public information precision $\hat{\beta}_n$. The model's structural parameters are α , β , and r . For ease of reference, we group all primitive parameter definitions and values in Table 5.1. We avoid extreme values of r even though results don't change dramatically in these cases. We comment about the extreme cases $r = 0$ and $r = 1$ below.

$\hat{\beta}^*$ limit

Substitute the lag of the expression for $\hat{\lambda}_n$ at (1-13) in (1-12) to obtain a recursive expression for $\hat{\beta}_n$

$$\hat{\beta}_n = \frac{\left[\beta \left(\alpha + \hat{\beta}_{n-1} \right) \right]^2 \hat{\beta}_{n-1}}{\beta \left(\alpha + \hat{\beta}_{n-1} \right)^2 \hat{\beta}_{n-1} + \left[(\beta - r\alpha) \hat{\beta}_{n-1} + r\alpha\beta \right]^2}. \quad (5-4)$$

If we set $\hat{\beta}_n = \hat{\beta}_{n-1} = \hat{\beta}^*$, we obtain

$$\hat{\beta}^* = \frac{[\beta(\alpha + \hat{\beta}^*)]^2 \hat{\beta}^*}{\beta(\alpha + \hat{\beta}^*)^2 \hat{\beta}^* + [(\beta - r\alpha)\hat{\beta}^* + r\alpha\beta]^2},$$

which is a root of the third degree equation

$$f(X) \equiv (X)^3 + c_1(X)^2 + c_2X + c_3, \quad (5-5)$$

where

$$\begin{aligned} c_1 &\equiv \frac{\alpha}{\beta} [2\beta(1-r) + r^2\alpha] > 0, \\ c_2 &\equiv \alpha [\alpha(1-2r^2) - 2\beta(1-r)], \\ c_3 &\equiv -\alpha^2\beta(1-r^2) < 0. \end{aligned}$$

Considering the parameter values of Table 3.1, $\hat{\beta}_n$ decreases monotonically to $\hat{\beta}^*$, the unique positive root of (5-5) when stage n increases.

Extreme values of r

If $r = 1$, zero is necessarily one of the roots of $f(X)$ and thus we must define $\hat{\beta}^*$ as the unique strict positive root of (5-5). For values of r very close to zero, we need an extra condition regarding the exogenous precisions α , β , and r to guarantee monotonicity. If $r = 0$, for example, we need $\alpha \leq \frac{3}{4}\beta$. To see this, consider (5-5) when $r = 0$

$$f(X) \equiv (X)^3 + 2\alpha(X)^2 + \alpha(\alpha - 2\beta)X - \alpha^2\beta. \quad (5-6)$$

In order to $\hat{\beta}_n$ converges monotonically to $\hat{\beta}^*$, the first derivative of (5-4) must be non-negative on $\hat{\beta}^*$

$$\begin{aligned} \frac{\partial \hat{\beta}_n}{\partial \hat{\beta}_{n-1}} \Big|_{\hat{\beta}_{n-1}=\hat{\beta}^*} &= \frac{\beta^2((\hat{\beta}^*)^2 - \alpha^2)}{[(\alpha + \hat{\beta}^*)^2 + \beta\hat{\beta}^*]^2} \geq 0 \\ &\Rightarrow \hat{\beta}^* \geq \alpha. \end{aligned}$$

If we substitute $\hat{\beta}^* = \alpha$ in the third degree equation (5-6), we obtain the result

$$f(\alpha) = 0 \Rightarrow \alpha = \frac{3}{4}\beta.$$

5.2

Appendix of Chapter 2

5.2.1

Proofs of Results

Proof 19 (Proof of Result 12: ERPT under Complete Information)

If information is complete, the external price of a given exporter is

$$p_f^* = (1 - r^*)\theta^* + r^*P_F^*.$$

Because firms are identical, all set the same price. As a result,

$$P_F^* = (1 - r^*)\theta^* + r^*P_F^* \Rightarrow P_F^* = \theta^*,$$

and the price of an imported good is $p_f = P_F = e + \theta^*$. Now, consider the prices of domestic firms. Under complete information, all final goods' firms set the same price

$$p_h = (1 - r)\theta + r[e + \theta^*].$$

Setting $r = 1$ establishes the result

$$p_h = P_F = e + \theta^*.$$

Proof 20 (Proof of Result 13: Incomplete ERPT (Morris and Shin (2002)))

Under dispersed information, the optimal response for an exporter is

$$p_f^* = E[(1 - r^*)\theta^* + r^*P_F^* | \mathfrak{S}_f^*].$$

After iterating and writing $E[\theta^* | \mathfrak{S}^*]$ for the average expectation of θ^* across exporters, we have

$$p_f^* = (1 - r^*) \sum_{k=0}^{\infty} (r^*)^k E[E^k[\theta^* | \mathfrak{S}^*] | \mathfrak{S}_f^*]. \quad (5-7)$$

Given the signals, it is straightforward to compute the high-order beliefs. The average expectation of θ^* across exporters is

$$\begin{aligned} E[\theta^* | \mathfrak{S}^*] &\equiv \int_{f \in [0,1]} E[\theta^* | \mathfrak{S}_f^*] df \\ &= (1 - \lambda^*) \int_{f \in [0,1]} x_f^* df + \lambda^* \left[\int_{f \in [0,1]} y_f^* df - e \right] \\ &= (1 - \lambda^*)\theta^* + \lambda^*[\theta - e]. \end{aligned}$$

Now, the expected value of the average expectation for a given exporter is

$$\begin{aligned}
 E [E [\theta^* | \mathfrak{S}^*] | \mathfrak{S}_f^*] &= E [(1 - \lambda^*) \theta^* + \lambda^* [\theta - e] | \mathfrak{S}_f^*] \\
 &= (1 - \lambda^*) E [\theta^* | \mathfrak{S}_f^*] + \lambda^* [E [\theta | \mathfrak{S}_f^*] - e] \\
 &= \left[1 - \lambda^* \left(\frac{1 - (1 - \lambda^* - \eta^*)^2}{\lambda^* + \eta^*} \right) \right] x_f^* \\
 &\quad + \lambda^* \left(\frac{1 - (1 - \lambda^* - \eta^*)^2}{\lambda^* + \eta^*} \right) [y_f^* - e].
 \end{aligned}$$

More generally, we have

$$\begin{aligned}
 E [E^k [\theta^* | \mathfrak{S}^*] | \mathfrak{S}_f^*] &= \left[1 - \lambda^* \left(\frac{1 - (1 - \lambda^* - \eta^*)^{k+1}}{\lambda^* + \eta^*} \right) \right] x_f^* \\
 &\quad + \lambda^* \left(\frac{1 - (1 - \lambda^* - \eta^*)^{k+1}}{\lambda^* + \eta^*} \right) [y_f^* - e]. \quad (5-8)
 \end{aligned}$$

If we replace (5-8) in (5-7), we obtain

$$\begin{aligned}
 p_f^* &= (1 - r^*) \sum_{k=0}^{\infty} (r^*)^k \left[1 - \lambda^* \left(\frac{1 - (1 - \lambda^* - \eta^*)^{k+1}}{\lambda^* + \eta^*} \right) \right] x_f^* \\
 &\quad + (1 - r^*) \sum_{k=0}^{\infty} (r^*)^k \lambda^* \left(\frac{1 - (1 - \lambda^* - \eta^*)^{k+1}}{\lambda^* + \eta^*} \right) [y_f^* - e] \\
 &= (1 - \delta^*) x_f^* + \delta^* [y_f^* - e].
 \end{aligned}$$

where

$$\begin{aligned}
 \delta^* &\equiv \frac{\lambda^*}{1 - r^* (1 - \lambda^* - \eta^*)} \\
 &= \frac{\sigma_x^{2*}}{\underbrace{\sigma_x^{2*} + \sigma_y^{2*} + (1 - r^*) \sigma_e^2}_{\in (0, 1)}} \in (0, 1) \\
 &\quad : \text{signal effect for exporters}
 \end{aligned}$$

As a result, import prices are

$$\begin{aligned}
 p_f &= e + p_f^* \\
 &= e + (1 - \delta^*) x_f^* + \delta^* [y_f^* - e] \\
 &= (1 - \delta^*) [x_f^* + e] + \delta^* y_f^*.
 \end{aligned}$$

Setting $r^* = 0$ establishes our result.

Proof 21 (Proof of Result 14: ERCP Puzzle) Given Result 13, aggre-

gate imports' prices are

$$\begin{aligned} P_F &\equiv \int_{f \in [0,1]} p_f df \\ &= (1 - \delta^*) \left[\int_{f \in [0,1]} x_f^* df + e \right] + \delta^* \int_{f \in [0,1]} y_f^* df \\ &= (1 - \delta^*) [\theta^* + e] + \delta^* \theta. \end{aligned}$$

As a result, domestic prices are

$$\begin{aligned} p_h &= E[(1 - r)\theta + rP_F \mid \mathfrak{S}_h] \\ &= E[(1 - r)\theta + r[(1 - \delta^*)[\theta^* + e] + \delta^*\theta] \mid \mathfrak{S}_h] \\ &= [1 - r(1 - \delta^*)] E[\theta \mid \mathfrak{S}_h] + r(1 - \delta^*) E[\theta^* + e \mid \mathfrak{S}_h] \\ &= (1 - \delta) x_h + \delta [y_h + e], \end{aligned}$$

where

$$\begin{aligned} \delta &\equiv \lambda + r(1 - \delta^*)(1 - \lambda - \eta) \\ &= 1 - \underbrace{\frac{\sigma_y^2 + [1 - r(1 - \delta^*)]\sigma_e^2}{\sigma_x^2 + \sigma_y^2 + \sigma_e^2}} \\ &: \text{signal effect for domestic firms} \end{aligned}$$

Setting $r = 1$ establishes our result.

Proof 22 (Proof of Result 15: Macroeconomic Stability and ERPT)

Given Result 14, the first derivative of δ with respect to σ_e^2 is

$$\frac{\partial \delta}{\partial \sigma_e^2} = \frac{r(1 - \delta^*)(\sigma_x^2 + \sigma_y^2) - r \frac{\partial \delta^*}{\partial \sigma_e^2} \sigma_e^2 (\sigma_x^2 + \sigma_y^2 + \sigma_e^2) - \sigma_x^2}{(\sigma_x^2 + \sigma_y^2 + \sigma_e^2)^2},$$

where

$$\frac{\partial \delta^*}{\partial \sigma_e^2} = - \frac{(1 - r^*) \sigma_{x^*}^2}{[\sigma_{x^*}^2 + \sigma_{y^*}^2 + (1 - r^*) \sigma_e^2]^2}.$$

If we set $\sigma_x^2 = \sigma_{x^*}^2$, $\sigma_y^2 = \sigma_{y^*}^2$, we obtain

$$\frac{\partial \delta}{\partial \sigma_e^2} > 0 \Leftrightarrow r \left\{ [\sigma_y^2 + (1 - r^*) \sigma_e^2] \left(1 + \frac{\sigma_y^2}{\sigma_x^2} \right) + \frac{(1 - r^*) \sigma_e^2 (\sigma_x^2 + \sigma_y^2 + \sigma_e^2)}{[\sigma_x^2 + \sigma_y^2 + (1 - r^*) \sigma_e^2]} \right\} > 1.$$

If we also set $r = 1$, $r^* = 0$, we obtain

$$\frac{\partial \delta}{\partial \sigma_e^2} = \frac{(\sigma_y^2 - \sigma_x^2 + \sigma_e^2) (\sigma_x^2 + \sigma_y^2)}{(\sigma_x^2 + \sigma_y^2 + \sigma_e^2)^3} > 0 \Leftrightarrow \sigma_e^2 > \sigma_x^2 - \sigma_y^2.$$

If instead we have $r = r^*$

$$\frac{\partial \delta}{\partial \sigma_e^2} = \frac{1}{(\Delta)^2 (\Lambda)^2} \left[2r (1-r) \sigma_x^2 \sigma_e^2 \left(\Delta - \frac{r}{2} \sigma_e^2 \right) + (r \sigma_y^2 - \sigma_x^2) (\Lambda)^2 \right],$$

where

$$\begin{aligned} \Delta &\equiv \sigma_x^2 + \sigma_y^2 + \sigma_e^2, \\ \Lambda &\equiv \sigma_x^2 + \sigma_y^2 + (1-r) \sigma_e^2. \end{aligned}$$

Note that if

$$r > \frac{\sigma_x^2}{\sigma_y^2} \in (0, 1) \Rightarrow \frac{\partial \delta}{\partial \sigma_e^2} > 0.$$

Note also that

$$\begin{aligned} \frac{\partial^2 \delta}{\partial r \partial \sigma_e^2} &= \frac{(1-\delta^*) (\sigma_x^2 + \sigma_y^2) - \frac{\partial \delta^*}{\partial \sigma_e^2} \sigma_e^2 (\sigma_x^2 + \sigma_y^2 + \sigma_e^2)}{(\sigma_x^2 + \sigma_y^2 + \sigma_e^2)^2} > 0, \\ \frac{\partial^2 \delta}{\partial r^* \partial \sigma_e^2} &= \frac{r \sigma_{x^*}^2 \sigma_e^2}{[\sigma_{x^*}^2 + \sigma_{y^*}^2 + (1-r^*) \sigma_e^2]^2 (\sigma_x^2 + \sigma_y^2 + \sigma_e^2)^2} \\ &\quad \times \left[\frac{(1-r^*) \sigma_e^4 - [\sigma_{x^*}^2 + \sigma_{y^*}^2] (2\sigma_x^2 + 2\sigma_y^2 + \sigma_e^2)}{[\sigma_{x^*}^2 + \sigma_{y^*}^2 + (1-r^*) \sigma_e^2]} \right]. \end{aligned}$$

5.2.2 Production Chains

For $n = 1$, domestic prices are

$$\begin{aligned} p_{H,1} &= E[\theta \mid \mathfrak{S}_1] = (1-\lambda) x_1 + \lambda [y_1 + e], \\ p_{F,1}^* &= E[\theta^* \mid \mathfrak{S}_1^*] = (1-\lambda^*) x_1^* + \lambda^* [y_1^* - e]. \end{aligned}$$

Then, foreign prices are

$$\begin{aligned} p_{H,1}^* &= p_{H,1} - e = (1-\lambda) [x_1 - e] + \lambda y_1, \\ p_{F,1} &= e + p_{F,1}^* = (1-\lambda^*) [x_1^* + e] + \lambda^* y_1^*. \end{aligned}$$

If we aggregate:

$$\begin{aligned} P_{H,1} &\equiv \frac{1}{m} \int_0^m p_{H,1}(j) dj = (1-\lambda) \theta + \lambda [\theta^* + e], \\ P_{F,1}^* &\equiv \frac{1}{1-m} \int_m^1 p_{F,1}^*(j) dj = (1-\lambda^*) \theta^* + \lambda^* [\theta - e], \end{aligned}$$

and then

$$\begin{aligned} P_{H,1}^* &\equiv P_{H,1} - e = (1 - \lambda) [\theta - e] + \lambda \theta^*, \\ P_{F,1} &\equiv e + P_{F,1}^* = (1 - \lambda^*) [\theta^* + e] + \lambda^* \theta. \end{aligned}$$

The countries price levels are

$$\begin{aligned} P_1 &\equiv (1 - \chi) P_{H,1} + \chi P_{F,1} \\ &= (1 - \chi) [(1 - \lambda) \theta + \lambda [\theta^* + e]] + \chi [(1 - \lambda^*) [\theta^* + e] + \lambda^* \theta] \\ &= (1 - \delta_1) \theta + \delta_1 [\theta^* + e], \\ P_1^* &\equiv (1 - \chi^*) P_{F,1}^* + \chi^* P_{H,1}^* \\ &= (1 - \chi^*) [(1 - \lambda^*) \theta^* + \lambda^* [\theta - e]] + \chi^* [(1 - \lambda) [\theta - e] + \lambda \theta^*] \\ &= (1 - \delta_1^*) \theta^* + \delta_1^* [\theta - e], \end{aligned}$$

where

$$\begin{aligned} \delta_1 &\equiv (1 - \chi) \lambda + \chi (1 - \lambda^*), \\ \delta_1^* &\equiv (1 - \chi^*) \lambda^* + \chi^* (1 - \lambda). \end{aligned}$$

Now, consider $n = 2$:

$$\begin{aligned} p_{H,2} &= E [(1 - r) \theta + r P_1 \mid \mathfrak{S}_2] \\ &= E [(1 - r) \theta + r [(1 - \delta_1) \theta + \delta_1 [\theta^* + e]] \mid \mathfrak{S}_2] \\ &= (1 - r \delta_1) E [\theta \mid \mathfrak{S}_2] + r \delta_1 E [\theta^* + e \mid \mathfrak{S}_2] \\ &= (1 - \lambda_2) x_2 + \lambda_2 [y_2 + e], \\ p_{F,2}^* &= E [(1 - r^*) \theta^* + r^* P_1^* \mid \mathfrak{S}_2^*] \\ &= E [(1 - r^*) \theta^* + r^* [(1 - \delta_1^*) \theta^* + \delta_1^* [\theta - e]] \mid \mathfrak{S}_2^*] \\ &= (1 - r^* \delta_1^*) E [\theta^* \mid \mathfrak{S}_2^*] + r^* \delta_1^* E [\theta - e \mid \mathfrak{S}_2^*] \\ &= (1 - \lambda_2^*) x_2^* + \lambda_2^* [y_2^* - e]. \end{aligned}$$

Foreign prices are

$$\begin{aligned} p_{H,2}^* &= p_{H,2} - e = (1 - \lambda_2) [x_2 - e] + \lambda_2 y_2, \\ p_{F,2} &= e + p_{F,2}^* = (1 - \lambda_2^*) [x_2^* + e] + \lambda_2^* y_2^*, \end{aligned}$$

where

$$\begin{aligned}\lambda_2 &\equiv (1 - r\delta_1)\lambda + r\delta_1(1 - \eta), \\ \lambda_2^* &\equiv (1 - r^*\delta_1^*)\lambda^* + r^*\delta_1^*(1 - \eta^*).\end{aligned}$$

If we aggregate

$$\begin{aligned}P_{H,2} &\equiv \frac{1}{m} \int_0^m p_{H,2}(j) dj = (1 - \lambda_2)\theta + \lambda_2[\theta^* + e], \\ P_{F,2}^* &\equiv \frac{1}{1 - m} \int_m^1 p_{F,2}^*(j) dj = (1 - \lambda_2^*)\theta^* + \lambda_2^*[\theta - e],\end{aligned}$$

and then

$$\begin{aligned}P_{H,2}^* &\equiv P_{H,2} - e = (1 - \lambda_2)[\theta - e] + \lambda_2\theta^*, \\ P_{F,2} &\equiv e + P_{F,2}^* = (1 - \lambda_2^*)[\theta^* + e] + \lambda_2^*\theta.\end{aligned}$$

The countries price levels are

$$\begin{aligned}P_2 &\equiv (1 - \chi)P_{H,2} + \chi P_{F,2} \\ &= (1 - \chi)[(1 - \lambda_2)\theta + \lambda_2[\theta^* + e]] + \chi[(1 - \lambda_2^*)[\theta^* + e] + \lambda_2^*\theta] \\ &= (1 - \delta_2)\theta + \delta_2[\theta^* + e], \\ P_2^* &\equiv (1 - \chi^*)P_{F,2}^* + \chi^*P_{H,2}^* \\ &= (1 - \chi^*)[(1 - \lambda_2^*)\theta^* + \lambda_2^*[\theta - e]] + \chi^*[(1 - \lambda_2)[\theta - e] + \lambda_2\theta^*] \\ &= (1 - \delta_2^*)\theta^* + \delta_2^*[\theta - e],\end{aligned}$$

where

$$\begin{aligned}\delta_2 &\equiv (1 - \chi)\lambda_2 + \chi(1 - \lambda_2^*), \\ \delta_2^* &\equiv (1 - \chi^*)\lambda_2^* + \chi^*(1 - \lambda_2).\end{aligned}$$

For a generic $n \geq 1$:

$$\begin{aligned}p_{H,n} &= (1 - \lambda_n)x_n + \lambda_n[y_n + e], \\ p_{F,n}^* &= (1 - \lambda_n^*)x_n^* + \lambda_n^*[y_n^* - e], \\ p_{H,n}^* &= (1 - \lambda_n)[x_n - e] + \lambda_n y_n, \\ p_{F,n} &= (1 - \lambda_n^*)[x_n^* + e] + \lambda_n^* y_n^*,\end{aligned}$$

where

$$\begin{aligned}\lambda_n &\equiv \lambda + r(1 - \lambda - \eta)\delta_{n-1}, \quad \lambda_1 = \lambda, \\ \lambda_n^* &\equiv \lambda^* + r^*(1 - \lambda^* - \eta^*)\delta_{n-1}^*, \quad \lambda_1^* = \lambda,\end{aligned}$$

and

$$\begin{aligned}\delta_n &\equiv (1 - \chi)\lambda_n + \chi(1 - \lambda_n^*), \\ \delta_n^* &\equiv (1 - \chi^*)\lambda_n^* + \chi^*(1 - \lambda_n).\end{aligned}$$

If we aggregate

$$\begin{aligned}P_{H,n} &= (1 - \lambda_n)\theta + \lambda_n[\theta^* + e], \\ P_{F,n}^* &= (1 - \lambda_n^*)\theta^* + \lambda_n^*[\theta - e], \\ P_{H,n}^* &= (1 - \lambda_n)[\theta - e] + \lambda_n\theta^*, \\ P_{F,n} &= (1 - \lambda_n^*)[\theta^* + e] + \lambda_n^*\theta,\end{aligned}$$

which yields

$$\begin{aligned}P_n &\equiv (1 - \delta_n)\theta + \delta_n[\theta^* + e], \\ P_n^* &\equiv (1 - \delta_n^*)\theta^* + \delta_n^*[\theta - e].\end{aligned}$$

We can obtain recursive expressions for (λ_n, λ_n^*)

$$\begin{aligned}\lambda_n &\equiv \lambda + r(1 - \lambda - \eta)\delta_{n-1} \\ &= \lambda + r(1 - \lambda - \eta)[(1 - \chi)\lambda_{n-1} + \chi(1 - \lambda_{n-1}^*)], \\ \lambda_n^* &\equiv \lambda^* + r^*(1 - \lambda^* - \eta^*)\delta_{n-1}^* \\ &= \lambda^* + r^*(1 - \lambda^* - \eta^*)[(1 - \chi^*)\lambda_{n-1}^* + \chi^*(1 - \lambda_{n-1})].\end{aligned}$$

Write the weights in matrix format

$$\begin{aligned}\Delta_n &= \Delta + A\Lambda_n, \\ \Lambda_n &= \Lambda + B\Delta_{n-1},\end{aligned}$$

where

$$\begin{aligned}\Delta_n &\equiv \begin{bmatrix} \delta_n \\ \delta_n^* \end{bmatrix}, \quad \Lambda_n \equiv \begin{bmatrix} \lambda_n \\ \lambda_n^* \end{bmatrix}, \\ \Delta &\equiv \begin{bmatrix} \chi \\ \chi^* \end{bmatrix} \neq \Delta_1 \equiv \begin{bmatrix} \delta_1 \\ \delta_1^* \end{bmatrix}, \quad \Lambda \equiv \begin{bmatrix} \lambda \\ \lambda^* \end{bmatrix} = \Lambda_1, \\ A &\equiv \begin{bmatrix} 1 - \chi & -\chi \\ -\chi^* & 1 - \chi^* \end{bmatrix}, \dots B \equiv \begin{bmatrix} r(1 - \lambda - \eta) & 0 \\ 0 & r^*(1 - \lambda^* - \eta^*) \end{bmatrix}.\end{aligned}$$

If we combine the two equations

$$\begin{aligned}\Delta_n &= \Delta + A[\Lambda + B\Delta_{n-1}] \\ &= \sum_{j=0}^{n-1} C^j \Delta_1,\end{aligned}$$

where

$$\begin{aligned}\Delta_1 &\equiv \Delta + A\Lambda. \\ C &\equiv AB.\end{aligned}$$

In order to solve the matricial difference equation, we must obtain the inverse of $I - C$ and calculate the powers of C . Under our simplifying assumptions ($r = r^*, \sigma_x^2 = \sigma_{x^*}^2, \sigma_y^2 = \sigma_{y^*}^2$), the weights on e are $\lambda = \lambda$ and $\eta = \eta^*$. As a result

$$C = \tilde{r} \begin{bmatrix} 1 - \chi & -\chi \\ -\chi^* & 1 - \chi^* \end{bmatrix},$$

where $\tilde{r} \equiv r(1 - \lambda - \eta)$.

We can show that matrix $I - C$ is invertible. Just note that

$$|I - C| = \begin{vmatrix} 1 - \tilde{r}(1 - \chi) & \tilde{r}\chi \\ \tilde{r}\chi^* & 1 - \tilde{r}(1 - \chi^*) \end{vmatrix} = 0$$

if and only if

$$(1 - \tilde{r})[1 - \tilde{r}(1 - \chi - \chi^*)] = 0.$$

Thus, matrix $I - C$ is invertible as long as $\tilde{r} \neq \{1, (1 - \chi - \chi^*)^{-1}\}$. This condition is not restrictive because $\tilde{r} \in (0, 1)$. In this case

$$(I - C)^{-1} = \frac{1}{(1 - \tilde{r})[1 - \tilde{r}(1 - \chi - \chi^*)]} \begin{bmatrix} 1 - \tilde{r}(1 - \chi^*) & -\tilde{r}\chi \\ -\tilde{r}\chi^* & 1 - \tilde{r}(1 - \chi) \end{bmatrix}.$$

We must also calculate the powers of C . The eigenvalue ϕ is such that

$$\begin{vmatrix} \tilde{r}(1-\chi) - \phi & -\tilde{r}\chi \\ -\tilde{r}\chi^* & \tilde{r}(1-\chi^*) - \phi \end{vmatrix} = 0,$$

or

$$[\tilde{r}(1-\chi-\chi^*) - \phi][\tilde{r} - \phi] = 0.$$

As a result, we obtain

$$\phi = \{\tilde{r}, \tilde{r}(1-\chi-\chi^*)\}.$$

The eigenvectors $\{v\}$ are

$$\begin{aligned} \tilde{r} \begin{bmatrix} 1-\chi & -\chi \\ -\chi^* & 1-\chi^* \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} &= \tilde{r} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix}, \\ \tilde{r} \begin{bmatrix} 1-\chi & -\chi \\ -\chi^* & 1-\chi^* \end{bmatrix} \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} &= \tilde{r}(1-\chi-\chi^*) \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix}, \end{aligned}$$

or

$$\begin{aligned} v_{21} &= -v_{11}, \\ v_{22} &= \frac{\chi^*}{\chi} v_{12}. \end{aligned}$$

If we set $v_{11} = v_{12} = 1$, we can rewrite C as

$$C \equiv TDT^{-1},$$

where

$$\begin{aligned} T &\equiv \begin{bmatrix} 1 & 1 \\ -1 & \frac{\chi^*}{\chi} \end{bmatrix}, T^{-1} \equiv \frac{\chi}{\chi + \chi^*} \begin{bmatrix} \frac{\chi^*}{\chi} & -1 \\ 1 & 1 \end{bmatrix} \\ D &\equiv \begin{bmatrix} \tilde{r} & 0 \\ 0 & \tilde{r}(1-\chi-\chi^*) \end{bmatrix}, \end{aligned}$$

and then

$$\begin{aligned} C^n &= TD^nT^{-1} \\ &= \tilde{r}^n \frac{\chi}{\chi + \chi^*} \begin{bmatrix} \frac{\chi^*}{\chi} + (1-\chi-\chi^*)^n & -[1 - (1-\chi-\chi^*)^n] \\ -\frac{\chi^*}{\chi}[1 - (1-\chi-\chi^*)^n] & 1 + \frac{\chi^*}{\chi}(1-\chi-\chi^*)^n \end{bmatrix} \end{aligned}$$

Finally, we obtain

$$\Delta_n = (I - C^n)(I - C)^{-1} \Delta_1.$$

If we ignore home-bias ($\bar{\chi} = \bar{\chi}^* = 1$), then $\chi + \chi^* = 1$. We can rewrite $(I - C)^{-1}$ and C^n as

$$(I - C)^{-1} = \frac{1}{1 - \tilde{r}} \begin{bmatrix} 1 - \tilde{r}\chi & -\tilde{r}\chi \\ -\tilde{r}(1 - \chi) & 1 - \tilde{r}(1 - \chi) \end{bmatrix}$$

and

$$C^n = \tilde{r}^n \begin{bmatrix} (1 - \chi) & -\chi \\ -(1 - \chi) & \chi \end{bmatrix}.$$

After tedious calculation, we obtain

$$\begin{aligned} \delta_n &= \chi + (\chi^* - \chi) \left(\frac{1 - \tilde{r}^n}{1 - \tilde{r}} \right) \lambda \\ &< \delta_{n-1} \iff \chi^* < \chi, \\ \delta_n^* &= \chi^* - (\chi^* - \chi) \left(\frac{1 - \tilde{r}^n}{1 - \tilde{r}} \right) \lambda \\ &< \delta_{n-1}^* \iff \chi^* > \chi, \\ \lambda_n &\equiv \lambda + \tilde{r}\delta_{n-1} \\ &< \lambda_{n-1} \iff \delta_{n-1} < \delta_{n-2} \iff \chi^* < \chi, \\ \lambda_n^* &\equiv \lambda + \tilde{r}\delta_{n-1}^* \\ &< \lambda_{n-1}^* \iff \delta_{n-1}^* < \delta_{n-2}^* \iff \chi^* > \chi. \end{aligned}$$

5.3

Appendix of Chapter 3

Proof 23 (Proof of expression (3-21): Equilibrium Aggregate Price Level)

First, we replace (3-1) in (3-7) to obtain:

$$\begin{aligned} P_t &= \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E[rP_t + (1 - r)\theta_t \mid \mathfrak{S}_{t-j}(z)] dz \\ &= r \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E[P_t \mid \mathfrak{S}_{t-j}(z)] dz + (1 - r) \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E[\theta_t \mid \mathfrak{S}_{t-j}(z)] dz. \end{aligned}$$

From the definition of the average 1st order belief in (3-8):

$$P_t = r\bar{E}[P_t] + (1 - r)\bar{E}[\theta_t].$$

If we iterate one time, we obtain:

$$\begin{aligned} P_t &= r\bar{E} [r\bar{E} [P_t] + (1-r)\bar{E} [\theta_t]] + (1-r)\bar{E} [\theta_t] \\ &= r^2\bar{E} [\bar{E} [P_t]] + r(1-r)\bar{E} [\bar{E} [\theta_t]] + (1-r)\bar{E} [\theta_t] \\ &= r^2\bar{E}^2 [P_t] + r(1-r)\bar{E}^2 [\theta_t] + (1-r)\bar{E} [\theta_t]. \end{aligned}$$

If we iterate N times:

$$P_t = r^N \bar{E}^N [P_t] + (1-r) \sum_{k=1}^N r^{k-1} \bar{E}^k [\theta_t].$$

Taking the limit as $N \rightarrow \infty$, we obtain expression (3-9):

$$P_t = (1-r) \sum_{k=1}^{\infty} r^{k-1} \bar{E}^k [\theta_t],$$

which proves the result.

Proof 24 (Obtaining the Expectations) $E[\theta_{t-m} | \mathfrak{S}_{t-j}(z)]$: First, we calculate the distribution of the fundamental θ_{t-j} given that the firm updated its information set at period $t-j$. We can compute $f(\theta_{t-j} | \Theta_{t-j-1}, x_{t-j})$ as

$$\begin{aligned} f(\theta_{t-j} | \theta_{t-j-1}, x_{t-j}) &= \frac{f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j})}{\int_{-\infty}^{\infty} f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}) d\theta_{t-j}} \\ &= \frac{f(\theta_{t-j-1}, x_{t-j} | \theta_{t-j}) f(\theta_{t-j})}{\int_{-\infty}^{\infty} f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}) d\theta_{t-j}} \\ &= \frac{f(\theta_{t-j-1} | \theta_{t-j}) f(x_{t-j} | \theta_{t-j}) f(\theta_{t-j})}{\int_{-\infty}^{\infty} f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}) d\theta_{t-j}} \end{aligned}$$

where the last equality holds due to the independence of $\xi_t(z)$ and ϵ_{t-j} . As

$$\begin{aligned} x_{t-j}(z) &= \theta_{t-j} + \xi_{t-j}(z), \\ \theta_{t-j-1} &= \theta_{t-j} - \epsilon_{t-j}, \end{aligned}$$

where $\xi_t(z) \sim N(0, \beta^{-1})$ and $\epsilon_{t-j} \sim N(0, \alpha^{-1})$, we have that $f(x_{t-j} | \theta_{t-j}) = N(\theta_{t-j}, \beta^{-1})$ and $f(\theta_{t-j-1} | \theta_{t-j}) = N(\theta_{t-j}, \alpha^{-1})$. If the dynamics of θ_t was

$$\theta_{t-j-1} = \rho\theta_{t-j} - \epsilon_{t-j},$$

we would have

$$E[\theta_{t-j}] = E[\theta_t] = \frac{E[\epsilon_t]}{1-\rho} = 0,$$

$$\text{Var}[\theta_{t-j}] = \text{Var}[\theta_t] = \frac{\text{Var}[\epsilon_t]}{1-\rho^2} = \frac{\alpha^{-1}}{1-\rho^2}.$$

Therefore, the distribution of θ_{t-j} would be given by $f(\theta_{t-j}) = N(0, \Psi^{-1})$ where $\Psi = \alpha(1-\rho^2)$. Thus, we would obtain

$$\begin{aligned} f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}) &= c \times \exp \left\{ -\frac{1}{2} \left[\frac{(x_{t-j}(z) - \theta_{t-j})^2}{\beta^{-1}} + \frac{(\theta_{t-j-1} - \rho^{-1}\theta_{t-j})^2}{(\rho^2\alpha)^{-1}} + \frac{\theta_{t-j}^2}{\Psi^{-1}} \right] \right\} \\ &= c \times \exp \left\{ -\frac{1}{2} [(\beta + \alpha + \Psi)\theta_{t-j}^2 - 2(\beta x_{t-j}(z) + \alpha\rho\theta_{t-j-1})\theta_{t-j}] \right\} \\ &\times \exp \left\{ -\frac{1}{2} [\beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2] \right\} \\ &= c \times d \times \frac{1}{\sqrt{2\pi\sigma\Sigma}} \times \exp \left\{ -\frac{1}{2} \frac{(\theta_{t-j} - \mu)^2}{\Sigma^2} \right\}, \end{aligned}$$

where

$$\begin{aligned} c &= (2\pi)^{-3/2} (\beta\alpha\Psi)^{1/2}, & d &= \sqrt{2\pi}\sigma \exp \left\{ -\frac{1}{2} [-\mu^2\Sigma^{-2} + \beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2] \right\}, \\ \mu &= [\Delta x_{t-j}(z) + (1-\Delta)z_{t-j-1}], & \Delta &= \beta(\beta + \alpha + \Psi)^{-1}, \\ z_{t-j-1} &= \Lambda\rho\theta_{t-j-1}, & \Lambda &= \alpha(\beta + \alpha)^{-1}. \\ \Sigma^2 &= (\beta + \alpha + \Psi)^{-1}, \end{aligned}$$

As $\rho \rightarrow 1$, we have $\Psi \rightarrow 0$, $\Delta \rightarrow \delta$, and $\Sigma^2 \rightarrow (\beta + \alpha)^{-1}$. Thus $f(\theta_{t-j} | \theta_{t-j-1}, x_{t-j}) = N(\mu, \sigma^2)$ where $\mu = [\delta x_{t-j}(z) + (1-\delta)\theta_{t-j-1}]$, and $\sigma^2 = (\beta + \alpha)^{-1}$.

Proof 25 (Proof of Lemma 17: Higher Order Beliefs) In this Appendix we derive the general formula of the k -th order average expectation

$$\bar{E}^k[\theta_t] = \lambda \sum_{m=0}^{\infty} (1-\lambda)^m [\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}]$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ recursively defined for $k \geq 1$

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} [1 - (1-\lambda)^m]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} [(1-\delta)[1 - (1-\lambda)^{m+1}] + \delta[1 - (1-\lambda)^m]] & 0 \\ \delta[[1 - (1-\lambda)^{m+1}] - [1 - (1-\lambda)^m]] & [1 - (1-\lambda)^{m+1}] \end{bmatrix},$$

and the initial weights are $(\kappa_{1,k}, \delta_{1,k}) \equiv (1 - \delta, \delta)$.

We start by computing $\bar{E}^1[\theta_t]$ as

$$\begin{aligned}\bar{E}^1[\theta_t] &= \sum_{j=0}^{\infty} \int_{\Lambda_j} E[\bar{E}^0[\theta_t] | \mathfrak{S}_{t-j}(z)] dz \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_j} E[\theta_t | \mathfrak{S}_{t-j}(z)] dz \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_j} [(1 - \delta)x_{t-j}(z) + \delta\theta_{t-j-1}] dz \\ &= \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j [(1 - \delta)\theta_{t-j} + \delta\theta_{t-j-1}].\end{aligned}$$

We can use this result to obtain $\bar{E}^2[\theta_t]$ as

$$\begin{aligned}\bar{E}^2[\theta_t] &= \sum_{m=0}^{\infty} \int_{\Lambda_m} E[\bar{E}^1[\theta_t] | \mathfrak{S}_{t-m}(z)] dz \\ &= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{\infty} (1 - \lambda)^j E[(1 - \delta)\theta_{t-j} + \delta\theta_{t-j-1} | \mathfrak{S}_{t-m}(z)] dz.\end{aligned}$$

We know that

$$E[\theta_{t-j} | \mathfrak{S}_{t-m}(z)] = \begin{cases} (1 - \delta)x_{t-m}(z) + \delta\theta_{t-m-1} & : m \geq j, \\ \theta_{t-j} & : m < j. \end{cases}$$

Thereafter,

$$\begin{aligned}\bar{E}^2[\theta_t] &= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1 - \lambda)^j \{(1 - \delta)E[\theta_{t-j} | \mathfrak{S}_{t-m}(z)] + \delta E[\theta_{t-j-1} | \mathfrak{S}_{t-m}(z)]\} dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} (1 - \lambda)^m \{(1 - \delta)E[\theta_{t-m} | \mathfrak{S}_{t-m}(z)] + \delta\theta_{t-m-1}\} dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1 - \lambda)^j [(1 - \delta)\theta_{t-j} + \delta\theta_{t-j-1}] dz \\ &= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1 - \lambda)^j [(1 - \delta)x_{t-m}(z) + \delta\theta_{t-m-1}] dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} (1 - \lambda)^m [(1 - \delta)[(1 - \delta)x_{t-m}(z) + \delta\theta_{t-m-1}] + \delta\theta_{t-m-1}] dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1 - \lambda)^j [(1 - \delta)\theta_{t-j} + \delta\theta_{t-j-1}] dz \\ &= \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^m [(1 - \delta)\theta_{t-m} + \delta\theta_{t-m-1}] \sum_{j=0}^{m-1} (1 - \lambda)^j \\ &\quad + \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^{2m} [(1 - \delta)^2\theta_{t-m} + [1 - (1 - \delta)^2]\theta_{t-m-1}] \\ &\quad + \lambda^2 \sum_{j=1}^{\infty} (1 - \lambda)^j [(1 - \delta)\theta_{t-j} + \delta\theta_{t-j-1}] \sum_{m=0}^{j-1} (1 - \lambda)^m \\ &= \lambda \sum_{m=0}^{\infty} (1 - \lambda)^m [(1 - \delta)\theta_{t-m} + \delta\theta_{t-m-1}] [1 - (1 - \lambda)^m] \\ &\quad + \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^{2m} [(1 - \delta)^2\theta_{t-m} + [1 - (1 - \delta)^2]\theta_{t-m-1}] \\ &\quad + \lambda \sum_{j=1}^{\infty} (1 - \lambda)^j [(1 - \delta)\theta_{t-j} + \delta\theta_{t-j-1}] [1 - (1 - \lambda)^j] \\ &= \lambda \sum_{m=0}^{\infty} (1 - \lambda)^m 2[1 - (1 - \lambda)^m] [(1 - \delta)\theta_{t-m} + \delta\theta_{t-m-1}] \\ &\quad + \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^{2m} [(1 - \delta)^2\theta_{t-m} + [1 - (1 - \delta)^2]\theta_{t-m-1}].\end{aligned}$$

We can write this expression as

$$\bar{E}^2[\theta_t] = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j [\kappa_{j,2}\theta_{t-j} + \delta_{j,2}\theta_{t-j-1}],$$

where

$$\begin{aligned}\kappa_{j,2} &= (1 - \delta^2) \left[1 - (1 - \lambda)^j \right] + (1 - \delta)^2 \left[1 - (1 - \lambda)^{j+1} \right] \\ &= \left[1 - (1 - \lambda)^{j+1} \right] \kappa_{j,1}^2 + \left[1 - (1 - \lambda)^j \right] (1 - \delta_{j,1}^2), \\ \delta_{j,2} &= \delta^2 \left[1 - (1 - \lambda)^j \right] + \left[1 - (1 - \delta)^2 \right] \left[1 - (1 - \lambda)^{j+1} \right] \\ &= \left[1 - (1 - \lambda)^{j+1} \right] (1 - \kappa_{j,1}^2) + \left[1 - (1 - \lambda)^j \right] \delta_{j,1}^2.\end{aligned}$$

Note that

$$\kappa_{j,2} + \delta_{j,2} = \sum_{n=0}^1 \left[1 - (1 - \lambda)^j \right]^n \left[1 - (1 - \lambda)^{j+1} \right]^{1-n}.$$

We use induction to obtain the general case. Suppose that (3-18) holds for $k - 1$. Then

$$\bar{E}^{k-1} [\theta_t] = \lambda \sum_{m=0}^{\infty} (1 - \lambda)^m [\kappa_{m,k-1} \theta_{t-m} + \delta_{m,k-1} \theta_{t-m-1}],$$

where

$$\sum_{j=0}^{m-1} (1 - \lambda)^j (\kappa_{j,k-1} + \delta_{j,k-1}) = \frac{1}{\lambda} [1 - (1 - \lambda)^m]^{k-1}.$$

As a result,

$$\begin{aligned}\bar{E}^k [\theta_t] &= \sum_{m=0}^{\infty} \int_{\Lambda_m} E [\bar{E}^{k-1} [\theta_t] | \mathfrak{S}_{t-m}(z)] dz \\ &= \sum_{m=0}^{\infty} \int_{\Lambda_m} E \left[\lambda \sum_{j=0}^{\infty} (1 - \lambda)^j [\kappa_{j,k-1} \theta_{t-j} + \delta_{j,k-1} \theta_{t-j-1}] | \mathfrak{S}_{t-m}(z) \right] dz \\ &= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1 - \lambda)^j \{ \kappa_{j,k-1} E [\theta_{t-j} | \mathfrak{S}_{t-m}(z)] + \delta_{j,k-1} E [\theta_{t-j-1} | \mathfrak{S}_{t-m}(z)] \} dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} (1 - \lambda)^m \{ \kappa_{m,k-1} E [\theta_{t-m} | \mathfrak{S}_{t-m}(z)] + \delta_{m,k-1} \theta_{t-m-1} \} dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1 - \lambda)^j [\kappa_{j,k-1} \theta_{t-j} + \delta_{j,k-1} \theta_{t-j-1}] dz \\ &= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1 - \lambda)^j (\kappa_{j,k-1} + \delta_{j,k-1}) [(1 - \delta) x_{t-m}(z) + \delta \theta_{t-m-1}] dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} (1 - \lambda)^m [\kappa_{m,k-1} [(1 - \delta) x_{t-m}(z) + \delta \theta_{t-m-1}] + \delta_{m,k-1} \theta_{t-m-1}] dz \\ &\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1 - \lambda)^j [\kappa_{j,k-1} \theta_{t-j} + \delta_{j,k-1} \theta_{t-j-1}] dz \\ &= \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^m [(1 - \delta) \theta_{t-m} + \delta \theta_{t-m-1}] \sum_{j=0}^{m-1} (1 - \lambda)^j (\kappa_{j,k-1} + \delta_{j,k-1}) \\ &\quad + \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^{2m} [\kappa_{m,k-1} (1 - \delta) \theta_{t-m} + [\kappa_{m,k-1} \delta + \delta_{m,k-1}] \theta_{t-m-1}] \\ &\quad + \lambda^2 \sum_{j=1}^{\infty} (1 - \lambda)^j [\kappa_{j,k-1} \theta_{t-j} + \delta_{j,k-1} \theta_{t-j-1}] \sum_{m=0}^{j-1} (1 - \lambda)^m \\ &= \lambda \sum_{m=0}^{\infty} (1 - \lambda)^m [1 - (1 - \lambda)^m]^{k-1} [(1 - \delta) \theta_{t-m} + \delta \theta_{t-m-1}] \\ &\quad + \lambda^2 \sum_{m=0}^{\infty} (1 - \lambda)^{2m} [\kappa_{m,k-1} (1 - \delta) \theta_{t-m} + [\kappa_{m,k-1} \delta + \delta_{m,k-1}] \theta_{t-m-1}] \\ &\quad + \lambda \sum_{m=0}^{\infty} (1 - \lambda)^m [1 - (1 - \lambda)^m] [\kappa_{m,k-1} \theta_{t-m} + \delta_{m,k-1} \theta_{t-m-1}].\end{aligned}$$

We can rewrite the last three lines above as

$$\bar{E}^k [\theta_t] = \lambda \sum_{m=0}^{\infty} (1 - \lambda)^m [\kappa_{m,k} \theta_{t-m} + \delta_{m,k} \theta_{t-m-1}],$$

where

$$\begin{aligned} \kappa_{m,k} &\equiv (1 - \delta) [1 - (1 - \lambda)^m]^{k-1} + [(1 - \delta) \lambda (1 - \lambda)^m + [1 - (1 - \lambda)^m]] \kappa_{m,k-1} \\ &= (1 - \delta) [1 - (1 - \lambda)^m]^{k-1} \\ &\quad + [(1 - \delta) [1 - (1 - \lambda)^{m+1}] + \delta [1 - (1 - \lambda)^m]] \kappa_{m,k-1}, \\ \delta_{m,k} &\equiv \delta [1 - (1 - \lambda)^m]^{k-1} + \delta \lambda (1 - \lambda)^m \kappa_{m,k-1} + [\lambda (1 - \lambda)^m + [1 - (1 - \lambda)^m]] \delta_{m,k-1} \\ &= \delta [1 - (1 - \lambda)^m]^{k-1} \\ &\quad + \delta [[1 - (1 - \lambda)^{m+1}] - [1 - (1 - \lambda)^m]] \kappa_{m,k-1} + [1 - (1 - \lambda)^{m+1}] \delta_{m,k-1}, \end{aligned}$$

since

$$\lambda (1 - \lambda)^m = [1 - (1 - \lambda)^{m+1}] - [1 - (1 - \lambda)^m].$$

Rewriting these weights in matrix format, we obtain

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1 - \delta) \\ \delta \end{bmatrix} [1 - (1 - \lambda)^m]^{k-1} + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} [(1 - \delta) [1 - (1 - \lambda)^{m+1}] + \delta [1 - (1 - \lambda)^m]] & 0 \\ \delta [[1 - (1 - \lambda)^{m+1}] - [1 - (1 - \lambda)^m]] & [1 - (1 - \lambda)^{m+1}] \end{bmatrix},$$

which is exactly our result.