4. Methodology

4.1 The Description of the Research

In this section two mathematical models will be proposed. The first one takes into consideration the concept of major/mini hub and makes an application for passengers using real data that considers 41 airports in Brazil and nine airports in other countries of South America. This model also considers direct links amongst nodes.

The first model's methodology consists of building a 50 x 50 flow matrix, which is shown in Figure 4.1, the application of a mathematical model to determine optimal locations for major and mini hubs as well as the identification of the direct flows, and a sensitivity analysis – through the variation of some parameters – which illustrates the pattern of the flows in the networks, visualized in four experiments.

The second model searches optimal location for hubs taking into consideration only the Brazilian air passenger market. A total of 135 airports will be considered and a methodology for solving it for such a big network will be presented – and given the complexity of this type of problem – which is divided into two phases. A network flow analysis will be also made, which helps the discussion of the optimal results obtained.

The concepts used to elaborate the two formulations in the case studies in this section are idealized, mostly because the existence of some peculiarities in South America and, mainly, in Brazil. There is a strong relationship, both in cultural and economical terms, which makes reasonable the planning of an integrated network, considering the main airports in Brazil and the main ones located in other countries of South America. The concept and the application are developed in the Case Study A.

In the Case Study B, the formulation was made taking into consideration of geographical and economical peculiarities of Brazil. The country has an uneven distribution of the operations in the industry and on the way that the population has settled down throughout it. The south and the southeast regions are the most developed ones, concentrating the most part of the population and the country's richness, as well as the qualified labor force. The north, middle-west, and some parts of the northeast regions have some problems of accessibility, especially when the subject is the planning of national transportation networks.

The intent of the two case studies is not to solve real problems found in the air transportation networks in South America and Brazil. For such a very complex problem, this would require the participation and the involvement of different segments of the society, ranging from governmental authorities, air transportation entities, such as ANAC, public enterprises, such as INFRAERO, the air companies and society representatives. The main objective here is, beyond the technical and conceptual contribution to the research field, to provide the decision makers some useful results found through the use of scientific methods, idealized for the environments discussed.

4.2 Case Study A

The main objective of this Case Study is to develop a location model using operations research to solve a hub location problem in South America, with focus in Brazil. In order to do this, real flow data amongst 50 airports in South America, concerning passengers, will be used. The number of Brazilian airports to be used in this study is 41 and for the other countries of South America is 9. Some of these flows are observed and some are estimated. Given the lack of specific information, a gravitational model will have to be applied to estimate flows among the main Brazilian airports and the other countries of South America. In proceeding this way, the total flow matrix among 50 airports might be obtained.

The major part of the data used in this study case was extracted from an official document published by ANAC entitled “Air Transportation Annual 2007”. This document provides detailed information about passenger and freight transportation in the year 2007 in the domestic and in the international market (in this latter case, having Brazil as an origin or destination).

In Brazil, the management of the airport infrastructure is made by a national public agency - INFRAERO (The Brazilian Enterprise of Airports Infrastructure), which has approximately 28,000 employees. In total, this agency is responsible for managing 67 airports, which accounts for around 97% of the regular movement in the country, and 33 logistical freight terminals. At the other hand, the Civil Aviation National Agency (ANAC) is responsible for the air traffic management such as the number of slots in an airport and the regulation of new air companies in the market.

The airports located in the main Brazilian cities as Sao Paulo, Rio de Janeiro, Brasilia, Curitiba, Belo Horizonte, Porto Alegre and Salvador have been facing serious congestion problems along the past few years. One typical example is the Congonhas (CGH) airport, located in downtown Sao Paulo. Sao Paulo is one of the biggest cities in the world and the most important pole of economic activities in Brazil and South America. Therefore, it is an important center for generation and attraction of passengers and freight. Moreover, it is the biggest airport in terms of passenger movement in Brazil. The inconvenience is that the airport is totally congested and in its vicinity there is no space available for further expansions.

One alternative to redirect some "in transit" flows would be the international airport of Guarulhos (GRU) located in the metropolitan area of Sao Paulo, but this airport also faces serious congestion problems. The Brasilia airport (BSB) would be another alternative, but it does not have enough infrastructure to handle all the traffic. A convenient alternative to catch this surplus seems to be the international airport of Rio de Janeiro – Galeao (GIG), which has additional available capacity.

In the field of ANAC management, two main air companies operating in Brazil achieve more than 85% of the total passenger market. They are TAM and Gol. TAM operates with fuel efficient planes in domestic routes – A-319, 320 – and with *wide bodies*[1] in international ones – A-340, A-330, Boeings-767 and 777. It relies on a full service basis. On the other hand, Gol operates with an unified fleet consisting of Boeings 737-700 and 800 and it is considered a *low cost carrier*.

Both TAM and Gol have their networks configured in a hub-and-spoke structure. A peculiarity is the fact that they do not use wide body aircrafts to make

[1] An aircraft with two passenger aisles and a typical fuselage diameter of 5 to 6 meters (source: www.wikipedia.org).

the *trunk routes*[2], as noticed in the domestic American market for the linkages between East and West Coast, for instance. These two companies focus their domestic operations mainly at Congonhas (CGH) airport, in Sao Paulo. TAM also concentrates its international operations at Guarulhos (GRU) airport, located in the metropolitan area of Sao Paulo.

An important trend has been occurring in the Air Transportation industry: mergers and alliances. Recently, two important companies in South America, Avianca and TACA, have joined their operations in an alliance. The new merger flies to 100 destinations in Europe and in the American Continent (South, Central and North America) and has become the major air company in South America in terms of number of aircrafts and routes. The combined network operates with 4 hubs: Bogota, San Salvador, San Jose de Costa Rica and Lima.

4.2.1 The Data

The number of airports used in this study is fifty: 41 of them are Brazilians and nine are located in other countries of South America. These South American countries and their respective airports are listed in Table 4.1.

Table 4.1: List of South American Airports (excluding the Brazilian ones) used in the Study

Country	City	Airport Name	IATA
Argentina	Buenos Aires	Ezeiza International	EZE
Bolivia	Santa Cruz de La Sierra	Viru Viru International	VVI
Chile	Santiago	Arturo Merino Benitez International	SCL
Colombia	Bogota	El Dorado International	BOG
Ecuador	Quito	Mariscal Sucre International	UIO
Paraguay	Asuncion	Selvio Pettirossi International	ASU
Peru	Lima	Jorge Chavez International	LIM
Uruguay	Montevideo	Carrasco International	MVD
Venezuela	Caracas	Simon Bolivar International	CCS

Source: ACI 2007 (World Airport Traffic Report 2007)

[2] Strategic route, recommended for long distances and high load factors.

The 50 x 50 distance matrix was obtained using the software TRANSCAD version 4.5 and the values computed are given in kilometers. To denote the location of the airports, longitude and latitude data extracted from different sources were used. For the 41 Brazilian airports, a document published by ROTAER (1999) entitled `Brazil – Auxiliary Manual of Air Routes` was used. For the remaining South American airports, the data source of ACI (2007) was used.

On the other hand, it was not an easy task to obtain the W_{ij} matrix regarding the flow of passengers amongst 50 airports. In the ANAC data, only the data amongst Brazilian airports was available. The data between some Brazilian airports and the other South American airports was estimated using a gravitational mathematical model and the data amongst the other South American Airports were obtained from CLAC regarding the year 2007. Figure 4.1 sketches this scenario.

<table>
<tr><td>W_{ij}</td><td colspan="7">Destination</td></tr>
<tr><td rowspan="7">Origin</td><td></td><td>1</td><td>...</td><td>41</td><td>42</td><td>...</td><td>50</td></tr>
<tr><td>1</td><td colspan="3" rowspan="3">Data amongst Brazilian Airports: available from ANAC (2007) - $\boldsymbol{M_1}$</td><td colspan="3" rowspan="3">Data between Brazilian Airports and the Other of South American Airports: will be estimated through a gravitational model - $\boldsymbol{M_2}$</td></tr>
<tr><td>…</td></tr>
<tr><td>41</td></tr>
<tr><td>42</td><td colspan="3" rowspan="3">Data between South American Airports and the Brazilian Airports: will be estimated through a gravitational model - $\boldsymbol{M_3}$</td><td colspan="3" rowspan="3">Data amongst South American Airports: available from CLAC (2007) - $\boldsymbol{M_4}$</td></tr>
<tr><td>…</td></tr>
<tr><td>50</td></tr>
</table>

Figure 4.1: Sketch about the data to be used in the case study

In the case of the Brazilian airports, 41 airports with a significant amount of passenger movements were selected. For the other countries of South American just the main airport of each country has been used.

4.2.2 The Estimation of Data Using the Gravitational Model

Firstly, some assumptions about the flow patterns amongst airports had to be made. From 41 Brazilian airports, only 21 of them were considered to have international flows coming from/going to the other South American airports. The methodology used to estimate these flows will be shown in seven steps.

- ***Identification of International Movement for the Main Brazilian Airports***

An official document issued by INFRAERO about the operational movement in the year 2007 was taken as the basis for the application of this methodology. Just airports that summed more than a 1,000 units of passenger in international market were considered. These international passengers may be originated and destined to every part of the world but the concern of this study is only with movements within South America. Table 4.2 lists these Brazilian airports and their respective international movements.

Table 4.2: The International Passenger Movements in the year 2007 in some Brazilian Airports

Airport – State	IATA Code	International PAX (Units)	Estimated Influence (%) – *Factor B*
Guarulhos – SP	GRU	8,448,854	66.99
Galeao – RJ	GIG	2,178,147	17.27
Salvador – BA	SSA	412,920	3.27
Brasilia – DF	BSB	72,831	0.58
Fortaleza – CE	FOR	267,881	2.12
Confins – MG	CNF	33,157	0.26
Pampulha – MG	PLU	1,508	0.01
Curitiba – PR	CWB	65,789	0.52
Manaus – AM	MAO	97,035	0.77
Recife – PE	REC	177,149	1.40
Porto Alegre – RS	POA	367,717	2.92
Belem – PA	BEL	61,594	0.49
Campinas – SP	VCP	2,336	0.02
Sao Luiz – MA	SLZ	1,058	0.01
Maceio – AL	MCZ	20,135	0.16
Natal – RN	NAT	218,825	1.74
Campo Grande –	CGR	11,808	0.09
Florianopolis – SC	FLN	157,801	1.25
Macapa – AP	MCP	6,812	0.05
Foz do Iguacu –	IGU	3,958	0.03
Boa Vista – RR	BVB	4,271	0.03
Total		**12,611,586**	**100**

Source: INFRAERO (2007)

- ***Estimation of the Influence of each Brazilian Airport in the International Market***

In the previous part of this Dissertation the influence of each airport in the international movement of passengers was defined. The sum of all of these flows is shown in Table 4.2 (summing over the third column) is 12,611,586. Dividing the movement of each airport by the total movement, then the estimated influence, called *Factor B*, is achieved, which is shown in the fourth column of Table 4.2.

- ***Identification of Total Inflow and Outflow for every South American airport having Brazil as Origin and Destination***

The official document issued by ANAC – Air Transportation Annual 2007 – informs the total flow of passenger to/from Brazil and the other countries of South America. Table 4.3 summarizes that information.

Table 4.3: The Flow of Passenger between Brazil and the Other Countries of South America

Country	Origin (from Brazil)	Destination (to Brazil)
Argentina	1,023,647	1,028,182
Bolivia	56,099	67,077
Chile	412,472	393,475
Colombia	71,941	72,522
Ecuador	Not available	Not available
Paraguay	122,910	145,865
Peru	125,140	128,155
Uruguay	147,070	139,649
Venezuela	42,314	43,599
Total (PAX)	**2,001,593**	**2,018,524**

Source: Air Transportation Annual 2007 - ANAC

The interpretation of Table 4.3 is straightforward. It takes into account Brazil as the origin and destination of all flows. For example, the total flow originated in Brazil and having Argentina as destination was 1,023,647 PAX and the total flow destined to Brazil having Argentina as origin was 1,028,182. These values will be taken into consideration to calculate the estimates that will be shown in the next step.

- ***Estimation of Total Inflow and Outflow for every Brazilian Airport***

The last line of Table 4.3 provides the total flow of passengers to/from nine other countries of South America having Brazil as origin and destination, respectively. The aim now is to estimate the total flow originated and destined from/to 21 Brazilian airports to/from the other South America airports (the airports listed in Table 4.2), O_i and D_j, respectively.

For instance, to estimate the total flow originated in the GRU airport to the other South American airports, the *Factor B* will be multiplied by the total flow originated in Brazil and having as destination the other South American airports. In this case, the Factor B for GRU is 0.6699 and the Total Flow is 2,001,593. Therefore, the total flow originated in GRU, the O_i value, will be the product of this multiplication, which is approximately 1,340,923. The total flow destined to GRU, the D_j value, will be estimated in the same way. The total flow having Brazil as destination coming from South America countries is 2,018,524. Then, the Factor B was multiplied by this total flow and the value achieved was 1,352,266, being now considered the D_j estimated value for GRU. The same methodology was used to estimate the other O_i and D_j values for the Brazilian airports, which is shown in Table 4.4.

Table 4.4: Estimation to the O_i and D_j values

Airport – State	**IATA Code**	O_i	D_j
Guarulhos – SP	GRU	1,340,923	1,352,266
Galeao – RJ	GIG	345,965	348,619
Salvador – BA	SSA	65,535	66,089
Brasilia – DF	BSB	11,559	11,657
Fortaleza – CE	FOR	42,516	42,875
Confins – MG	CNF	5,262	5,307
Pampulha – MG	PLU	239	241
Curitiba – PR	CWB	10,441	10,530
Manaus – AM	MAO	15,400	15,531
Recife – PE	REC	28,115	28,353
Porto Alegre – RS	POA	58,361	58,854
Belem – PA	BEL	9,776	9,858
Campinas – SP	VCP	371	374
Sao Luiz – MA	SLZ	168	169
Maceio – AL	MCZ	3,196	3,223
Natal – RN	NAT	34,730	35,024
Campo Grande – MS	CGR	1,874	1,890
Florianopolis – SC	FLN	25,045	25,257
Macapa – AP	MCP	1,081	1,090
Foz do Iguacu – PR	IGU	628	633
Boa Vista – RR	BVB	678	684
Total (PAX)		**2,001,593**	**2,018,524**

It is important to notice that these estimated flows are just within the South American continent. Although the data base used as the main source was related to the international market in general, the aim was to generate the flow to/from the other South American countries having Brazil as an origin and destination. It is also interesting to notice that the estimated flows in Table 4.4 match the observed flows in Table 4.3. This is because the estimated flow data is derived from the observed data. The estimated flow data in Table 4.4 is essential for the application of a gravitational model.

- ***Identification of O_i and D_j for the other South American Airports***

After the determination of the originated and destined flows from/to the 21 Brazilian airports was made, the same was to be done for the nine airports of the South American countries. This set of data is already available from Table 4.3, using the corresponding airport for each city shown. This correspondence is made on Table 4.1.

With these computations, the main data necessary to make the application of the gravitational model are available. In the next section, a description of the gravitational model will be made followed by its application using the data obtained in this section.

- ***Estimation of Flows Amongst Brazilian and South American Airports through a Gravitational Model***

Gravitational models are usually used for estimating the flow of people and/or goods amongst different regions in a circumscribed area. The model to be applied in this case study is defined below. In this specific case, the aim is to estimate a flow of passengers between two points *i* and *j*. In order to do this, some data has to be known: the distance between these points and the total flow originated in point *i* and the total flow destined to point *j*. The model to be applied is shown in Novaes (1986). The objective function (4.1) aims to estimate the flow between two points, *i* and *j* and is denoted by F_{ij}. This process is interactivity, as shown below.

$$F_{ij} = \lambda_i \mu_j \frac{O_i D_j}{(R_{ij})^{\beta}} \qquad (4.1)$$

Where:

O_i = Total flow originated in *i*;

D_j = Total flow destinated to *j*;

R_{ij} = Distance between *i* and *j*;

λ_i = Calibration coefficient, having a different value for every matrix row;

μ_j = Calibration coefficient, having a different value for every matrix column ;

β = Constant, with an adjustable value through calibration;

F_{ij} = Distributed trips;

According to the referred author, the summation might be done along rows or columns, summing along every row *i*, for example. It results in (Equation 4.2):

$$\sum_j F_{ij} = O_i = \lambda_i O_i \sum_j \frac{\mu_j D_j}{(R_{ij})^{\beta}} \qquad (4.2)$$

This produces (Equation 6.3):

$$\lambda_i = \left[\sum_j \frac{\mu_j D_j}{(R_{ij})^{\beta}} \right]^{-1} \text{ for } (i = 1, 2, ..., n) \qquad (4.3)$$

Summing through every column j, results in Equation 4.4:

$$\sum_i F_{ij} = D_j = \mu_j D_j \sum_i \frac{\lambda_i O_i}{(R_{ij})^{\beta}} \qquad (4.4)$$

Considering:

$$\mu_j = \left[\sum_i \frac{\lambda_i O_i}{(R_{ij})^{\beta}} \right]^{-1} \quad (4.5)$$

The precision levels are determined by the inequalities (4.6) and (4.7):

$$\left| \frac{\lambda_i^{(k)} - \lambda_i^{(k-1)}}{\lambda_i^{(k-1)}} \right| \leq \varepsilon \quad (4.6),$$

$$\left| \frac{\mu_j^{(k)} - \mu_j^{(k-1)}}{\mu_j^{(k-1)}} \right| \leq \varepsilon \quad (4.7)$$

The application of this model suggests the execution of the following steps (Novaes – 1986):

- Step 1: A value greater than 1 is given to the constant β ;
- Step 2: The value 1 is given for all of the coefficients μ_j, for $j = 1, 2, ..., n$;
- Step 3: The value of every λ_i is determined through Equation (4.3), using the initial values of μ_j (equal 1);
- Step 4: Calculate the new values of μ_j through the Equation (4.5);
- Step 5: The precision level is verified. In the case that this level does not achieve a pre-defined value, a new iteration is made. This process is repeated until the desired precision level is achieved.

The implementation of this model was made using the software Microsoft Office Excel 2007. An empirical value for β = 0.1 was used. A total of three iterations were necessary to achieve the precision level ε, which in the Case Study was set empirically to 0.1%, meaning that the overall precision was 99.9%. The final values of λ_i and μ_j achieved in the third interaction are shown in Table 4.5.

Table 4.5: Final Values of λ_i and μ_j in the Last Iteration

Code	λ_i (x 10^{-7})	μ_i		Code	λ_i (x 10^{-7})	μ_i
GRU	1.0699	1.8890		NAT	1.1469	2.0257
GIG	1.0849	1.9156		CGR	1.0544	1.8602
SSA	1.1242	1.9854		FLN	1.0460	1.8469
BSB	1.0888	1.9221		MCP	1.1345	2.0034
FOR	1.14773	2.0270		IGU	1.0244	1.8068
CNF	1.0905	1.9255		BVB	1.1159	1.9704
PLU	1.0874	1.9199		EZE	0.5539	0.9863
CWB	1.0489	1.8517		MVD	0.5492	0.9780
MAO	1.1159	1.9704		SCL	0.5762	1.0259
REC	1.1426	2.0181		ASU	0.5347	0.9520
POA	1.0148	1.7924		VVI	0.5572	0.9922
BEL	1.1359	2.0059		LIM	0.5907	1.0518
VCP	1.0658	1.8816		UIO	0.6021	1.0722
SLZ	1.1393	2.0120		BOG	0.6013	1.0707
MCZ	1.1405	2.0143		CCS	0.6007	1.0697

4.2.3 Identification of Flows amongst the South American Airports

In this part, data extracted from CLAC (2007) in relation to the passenger movement in the year 2007 will be used. This set of data is shown in Table 4.6 and represents the part M_4 illustrated in Figure 4.1.

Table 4.6: Sub-Matrix M_4 - Flow amongst 9 Remaining of South American Airports

W_{ij} (10^3)	EZE	MVD	SCL	ASU	VVI	LIM	UIO	BOG	CCS
EZE	0	214.4	584.1	124.2	59.7	200.5	5.2	47.2	24.7
MVD	214.4	0	55.4	9.4	0	0	0	0	0
SCL	584.1	55.4	0	18.6	6.1	207.5	20.1	47.7	15.7
ASU	124.3	9.4	18.6	0	16.9	0	0	0	0
VVI	59.8	0	6.1	16.9	0	15.1	0	708	340
LIM	200.5	0	207.5	0	15.1	0	77.8	91.6	135.5
UIO	5.2	0	20.1	0	0	77.8	0	149.2	20.6
BOG	47.2	0	47.7	0	708	91.6	149.2	0	235.4
CCS	24.7	0	15.7	0	30	135.5	20.6	235.4	0

Source: CLAC (2007)

4.2.4 The Major/Mini/Direct p-Hub Location Model

The mathematical modeling in this section is strongly based on the study made by O`Kelly (1998). In contrast with this referred study, only the flow of passengers will be considered. The modeling in this section will consider just passenger movements. The number of Brazilian airports to be used in this study is 41 and for the other countries of South America is nine. Table 4.7 below lists all the airports of this Case Study.

Table 4.7: List of Airports in the Case Study

#	Airport Name/City/State/Country	IATA Code	#	Airport Name/City/State/Country	IATA Code
01	Presidente Medice – Rio Branco – AC – Brazil	RBR	26	Afonso Pena – Curitiba – PR – Brazil	CWB
02	Campo dos Palmares – Maceio – AL – Brazil	MCZ	27	Londrina – PR – Brazil	LDB
03	Eduardo Gomes – Manaus – AM – Brazil	MAO	28	Galeao – Rio de Janeiro – RJ – Brazil	GIG
04	Macapa – AP – Brazil	MCP	29	Santos Dumont – Rio de Janeiro – RJ – Brazil	SDU
05	Ilheus – BA – Brazil	IOS	30	Augusto Severo – Natal – RN – Brazil	NAT
06	Dois de Julho – Salvador – BA	SSA	31	Porto Velho – RO – Brazil	PVH

	– Brazil				
07	Pinto Martins – Fortaleza – CE – Brazil	FOR	32	Boa Vista – RR – Brazil	BVB
08	Brasilia – DF – Brazil	BSB	33	Salgado Filho – Porto Alegre – RS – Brazil	POA
09	Goiabeiras – Vitoria – ES – Brazil	VIX	34	Hercilio Luz – Florianopolis – SC – Brazil	FLN
10	Santa Genoveva – Goiania – GO – Brazil	GYN	35	Joinville – SC – Brazil	JOI
11	Imperatriz – MA – Brazil	IMP	36	Navegantes – SC – Brazil	NVT
12	Marechal Cunha Machado – Sao Luis – MA – Brazil	SLZ	37	Santa Maria – Aracaju – SE – Brazil	AJU
13	Tancredo Neves – Belo Horizonte – MG – Brazil	CNF	38	Congonhas – Sao Paulo – SP – Brazil	CGH
14	Pampulha – Belo Horizonte – MG – Brazil	PLU	39	Guarulhos – SP – Brazil	GRU
15	Uberlandia – MG – Brazil	UDI	40	Viracopos – Campinas – SP – Brazil	VCP
16	Campo Grande – MS – Brazil	CGR	41	Palmas – TO – Brazil	PMW
17	Marechal Rondon – Cuiaba – MT – Brazil	CGB	42	Buenos Aires – Argentina	EZE
18	Val de Caes – Belem – PA – Brazil	BEL	43	Montevideo – Uruguay	MVD
19	Maraba – PA – Brazil	MAB	44	Santiago – Chile	SCL
20	Santarem – PA – Brazil	STM	45	Assuncion – Paraguay	ASU
21	Castro Pinto – Joao Pessoa – PB – Brazil	JPA	46	Santa Cruz de La Sierra – Bolivia	VVI
22	Guararapes – Recife – PE – Brazil	REC	47	Lima – Peru	LIM
23	Petrolina – PE – Brazil	PNZ	48	Quito – Ecuador	UIO
24	Teresina – PI – Brazil	THE	49	Bogota – Colombia	BOG
25	Cataratas – Foz do Iguacu – PR – Brazil	IGU	50	Caracas – Venezuela	CCS

This model allows three types of service: a) Directly or Non-Stop; b) through a major-hub k; and c) through a mini-hub m. When the demand between two points i and j is greater than a certain value, this flow can be sent without consolidation.

The objective function (4.8) aims the minimization of the overall costs considering three expressions: the first one regards the transportation flow through a major-hub; the second one regards a transportation flow using a mini-hub and the third expression regards the transportation flow that is made directly (non-stop). The constraint sets (4.9) and (4.10) determine the number of major-hubs and mini-hubs to be opened, respectively. Constraints sets (4.11) impose that the flow from $i \; to \; j$ is made through just only one path (through a major hub, through a mini hub or directly). Constraints (4.12) and (4.13) say that a path is valid only if a major-hub and a mini-hub is already established, respectively. Constraints (4.14) say that if the flow between *i* and *j* is greater or equal a minimum threshold Γ_s, this flow will be sent directly. The set of constraints (4.15) says that there must exist a maximum radius distance service S_1 for an operation of a mini-hub *m* and the constraints (4.16) impose that if the distance between a pair of points (*i*,*j*) is smaller than a parameter S_2, this flow must be sent through a mini-hub *m*.

Service types:

(a) Direct;

(b) Through a major-hub k;

(c) Through a mini-hub m.

$$Min\, Z = \sum_{i\in N}\sum_{j\in N}\sum_{k\in N} W_{ij}Y_{ijk}\alpha\left(C_{ik}+C_{kj}\right) + \sum_{i\in N}\sum_{j\in N}\sum_{m\in N} W_{ij}X_{ijm}\gamma\left(C_{im}+C_{mj}\right) + \sum_{i\in N}\sum_{j\in N} W_{ij}C_{ij}Z_{ij} \quad (4.8)$$

s.t.:

$$\sum_{k\in N} H_k = p \qquad (4.9)$$

$$\sum_{m\in N} T_m = q \qquad (4.10)$$

$$\sum_{k\in N} Y_{ijk} + \sum_{m\in N} X_{ijm} + Z_{ij} = 1, \qquad \forall i,j \qquad (4.11)$$

$$Y_{ijk} - H_k \leq 0, \qquad \forall i,j,k \qquad (4.12)$$

$$X_{ijm} - T_m \leq 0, \qquad \forall i,j,m \qquad (4.13)$$

$$W_{ij} - \Gamma_s Z_{ij} \geq 0, \qquad \forall i,j \qquad (4.14)$$

$$X_{ijm}\left[C_{im} + C_{mj}\right] \leq S_1, \qquad \forall i,j,m \qquad (4.15)$$

$$X_{ijm}\left(C_{ij}\right) \leq S_2\,, \qquad \forall i,j,m \qquad (4.16)$$

$$H_k, T_m, Y_{ijk}, X_{ijm}, Z_{ij} \in \{0,1\} \qquad \forall i,j,k,m \qquad (4.17)$$

Where:

W_{ij}: *flow between i and j*;

C_{ij}: *cost between i and j*;

C_{ik}: *cost between an origin i and a major hub k*;

C_{im}: *cost between an origin i and a mini hub m*;

α: *discount factor in the linkages using major hub(s)*. $(0 \leq \alpha \leq 1)$;

γ: *discount factor in the linkages using mini hub(s)*. $(\alpha < \gamma)$;

Y_{ijk}: 1, *if the flow from an origin i to a destination j is sent through a major hub k*;

0, *otherwise*;

X_{ijm}: 1, *if the flow from an origin i to a destination j is sent through a mini hub m*;

0, *otherwise*;

Z_{ij}: 1, *if the flow from an origin i to a destination j is sent directly (non stop)*;

0, *otherwise*;

p: *number of major hub(s) to be opened*;

q: *number of mini hub(s) to be opened*;

H_k: 1, *if the node k is a major hub*; 0, *otherwise*;

T_m: 1, *if the node m is a mini hub*; 0, *otherwise*;

Γ_S: minimum threshold to open a non stop route;

S_1: $radius\ maximum\ distance\ service\ for\ an\ operation\ of\ a\ mini - hub$;

S_2: $minimum\ distance\ between\ a\ pair\ (i, j)\ that\ determines\ an\ allocation\ to\ a\ mini - hub$

It is important to outline the role of the strategic parameter decisions. Table 4.8 shows the list of them, with their respective description.

Table 4.8: Description of Strategic Parameters Decisions

Strategic Parameter	Description
p	Number of major hubs to be located by the model.
q	Number of mini hubs to be located.
S_1	Maximum distance radius of a mini hub. An increase in its value means that the operations range of a mini hub also increases, bringing more nodes assignment to the mini hubs. It refers to the combination of the total distance transverse from an origin to a destination, through a mini hub.
S_2	Minimum separation distance between a pair of nodes (i,j), meaning that a separation distance greater than S_2 will disallow a pair of nodes to be assigned to a mini hub.
α	Discount factor used to denote gains in economies of scale in the operations of a major hub.
γ	Discount factor used to denote gains in economies of scale in the operations of a mini hub. It is assumed that $\alpha < \gamma$ because of the level of concentration in the major hubs.

4.2.5 The Results

To obtain the solution for the major and mini hub locations, the allocation of the spoke nodes to these hubs and the definition of which linkages would be made direct, the optimization software AIMMS – The Modeling System - Non-Commercial Educational version 3.9 was used, a registered license from the Industrial Engineering Department – Pontificial Catholic University of Rio de Janeiro (PUC-Rio) - Brazil. All the maps used to illustrate the allocation patterns and the visualization of the flows were made using the Transportation software

Transcad – Academic Version 4.5, a registered license from the Department of Geography – The Ohio State University (OSU) - USA.

In order to facilitate the analysis and the visualization of the flow and the allocation patterns, the set of 50 airports used in this Case Study (showed in Table X) was divided into five blocks, with 10 in each, as shown in the Figure 4.2 (with different colors identifying the nodes in each block), and outlined in Tables 4.9, 4.10 and 4.11 as follows. The block one represents the airports located in the south side of the South American Continent. Block two congregates the airports located in the southeast side. Blocks three, four and five represent the airports in the central-north, northeast and northwest sides, respectively. It is very important to outline that the 50 x 50 flow matrix built is sparse and asymmetric. As noticed, the number of cells in this matrix with flows greater than zero was just 1,038.

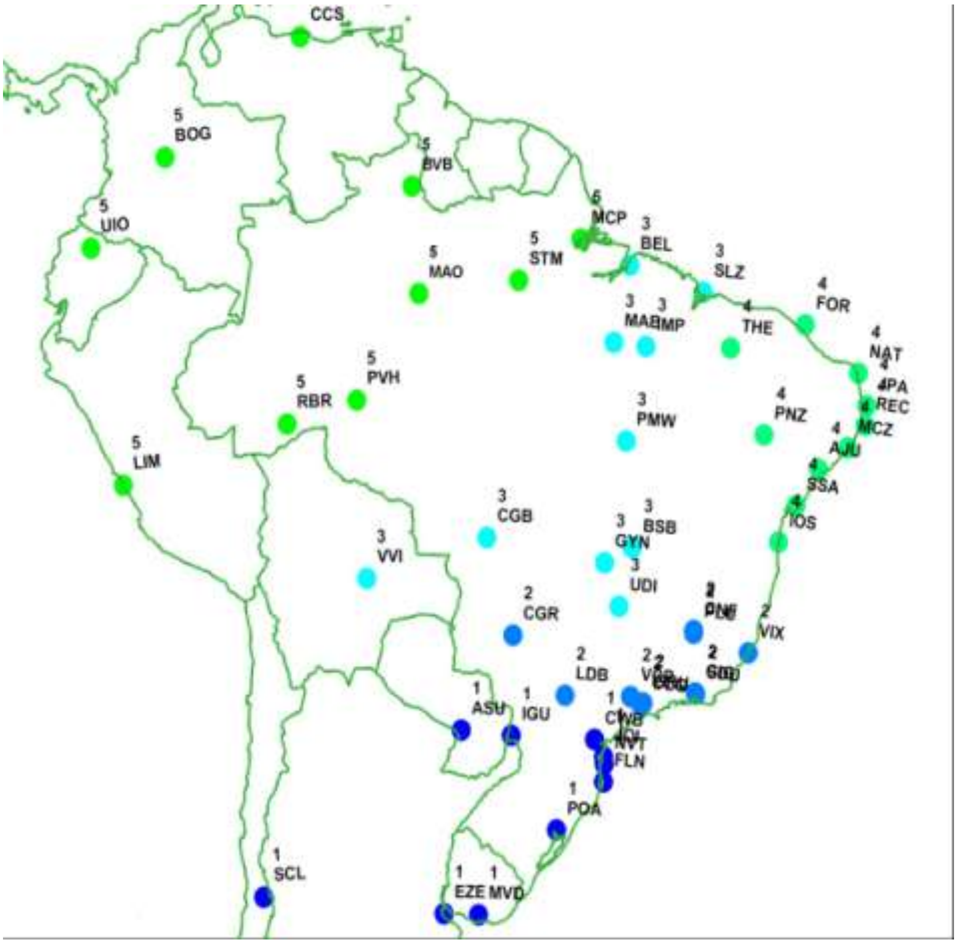

Figure 4.2: Set of Five Blocks and Its Airports

Legend:

Block 1: South Side; Block 2: Southeast Side; Block 3: Central-North Side

Block 4: Northeast Side; Block 5: Northwest Side.

Table 4.9: List of Airports in Blocks 1 (South Side) and 2 (Southeast Side)

Block 1 – South Side			Block 2 – Southeast Side	
IATA Code	**City/State/Country**		**IATA Code**	**City/State/Country**
ASU	Assuncion – Paraguay		CGH	São Paulo (Congonhas) – SP – Brazil
CWB	Curitiba – PR - Brazil		CGR	Campo Grande – MS – Brazil
EZE	Buenos Aires – Argentina		CNF	Belo Horizonte (Confins) – MG – Brazil
FLN	Florianópolis – SC – Brazil		GIG	Rio de Janeiro (Galeão) – RJ – Brazil
IGU	Foz do Iguaçu – PR – Brazil		GRU	Guarulhos – SP – Brazil
JOI	Joinville – SC – Brazil		LDB	Londrina – PR – Brazil
MVD	Montevideo – Uruguay		PLU	Belo Horizonte (Pampulha) – MG – Brazil
NVT	Navegantes – SC – Brazil		SDU	Rio de Janeiro (Santos Dumont) – RJ – Brazil
POA	Porto Alegre – RS – Brazil		VCP	Campinas (Viracopos) – SP – Brazil
SCL	Santiago – Chile		VIX	Vitória – ES – Brazil

Table 4.10: List of Airports in Blocks 3 (Central-North) and 4 (Northeast Side)

Block 3 – Central-North Side			Block 2 – Northeast Side	
IATA Code	**City/State/Country**		**IATA Code**	**City/State/Country**
BEL	Belém-PA – Brazil		AJU	Aracaju – SE – Brazil
BSB	Brasilia – DF – Brazil		FOR	Fortaleza – CE – Brazil
CGB	Cuiabá – MT – Brazil		IOS	Ilhéus – BA – Brazil
GYN	Goiânia – GO - Brazil		JPA	João Pessoa – PB – Brazil
IMP	Imperatriz – MA – Brazil		MCZ	Maceió – AL – Brazil
MAB	Marabá – PA – Brazil		NAT	Natal – RN – Brazil
PMW	Palmas – To – Brazil		PNZ	Petrolina – PE – Brazil
SLZ	São Luis – MA - Brazil		REC	Recife – PE – Brazil
UDI	Uberlândia – MG – Brazil		SSA	Salvador – BA – Brazil
VVI	Santa Cruz de La Sierra - Bolivia		THE	Teresina – PI – Brazil

Table 4.11: List of Airports in Block 5 – Northwest Side

IATA Code	City/State/Country
BOG	Bogotá – Colômbia
BVB	Boa Vista – RR – Brazil
CCS	Caracas – Venezuela
LIM	Lima – Peru
MAO	Manaus – AM – Brazil
MCP	Macapá – AP – Brazil
PVH	Porto Velho – RO – Brazil
RBR	Rio Branco – AC – Brazil
STM	Santarém – PA
UIO	Quito – Ecuador

In order to make a complete analysis of the results, four experiments were made. All the parameters were kept the same for the four trials, with an exception for the S_1 and S_2 values. Table 4.12 lists all these parameters and their respective values. The definition of these parameters was already shown in the model's description.

Table 4.12: List of the Parameters Used in the Model and Their Values

Parameter	Value
α	0.6
γ	0.8
p	2
q	3
Γ_s	200,000

Table 4.13 shows the variations in S_1 and S_2 for the four experiments.

Table 4.13: List of the Four Experiments and the S_1 and S_2 Values in Kilometers

Experiment	**S_1(km)**	**S_2(km)**
F1	1,000	1,250
F2	1,250	1,500
F3	1,500	1,750
F4	1,750	2,000

As already outlined in the model's description, the S_1 parameter in conjunction with the constraint (4.15) refers to the allowed distance to be transverse between two nodes using a mini-hub. If the sum of the distance from a spoke origin node to a mini-hub and from this mini-hub to a spoke destination node is less or equal the parameter S_1, this flow may be routed via this mini-hub. If the distance between two nodes is less or equal a parameter distance S_2, this flow may be routed via a mini-hub, as shown again in the constraints (4.15) e (4.16).

$$X_{ijm}\left[C_{im} + C_{mj}\right] \leq S_1, \qquad \forall i, j, m \qquad (4.15)$$

$$X_{ijm}\left(C_{ij}\right) \leq S_2\,, \qquad \forall i, j, m \qquad (4.16)$$

Table 4.14 shows the detailed results obtained for the four experiments: F1, F2, F3 and F4. In this table, the information about major hub location, mini hub location, the value of the objective function and the number of variables and constraints are shown. The number of variables and constraints are the same for the four experiments, which are 252,601 variables and 502,453 constraints, respectively.

Table 4.14: Detailed Information for the Four Experiments

Exp.	Major Hubs Chosen	Mini Hubs Chosen	Objective Function Value (x 10^{21})	S_1(km)	S_2(km)
F1	THE and CGH	MCZ, GYN and BOG	6.8159108160	1,000	1,250
F2	THE and CGH	SSA, BSB and BOG	6.8156409760	1,250	1,500
F3	THE and CGH	SSA, BSB and UIO	6.8150724350	1,500	1,750
F4	SLZ and CGH	SSA, BSB and UIO	6.8144306300	1,750	2,000

A remarkable difference between this type of model as compared to the traditional hub location problems is the number of variables in the optimal solution. In a study made by Figueiredo and Pizzolato (2009), in a network with just 25 nodes, the number of variables achieved in the optimal solution was 390,651 for the multiple assignment p-hub location problem and 391,276 for the single assignment p-hub location problem.

4.2.6 The Detailed Analysis of the Experiments

An important characteristic of this model is the allowance of direct routes between nodes, through some threshold. The value set for that was 200,000 passengers. This means that, if a flow between two airports *i* and *j* is greater or equal than 200,000 passengers a year, it will be made directly and therefore will not be necessary to make a stop or connection in a major or a mini hub.

As long as this value was set for the four experiments, the pattern of these flows will be equal for all of them. Figure 4.3 shows the linkages that were set to be made directly.

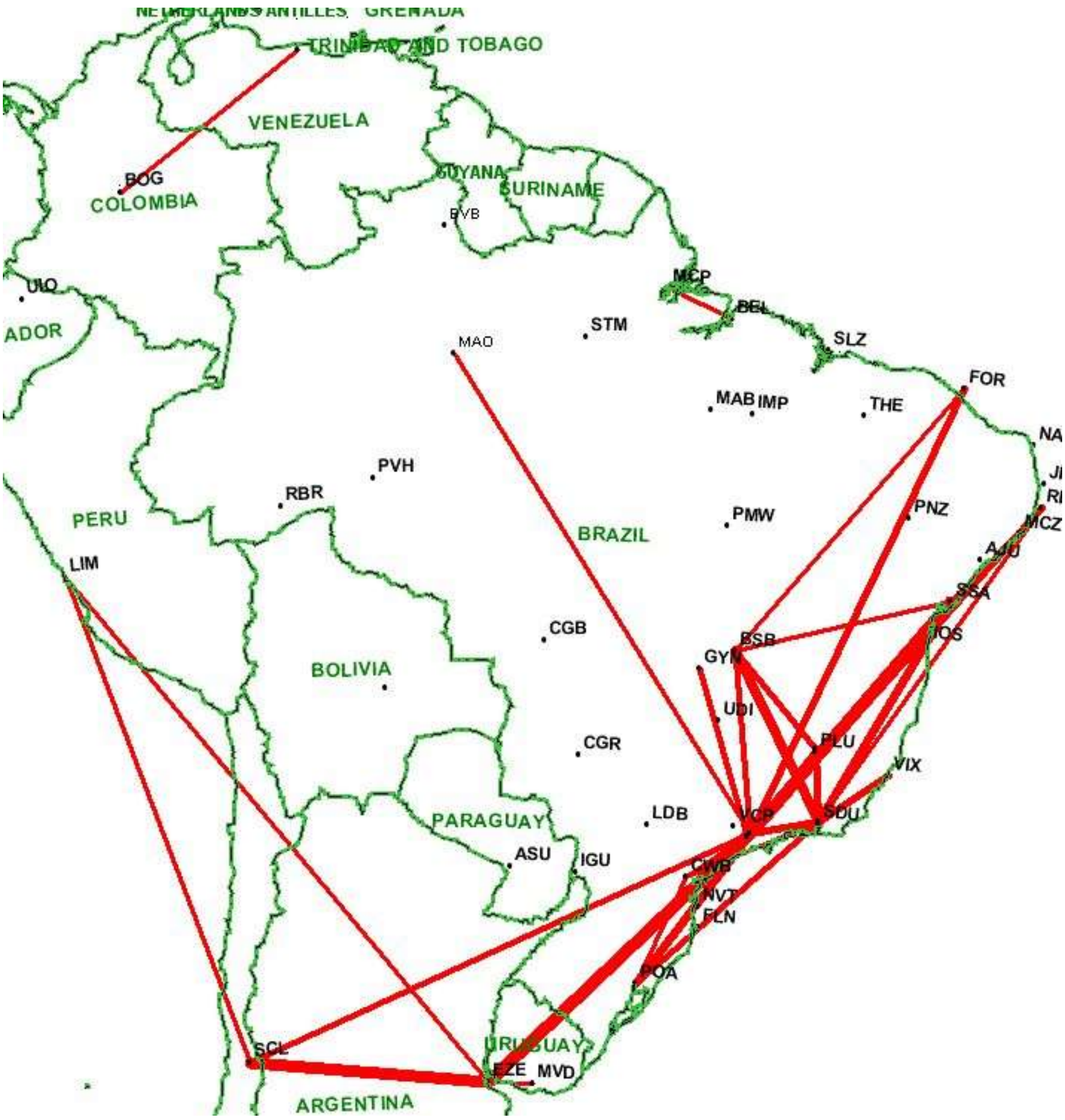

Figure 4.3: The Pattern of the Direct Links

In total, 74 linkages were set to be made directly. The main 16 linkages are shown in Table 4.15, with the thickness of the line representing the density of a linkage.

Table 4.15: The Main 16 Direct Linkages

Rank	Linkage		Rank	Linkage
1	CGH – SDU		9	EZE – SCL
2	SDU – CGH		10	SCL – EZE
3	GRU – EZE		11	GIG – BSB
4	EZE – GRU		12	BSB – GIG
5	CGH – BSB		13	CGH – POA
6	BSB – CGH		14	CGH – CWB
7	SSA – GRU		15	CGH – CNF
8	GRU – SSA		16	CWB – CGH

This type of configuration allows a less inconvenient travel for passengers in a dense linkage – set here as to be greater or equal 200,000 PAX/year - and enables an air company to increase the load factor of the airplanes. It also helps the splitting of the flows through the network, avoiding for passengers unnecessary stops in a major or mini hub, diminishing the congestion at these points.

- ***The F1 Experiment***

In the F1 experiment, as already shown in Table 4.14, the locations chosen to be major hubs were the airport of Teresina (THE), in the Brazilian state of Piaui (northeast side of the country) and the airport of Congonhas (CGH), located in the city of Sao Paulo, in the Brazilian state of Sao Paulo (southeast side of the South American continent). Mini hubs were chosen to be in the airports of Maceio (MCZ), in the Brazilian state of Alagoas, also in the northeast side of the South American continent, the airport of Goiania (GYN), in the state of Goias (central-north side of the country), and the airport of Bogota (BOG), located in Colombia, located in the northwest side of the South American continent. Only the block 1 did not have any major or mini hub located in this region. Figure 4.4 shows the allocation pattern for these major hubs.

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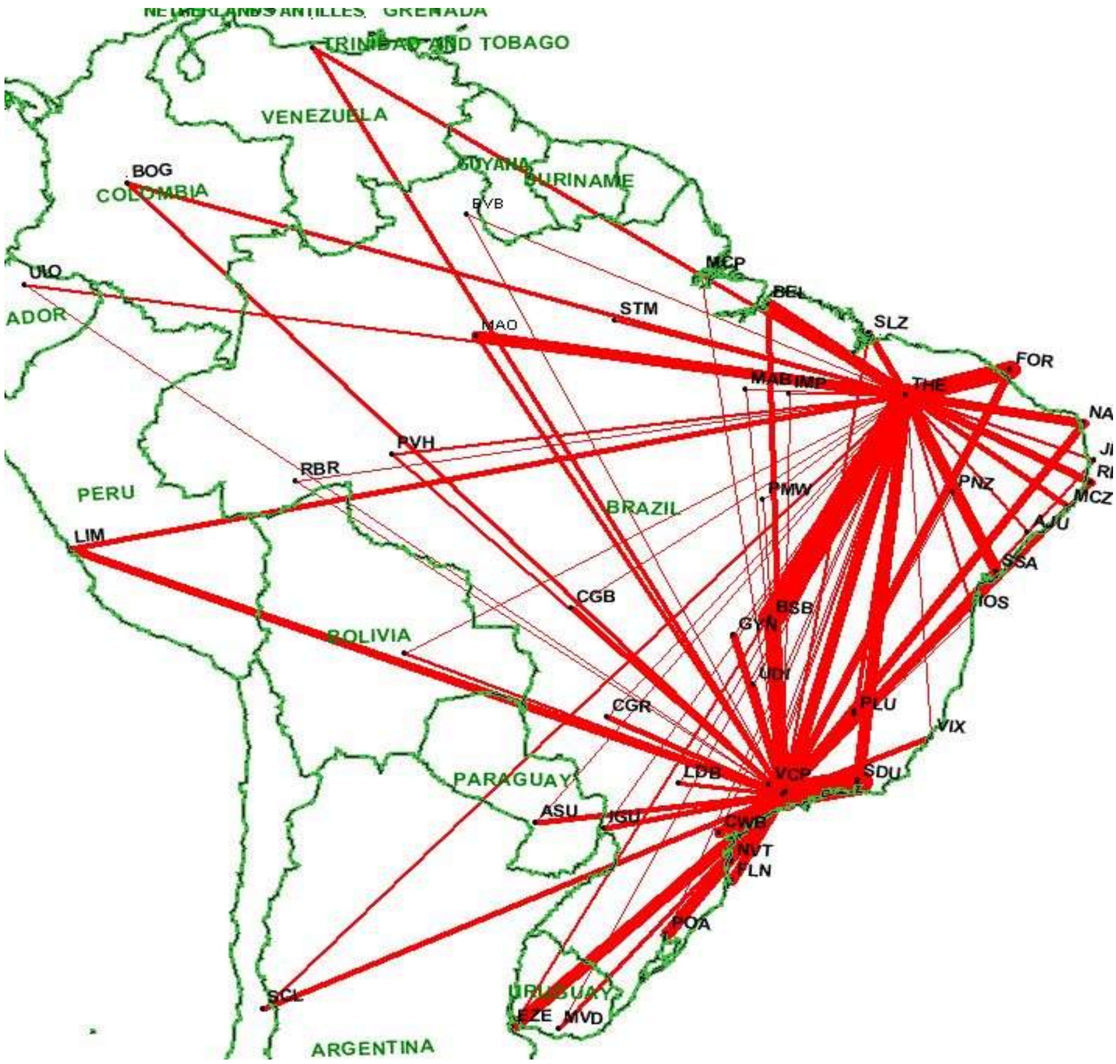

Figure 4.4: The Allocation and Flow Patterns for the Major Hubs – F1 Experiment

Table 4.16 shows the interaction amongst the five blocks in the F1 experiment. It is possible to notice that all the interactions between the following pairs of blocks is made through the major hub CGH. They are: from block 1 to blocks 1 and 2; from block 2 to blocks 1 and 2; from block 3 to block 2; from block 4 to blocks 1 and 2 and from block 5 to blocks 1 and 2. Just the flows that were chosen to be made directly (as shown in Figure 4.3) are exceptions.

The allocation pattern has some peculiarities. Regarding the flow interactions between blocks 1 and 3 (from block 1 to block 3), the linkages BEL-ASU and VVI-ASU were chosen to be routed through the major hub THE, in spite of CGH. If just the inflow patterns to the airport of Belem (BEL) – located in block 3 - are considered, there is a single allocation for this point to the major hub in THE. The outflow of this airport is made either to THE and CGH and it does not use any mini hub to interact with any other destination.

Table 4.16: The Interaction Amongst Blocks – F1 Experiment

Block	1	2	3	4	5
1	CGH	CGH	CGH, THE and GYN	THE	CGH and THE
2	CGH	CGH	CGH, THE and GYN	THE	CGH and THE
3	CGH and THE	CGH	CGH, THE and GYN	THE	CGH and THE
4	CGH	CGH	CGH and THE	THE and MCZ	CGH and THE
5	CGH	CGH	CGH and THE	THE	CGH, THE and BOG

Another interesting feature noticed is the flow interaction having block 4 as a destination. Blocks 1, 2, 3 and 5 interact with block 4 through the major hub THE (with exceptions for the direct linkages). Only the interactions within block 4 (from the nodes in block 4 to the nodes in block 4) are made using more than one node. The flow between JPA (origin) and AJU (destination) is made through CGH. If the inflow to the nodes of JPA and REC is taken into consideration, there are also single allocations to the mini hub MCZ. Block 5 only uses the major hubs CGH and THE to route its flows and does not use any mini hub for the interactions.

In total, 559 interactions were set to be made through the major hub CGH, representing 53.85 % of the pairs of flows to be routed, and 355 through the major hub THE, representing 34.2% the pairs of flows to be routed. Summing up all the pair of flows that uses major hubs CGH and THE, the percentage of 88.05 % is obtained. If the direct linkages are added to this number, only 4.82% of the remaining pairs of flows will be routed through the mini hubs.

Figure 4.5 shows the allocation patterns for the mini hubs MCZ, GYN and BOG.

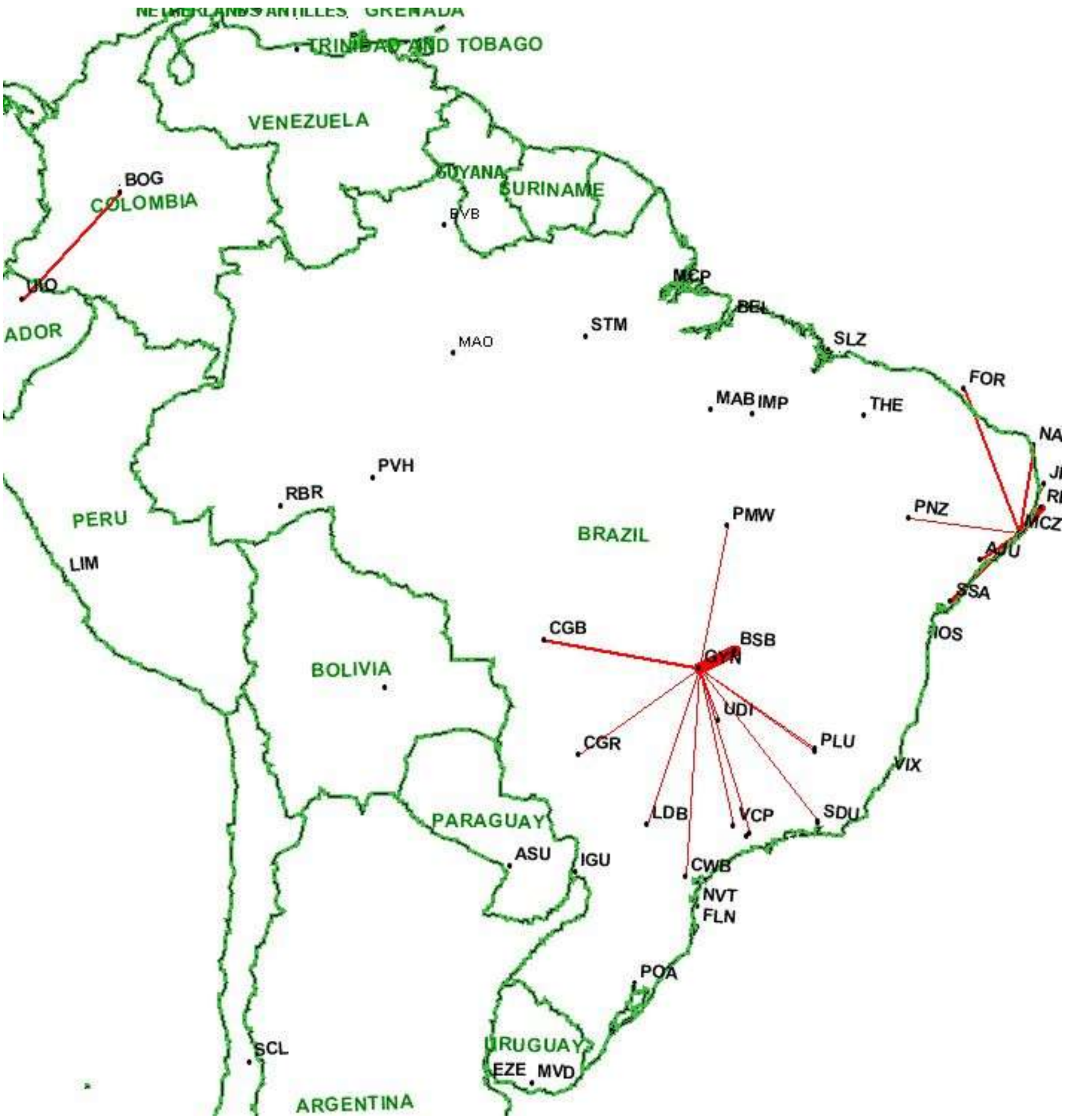

Figure 4.5: The Allocation Patterns for the Mini Hubs – F1 Experiment

The pair of flows that are routed through MCZ is 25 – which also include MCZ as either an origin or destination. This represents just 2.41% of the total pair of flows to be routed. The flows that are routed through GYN is 23 and represents 2.22% of the total pair of flows. The mini hub BOG was only used for the flows that had BOG as an origin or a destination. Table 4.17 lists the pair of nodes that are routed through MCZ and GYN.

Table 4.17: Routes that use the Mini Hubs MCZ and GYN in the F1 Experiment

Through MCZ		Through GYN
SSA – JPA	REC – NAT	BSB – UDI
SSA – NAT	REC – AJU	BSB – CGB
SSA – AJU	PNZ – REC	CNF – UDI
FOR – REC	NAT – SSA	PLU – BSB
FOR – AJU	NAT – JPA	UDI – BSB
JPA – SSA	NAT – REC	CGR – BSB
JPA – REC	AJU – SSA	CGB – BSB
JPA – NAT	AJU – JPA	LDB – BSB
REC – JPA	AJU – REC	VCP – BSB
-		PMW – BSB

The small number of mini hub allocations can be partially explained by the values set for the parameters S_1 and S_2. As long as the range of these values is increased, it is expected that the number of flows to be routed through the mini hubs also increases. In the following experiment, F2, the values of these parameters were increased to 1,250 and 1,500 km for S_1 and S_2, respectively. The analysis of this experiment is presented in the next section.

- ***The F2 Experiment***

In this experiment, the same locations chosen to be major hubs in the previous experiment were chosen to be major hubs in this one: CGH and THE. The mini hub locations chosen were: the airport of Salvador (SSA), in the Brazilian state of Bahia (northeast side of South American continent); the airport of Brasilia (BSB), the federal capital of the country, located in the center-north side of South American continent; and the airport of Bogota (BOG), in Colombia, in the north-west side of the South American continent.

Figure 4.6 shows all the allocations to the major hubs CGH and THE and the pattern of the flows.

Figure 4.6: The Allocation and Flow Patterns for the Major Hubs – F2 Experiment

Regarding the density of flows, there is a slight difference between the two experiments, especially for the following nodes (all of them located in block 4): IOS, SSA, REC, JPA and NAT. The linkage among these nodes and the major hub THE presents some changes. The linkages IOS-THE and SSA-THE in the F1 experiment is thinner than in the F2 experiment. On the other hand, the linkages REC-THE, JPA-THE and NAT-THE have become thicker. This can be partially explained by the change in the mini hub location for block 4: SSA, in spite of MCZ. Table 4.18 shows the interaction amongst the blocks in the experiment F2, with some exceptions for the direct linkages.

Table 4.18: The Interaction Amongst Blocks – F2 Experiment

Block	**1**	**2**	**3**	**4**	**5**
1	CGH	CGH	CGH and THE	THE	CGH and THE
2	CGH	CGH	CGH, THE and BSB	THE	CGH and THE
3	CGH and THE	CGH	CGH, THE and BSB	THE	CGH and THE
4	CGH	CGH	CGH and THE	CGH, THE and SSA	CGH and THE
5	CGH	CGH	CGH, THE and BSB	THE	CGH, THE and BOG

The increase in the parameters S_1 and S_2 was not sensitive for important changes in the allocation patterns. Figure 4.7 shows all allocation for the mini hubs SSA, BSB and BOG.

Figure 4.7: The Allocation and Flow Patterns for the Mini Hubs – F2 Experiment

Some important differences were noticed. In the F1 experiment, the group of airports located in Sao Paulo (CGH, GRU and VCP), Rio de Janeiro (GIG and SDU) and in Belo Horizonte (PLU and CNF) did not have a dense linkage with the mini hub GYN. In the F2 experiment, this linkage has increased considerably, as can be noticed in Figure 4.7. Despite these changes in the allocation, the overall scenario has not been changed much. Around 88% of all of the pair of flows was still routed either through CGH or THE. The remaining pairs of flows were being routed through the mini hubs SSA, BSB and BOG (with the last one accounting for only the flows that have BOG as an origin or a destination). Table 4.19 shows the linkages that use the mini hubs SSA and BSB for interactions – not considering those that have both SSA and BSB as origin or destination.

Table 4.19: Routes that use the Mini Hubs SSA and BSB in the F2 experiment

Through BSB		**Through SSA**
RBR - GYN	CGB – GYN	MCZ – REC
GYN – UDI	LDB – GYN	MCZ – AJU
GYN – CGB	GIG – GYN	REC – MCZ
CNF – UDI	CGH – UDI	REC – AJU
PLU – GYN	GRU – GYN	PNZ – REC
PLU – UDI	GRU – UDI	AJU – MCZ
UDI – GYN	VCP – UDI	AJU – REC
CGR – GYN	PMW – GYN	-

- ***The F3 Experiment***

In this experiment, the airports chosen to be major hubs were the same chosen by the first experiment: THE and CGH. The locations chosen to be mini hubs were SSA, BSB and UIO. Table 4.20 shows the interaction amongst blocks and Figure 4.8 shows the allocation and the flow pattern for the F3 experiment.

Table 4.20: The Interaction Amongst Blocks – F3 Experiment

Block	**1**	**2**	**3**	**4**	**5**
1	CGH	CGH	CGH, THE and BSB	CGH and THE	CGH and THE
2	CGH	CGH	CGH, THE and BSB	THE and SSA	CGH and THE
3	CGH	CGH	CGH, THE and BSB	THE	CGH and THE
4	CGH	CGH	CGH, THE and BSB	CGH, THE and SSA	CGH and THE
5	CGH	CGH	CGH and THE	THE	CGH, THE and UIO

Figure 4.8: The Allocation and Flow Patterns for the Major Hubs – F3 Experiment

In the interactions from Block 3 to Block 1, a slight change was noticed: the flow from VVI to ASU was not set to be routed through THE anymore. Now it uses CGH as an intermediate point, making the node ASU single allocated to CGH always when it is a destination. In the interactions from Block 3 to Block 3, the flow from GYN to PMW now uses the mini hub BSB, despite the major hub THE. The flows from CNF to AJU and IOS that were routed through THE in the F2 experiment, now are routed through the mini hub SSA.

Figure 4.9 shows the allocation for the mini hubs SSA, BSB and UIO.

Figure 4.9: The Allocation and Flow Patterns for the Mini Hubs – F3 Experiment

As expected, the number of allocations to the mini hubs has increased: from 45 allocations in the F2 experiment to 67 allocations in the F3 experiment. The nodes MAB, IMP, ASU, CWB and VIX started to use the mini hubs to route some flows – which was not being noticed in the F2 experiment. Even though these nodes had multiple allocations, they were only allocated to major hubs and did not have any allocation to mini hubs.

Another interesting feature noticed was the spreading of flows through mini hubs. The airports of Sao Paulo (VCP, CGH and GRU) and Minas Gerais (CNF and PLU) started to use the mini hub SSA to route some flows. The airports of AJU, MCZ, PNZ and VIX started out to route some flows through mini hub

BSB. All of these changes can be attributed to the increase in the service radius distance range for the mini hubs.

Table 4.21 shows the routes that use the mini hubs BSB and SSA in the F3 experiment.

Table 4.21: Routes that use the Mini Hubs SSA and BSB in the F3 experiment

Through BSB		**Through SSA**
GYN-CGB	GIG-GYN	MCZ-REC
GYN-PMW	CGH-UDI	MCZ-AJU
CNF-UDI	CGH-PMW	CNF-IOS
PLU-GYN	GRU-GYN	CNF-AJU
PLU-UDI	GRU-UDI	REC-MCZ
UDI-GYN	GRU-PMW	REC-AJU
CGR-GYN	VCP-UDI	PNZ-REC
CGB-GYN	PMW-GYN	AJU-MCZ
LDB-GYN	-	AJU-REC

- ***The F4 Experiment***

Unlike the other 3 experiments, the airports chosen to be major hub in this last F4 experiment were the nodes CGH and SLZ. The mini hubs chosen were SSA, BSB and UIO. Figure 4.10 shows the allocation and flow patterns for the major hubs in the F4 experiment. As can be noticed, the major change was in the decision about the location of the major hub: despite the choice of THE, the model has chosen the airport of SLZ, located in the city of Sao Luis, in the Brazilian state of Maranhao, in the northeast side of the South American continent. These airports, THE and SLZ, are relatively close to each other: about 350 km. The number of pair of nodes that use the major hubs to route its flows has decreased: from 897 in the F3 experiment to 841 in the F4 experiment.

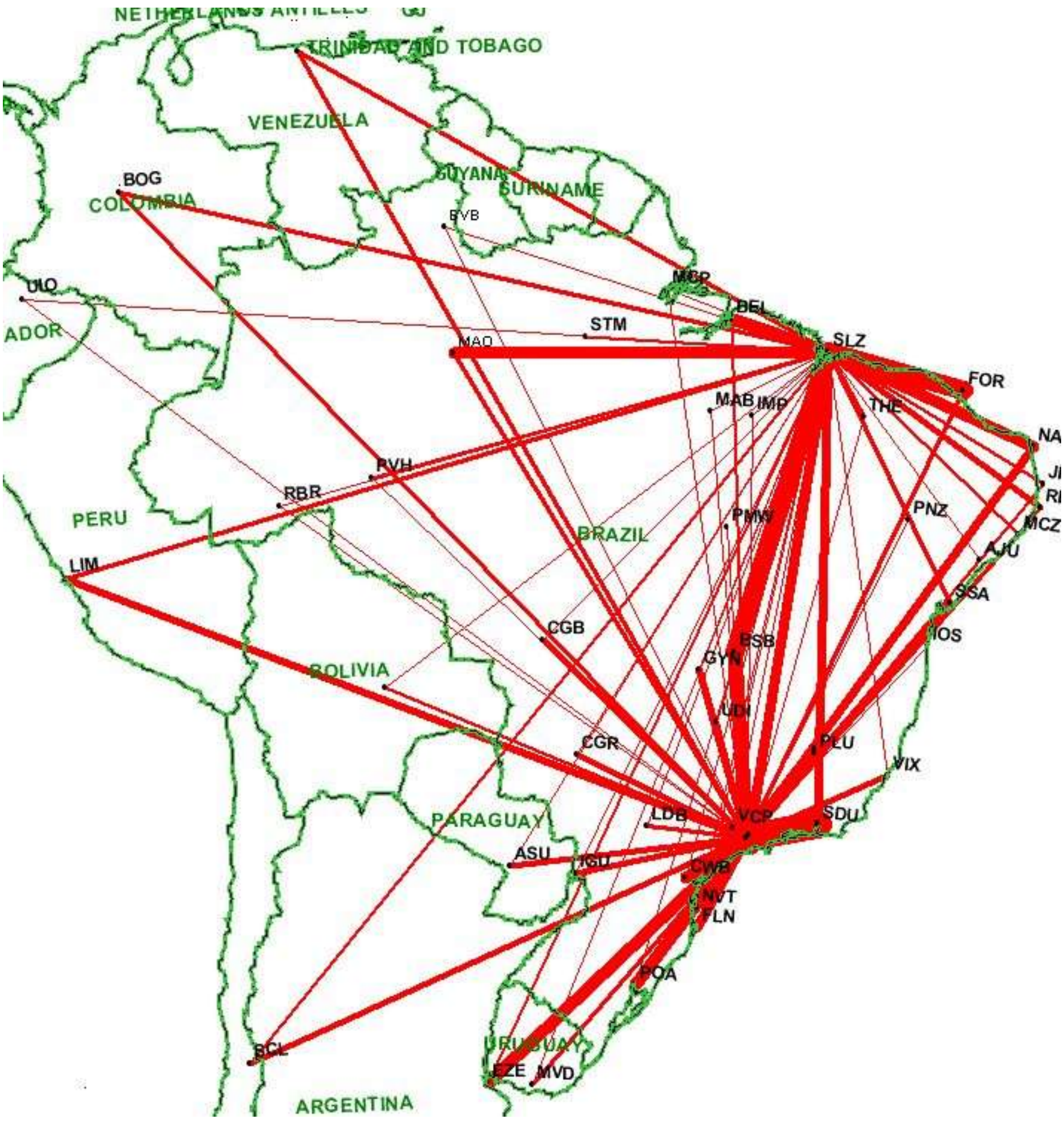

Figure 4.10: The Allocation and Flow Patterns for the Major Hubs – F4 Experiment

The allocation pattern of the mini hubs in the F4 experiment has showed some important changes. In this experiment, the number of pair of nodes that use some mini hubs to route their flows has almost doubled: from 67 in the F3 experiment to 123 in the F4 experiment. If the comparison is made with the F1 experiment, the change is even greater: from 50 allocations in F1 to 123 in F4. This fact can be specially attributed to the increase in the service radius distance range: in F1, the values of S_1 and S_2 were 1,000 km and 1,250 km respectively. These values were 1,750 km and 2,000 km in the F4 experiment, respectively. Figure 4.11 shows the allocation and the flow pattern for the mini hubs in the F4 experiment.

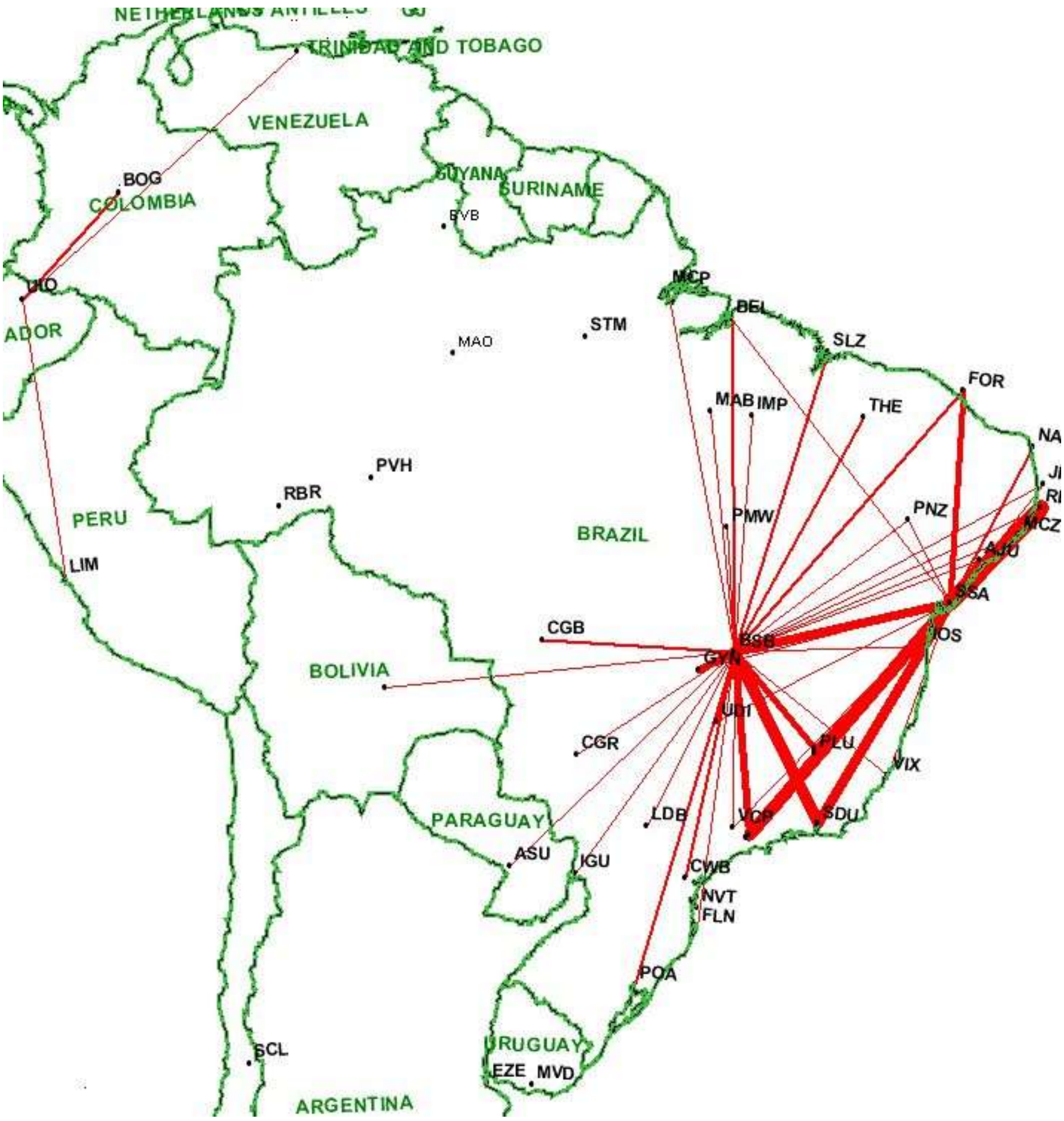

Figure 4.11: The Allocation and Flow Patterns for the Mini Hubs – F4 Experiment

Regarding the flow pattern, some dense linkages started to be noticed. In the northeast side, the flows of the airports FOR, NAT and REC that use the mini hub SSA got thicker. The airport BEL, that did not have any allocation to SSA, now has some – the flow from BEL to SSA that used to be routed through THE in the F3 experiment is now made directly to SSA. The airports of Rio de Janeiro, that did not have any allocations to the mini hub SSA, started to route some flows through this location. The flows from GIG to AJU, IOS, MCZ and PNZ that used to be routed through the major hub THE are now set to be routed through the mini hub SSA. The flow from SDU to AJU that used THE in F3 is now made through SSA in F4.

The allocations for the airports of Sao Paulo (CGH, GRU and VCP) to SSA have also increased. The flows from CGH and GRU to AJU and IOS, that used to be through the major hub THE is now made through the mini hub SSA as well as the flow from VCP to IOS. The airport BSB, which used to route any flow through SSA, is now using SSA as an intermediate point to route some flows: the flows from BSB to AJU, IOS, MCZ, PNZ and REC that used THE are now set to be routed through SSA. The flow from UDI to MCZ, which was routed in F3 through THE is now routed through SSA. The number of pair of routes that chose SSA to be a switching point was 58 in the F4 experiment and 24 in the F3 experiment (more than two times).

The number of allocations to the mini hub BSB has also increased. In the F3 experiment, there were 39 allocations while in the F4 it was 59 (20 allocations more). The airports MCP, BEL, VVI, IGU, SLZ, FOR, THE, JPA, REC, FLN and POA that did not have any flow routed through BSB, do it. The flows from IGU and THE to GYN, that were routed through the major hub CGH are now set to be routed through BSB. Table 4.22 lists all the pair of flows that use mini hubs SSA and BSB.

Table 4.22: Routes that use the Mini Hubs SSA and BSB in the F4 Experiment

Through BSB		Through SSA		
MCZ-GYN	CWB-GYN	MCZ-REC	GYN-MCZ	GIG-AJU
SSA-GYN	LDB-GYN	MCZ-AJU	CNF-MCZ	NAT-JPA
GYN-UDI	GIG-GYN	FOR-MCZ	CNF-IOS	NAT-REC
GYN-CGB	SDU-UDI	FOR-REC	CNF-JPA	AJU-MCZ
GYN-PMW	CGH-UDI	FOR-AJU	CNF-REC	AJU-JPA
CNF-UDI	CGH-PMW	BSB-MCZ	CNF-AJU	CGH-IOS
PLU-GYN	GRU-GYN	BSB-IOS	PLU-REC	CGH-AJU
PLU-UDI	GRU-UDI	BSB-REC	UDI-MCZ	GRU-IOS
UDI-GYN	GRU-PMW	BSB-PNZ	JPA-REC	GRU-AJU
UDI-CGB	VCP-UDI	BSBP-AJU	REC-JPA	VCP-IOS
CGR-GYN	VCP-CGB	REC-PNZ	REC-AJU	GIG-MCZ
CGB-GYN	PMW-GYN	GIG-IOS	GIG-PNZ	-
CGB-UDI	THE-GYN	-	-	-
IGU-GYN	-	-	-	-

4.2.7 The Sensitivity Analysis

To analyze the variations on the outputs of the model in the Case Study A (such as the major and mini hub locations and the objective function values) due to a systematic alteration in the strategic decision parameters, a sensitivity analysis was performed. It is interesting to notice that the variation in some parameters appear to be very sensitive, while the alterations in the others did not achieve the same results.

The process was divided into five subgroups: A, B, C, D and E. In the groups A and B (with two experiments in each one), the number of major and mini hubs was fixed, as well as the values of alpha, gamma, and the *minimum threshold*, while the values of the parameters S_1 and S_2 were varied. In the group C, the values of alpha and gamma were systematically varied, in the four experiments. In the groups D and E, the threshold values were varied, keeping the number of major and mini hubs constant, as well as the values of S_1 and S_2, *alpha* and *gamma*. Table 4.23 outlines these experiments.

Table 4.23: Results Found in the Sensitivity Analysis

	Exp.	p	q	S_1 (km)	S_2 (km)	α	γ	MT	Major Hubs	Mini Hubs
A	F5	2	2	1,000	1,250	0.6	0.8	200,000	THE and CGH	GYN and UIO
	F6	2	2	1,750	2,000	0.6	0.8	200,000	THE and CGH	BSB and UIO
B	F7	1	2	1,000	1,250	0.6	0.8	200,000	BSB	REC and CGH
	F8	1	2	1,750	2,000	0.6	0.8	200,000	BSB	AJU and CGH
C	F9	1	2	1,750	2,000	0.6	0.9	200,000	BSB	AJU and CGH
	F10	1	2	1,750	2,000	0.7	0.8	200,000	BSB	AJU and CGH
	F11	1	2	1,750	2,000	0.7	0.9	200,000	BSB	AJU and CGH
	F12	1	2	1,750	2,000	0.5	0.9	200,000	BSB	AJU and CGH
D	F13	2	2	1,750	2,000	0.6	0.8	300,000	MAO and GRU	BSB and AJU
	F14	2	2	1,750	2,000	0.6	0.8	250,000	MAO and GRU	BSB and AJU
	F15	2	2	1,750	2,000	0.6	0.8	150,000	MAO and GRU	THE and AJU
	F16	2	2	1,750	2,000	0.6	0.8	100,000	MAO and GRU	SLZ and AJU
	F17	2	2	1,750	2,000	0.6	0.8	50,000	BEL and CGH	BSB and AJU
E	F18	2	3	1,250	1,500	0.6	0.8	300,000	THE and CGH	SSA,BSB and BOG
	F19	2	3	1,250	1,500	0.6	0.8	150,000	MAO and GRU	SSA, FOR and BSB
	F20	2	3	1,250	1,500	0.6	0.8	100,000	MAO and GRU	MCZ, BSB and SLZ
	F21	2	3	1,250	1,500	0.6	0.8	50,000	SLZ and CGH	SSA, BSB and PVH
	F22	2	3	1,250	1,500	0.6	0.8	1,000,000	PNZ and GRU	BSB, EZE and BOG

In the two experiments of group A, keeping constant the values of *p*, *q*, *alpha*, *gamma* and the *minimum threshold* and varying the values of S_1 and S_2, a slight difference was found in regard to the locations of the mini hubs: GYN (Goiania-GO) in F5 to BSB (Brasilia-DF) in F6, airports quite close to each other. The other optimal locations found were kept the same. A similar result was found in the two experiments of group B, and using $p = 1$ (in spite of $p = 2$ in group A). For both experiments (F7 and F8), the major hub locations chosen were the same (BSB), and there was also a slight difference in the mini hub locations chosen – REC (Recife-PE) in F7 and AJU (Aracaju-SE) in F8, airports that are relatively close to each other.

In the experiments of group C, there was only a variation in *alpha* and *gamma* parameters, with all other parameters kept the same. The results found in all of the four experiments were the same, showing that a modification in *alpha* and *gamma* parameters is not so sensitive in this type of formulation. The results matched with the one in F8 experiment, with BSB as a major hub, and AJU and CGH (Congonhas – SP) as mini hubs.

A systematically variation on the *minimum threshold* parameter seemed to be much more sensitive than the others. In group D (F13, F14, F15, F16 and F17 experiments), keeping constant the values of $p = 2$, $q = 2$, $S_1 = 1{,}750$, $S_2 = 2{,}000$, *alpha* = 0.6 and *gamma* = 0.8, the results found within this group were very interesting. The airports of MAO (Manaus – AM) and GRU (Guarulhos – SP) were chosen to be major hubs in four experiments of this group (out of five), with the *minimum threshold* value ranging from 100,000 to 300,000. Only when the *minimum threshold* value was set to be 50,000 PAX that these locations have changed: from MAO and GRU to BEL (Belem – PA) and CGH. This means that the model kept a major hub within the set of airports in the north region and another in the set of airports of Sao Paulo, regardless of the *minimum threshold* value used in the experiments. In regard to the mini hub locations, AJU was always chosen to be a mini hub, in every five experiment. F13, F14 and F17 chosen BSB as the other mini hub, while F15 and F16 chosen THE (Teresina - PI) and SLZ (Sao Luis – MA) respectively, and emphasizing the importance of central and northeast regions of the continent.

In group E, containing five experiments, the difference in the results was a bit more notable than in group D. In this group (E), the parameters values were set in the following way: p = 2, q = 3, $S_1 = 1{,}250$, $S_2 = 1{,}500$, *alpha* = 0.6 and *gamma* = 0.8. For the smallest value of the *minimum threshold* (MT = 50,000 PAX – Experiment F21), the airports of SLZ and CGH were chosen to be the major hubs and the airports of BSB, SSA (Salvador – BA) and PVH (Porto Velho – RO) were the ones chosen to be the mini hubs. With an increase of 50,000 PAX in the *minimum threshold* parameter (MT = 100,000 PAX – Experiment F20), the major hub locations were chosen to be the airports of MAO and GRU, and the mini hubs to be MCZ (Maceio – AL), SLZ and BSB. A remarkable difference between these two experiments (F21 x F20) is that in spite of locating the major hubs in the northeast (SLZ) and in the southeast (CGH) regions, the model points out a different location for one major hub, now in the north region (MAO), and keeps the other in the set of airports of Sao Paulo (now being GRU, in spite of CGH). Also, in spite of having only one mini hub located in the northeast region, the solution for the F20 experiment locates two mini hubs there (SLZ and MCZ), keeping the airport of BSB as the mini hub of the central region.

Setting the value of the *minimum threshold* parameter to be 150,000 PAX (F19 Experiment), the major hub locations were the same as in the previous experiment (F20). The difference now regards the location of mini hubs. The airport of BSB continued to be the mini hub located in the central region, but MCZ and SLZ were no longer pointed out to be mini hubs, with the airports of SSA (Salvador – BA) and FOR (Fortaleza – CE) taking these positions. Using a value of 300,000 PAX for *minimum threshold* parameter, the solution found was the same as the one in F2. Setting this value to be a very big number (MT = 1,000,000 PAX) in the F22 experiment, which imposes all flows to be channeled through either a major or a mini hub, the results found were quite different. The airports of PNZ (Petrolina – PE) and GRU were the major hubs, while the airports of BSB, EZE (Buenos Aires, Argentina) and BOG (Bogota, Colombia) were the mini hubs.

Some conclusions can be made about the sensitivity analysis. The first one is about the sensitiveness of the parameters. The strategic decision parameter that appeared more sensitive to the changing in its values was the *minimum threshold*.

Making the comparison one by one in the ten experiments made in groups D and E, only the experiments F13 and F14 presented the same results for the location of major and mini hubs. The second one is in regard the parameters *alpha* and *gamma*. The variation in their values did not appear to be sensitive in the expected changes in the decisions about the location of the major and mini hubs, as can be notice in the experiments done in group C.

Table 4.24 shows the increment in the objective function (in percentage terms) in comparison with the lowest value found (F17 experiment).

Table 4.24: Increments in the Objective Function

Rank	Exp.	Inc. in the Objective Function (%)		Rank	Exp.	Inc. in the Objective Function (%)
1.	F11	14.467		12.	F1	0.116
2.	F10	7.323		13.	F2	0.112
3.	F9	7.299		14.	F6	0.107
4.	F22	0.228		15.	F3	0.104
5.	F7	0.176		16.	F4	0.095
6.	F13	0.172		17.	F19	0.085
7.	F18	0.171		18.	F15	0.081
8.	F8	0.156		19.	F20	0.047
9.	F14	0.155		20.	F16	0.044
10.	F12	0.131		21.	F21	0.002
11.	F5	0.122		22.	**F17**	0.000

The objective function only started to show an increase in its value – in comparison with experiment F17 – when the values of the strategic decision parameters *alpha* and *gamma* were changed. Mainly, when one of them had some increment (in respect to the values used in F7: *alpha* = 0.6 and *gamma* = 0.8). The greatest difference was found in the F11 experiment, where the values used for *alpha* and *gamma* were 0.7 and 0.9, respectively. When the other parameters were systematically changed, the values of the objective function did not show any important modification.

4.3 Case Study B

The Case Study B provides a different methodology in comparison to the one presented in the previous section. The aim now is to solve a new type of model for a greater number of nodes in the network and taking into consideration only the passenger movements in Brazil. In the following sections, the methodology will be described and a complete analysis of the results will be made.

4.3.1 Methodology for the Case Study B

Brazil has some singularities that just a few countries in the world may have. Its continental dimensions and the limitations in economic development that some regions face make the country almost unique in the world. These facts can be partially explained by historical development.

Brazil has a vast coast area where the Portuguese colonizers have settled, since the 16th century. At the beginning, all demographic and economic development was concentrated in this area. Such process has contributed to an uneven distribution of the population and an uneven development of the country. Nowadays, at least three main factors have defined new trends in the distribution of the population and the economic activities, which were: i) the change of the capital city from the coastal Rio de Janeiro to Brasilia, in the center of the country; ii) the saturation of the largest metropolitan areas along or next to the coast, has motivated people to occupy less expensive areas in the vast hinterland; and iii) the agribusiness sector has shown an impressive growth along the last few decades, suggesting that Brazil might soon become one of the largest food supplier in the world. This trend has created millions of new jobs in areas where, traditionally, there were no job demand.

A number of additional factors, such as the recent surge in economic development, the increase in the purchasing power, the relative decrease in the air tariffs, amongst others, have pushed the internal demand for air travel. At the same time, important infrastructure flaws have emerged, such as inappropriate airports, flight controls, air companies operations, and so on. In this sense, this doctoral thesis might contribute for improvements in airline operations.

It is well known that the achievement of optimal solutions for this type of problems in networks with more than 30 nodes is a difficult task, which restricts some real case applications. The objective of this Case Study is to provide a new mathematical programming technique for solving to optimality bigger instances of the hub-and-spoke problem. In this Case Study, a network with 135 Brazilian airports is going to be considered. Table 4.25 lists all of these airports.

Table 4.25: List of the 135 airports in the Case Study B

Code	State- Airport	Code	State- Airport	Code	State- Airport
SBCZ	AC-Cruzeiro do Sul	SBVG	MG-M. Trompowsky	SBCT	PR-Int. Afonso Pena
SBRB	AC–Presidente Medice	SBUR	MG-Uberaba	SBLO	PR-Londrina
SBMO	AL-C. dos Palmares	SBIP	MG-Usiminas	SBBZ	RJ-Umberto Modiano
SWBC	AM-Barcelos	SBMK	MG-Montes Claros	SBME	RJ-Macae
SWCA	AM-Carauari	SNPD	MG-Patos de Minas	SBCB	RJ-Cabo Frio
SWKO	AM-Coari	SBCF	MG-Int.T. Neves	SDAG	RJ-Angra dos Reis
SWEI	AM-Eirunepe	SBBH	MG-Pampulha	SBCP	RJ-B. Lisandro
SWOB	AM-Fonte Boa	SNJR	MG-S.J. Del rei	SBGL	RJ-Int. do R. de Janeiro
SWHT	AM-Humaita	SBUL	MG-Uberlandia	SBRJ	RJ-Santos Dumont
SBTT	AM-Int. Tabatinga	SBCR	MS-Corumba	SBNT	RN-Augusto Severo
SJQH	AM-Labrea	SSDO	MS-Dourados	SWJI	RO-Ji-Parana
SWMW	AM-Maues	SBCG	MS-Int. C. Grande	SBVH	RO-Vilhena
SWPI	AM-Parintins	SBAT	MT-Alta Floresta	SBPV	RO-Porto Velho
SWQE	AM-S.G da Cachoeira	SBBW	MT-Barra do Garcas	SBWQ	RR-Boa Vista

SWTP	AM-Tapuruquara	SJHG	MT-Confresa	SSZR	RS-Santa Rosa
SWTF	AM-Tefe	SJGP	MT-Rondonopolis	SSDM	RS-S. do Livramento
SWUY	AM-Urucu	SJGX	MT-S. F. do Araguaia	SBNM	RS-Santo Angelo
SBEG	AM-Eduardo Gomes	SWFF	MT-Sinop	SJRG	RS-Rio Grande
SBMQ	AP-Macapa	SBCY	MT-M. Rondon	SBUG	RS-Rubem Berta
SBLP	BA-B. Jesus da Lapa	SBHT	PA-Altamira	SBPK	RS-Pelotas
SJJL	BA-Barreiras	SNVS	PA-Breves	SBPF	RS-Lauro Kurtz
SNGI	BA-Guanambi	SBCJ	PA-Carajas	SSER	RS-Erechim
SBTC	BA-H. Transamerica	SBAA	PA-C. do Araguaia	SSHZ	RS-Horizontina
SNMU	BA-Mucuri	SNRB	PA-Itaituba	SBCX	RS-Campo dos Bugres
SBUF	BA-Paulo Afonso	SNMA	PA-Monte Alegre	SBPA	RS-Int. Salgado Filho
SBQV	BA-V. da Conquista	SBMD	PA-Monte Dourado	SBCH	SC-Chapeco
SBPS	BA-Porto Seguro	SNOX	PA-Orixima	SBCM	SC-Criciuma
SBIL	BA-Ilheus	SDOW	PA-O. do Norte	SSJA	SC-Joacaba
SBSV	BA-Int. Dois de Julho	SNMZ	PA-Porto de Moz	SBFL	SC-Hercilio Luz
SBFZ	CE-Pinto Martins	SNKE	PA-S. do Araguaia	SBJV	SC-Joinville
SBBR	DF-Int. de Brasilia	SBTB	PA-Trombetas	SBNF	SC-Navegantes
SBVT	ES-Goiabeiras	SBTU	PA-Tucurui	SBSM	SE-Santa Maria
SBFN	FN-F. de Noronha	SJRA	PA-S. Felix do Xingu	SBAU	SP-Aracatuba
SWGN	GO-Araguaina	SNDC	PA-Redencao	SBSJ	SP-S.J. dos Campos
SBMC	GO-Minacu	SBBE	PA-Int. Val de Caes	SBSR	SP-S.J. do Rio Preto
SWLC	GO-G. Leite de Castro	SBMA	PA-Maraba	SBXG	SP-Presidente Prudente

SBCN	GO-Caldas Novas	SBSN	PA-Santarem	SBML	SP-Marilia
SBGO	GO-Santa Genoveva	SBKG	PB-Joao Suassuna	SBRP	SP-Leite Lopes
SINM	MA-Balsas	SBJP	PB-P. Castro Pinto	SIMK	SP-Franca
SBIZ	MA-Imperatriz	SBRF	PE-Int, Guararapes	SBBU	SP-Bauru
SBSL	MA-M. C. Machado	SBPL	PE-Petrolina	SBAQ	SP-Araraquara
SBAX	MG-Araxa	SBTE	PE-Teresina	SBSP	SP-Congonhas
SIDV	MG-Divinopolis	SBCA	PR-Cascavel	SBGR	SP- Int. de Sao Paulo
SBJF	MG-F. de Assis	SBMG	PR-Maringa	SBKP	SP-Viracopos
SBGV	MG-G. Valadares	SBFI	PR-Cataratas	SWGI	TO-Gurupi

This list was extracted from the ANAC data considering the year 2007 (the same one used in the previous Case Study). It is possible to notice that some states have a great number of airports/aerodromes, such as: Amazonas - AM, 15; Bahia - BA, 10; Minas Gerais – MG, 13; Para - PA, 18; Rio Grande do Sul – RS, 11 and Sao Paulo – SP, with 12. The majority of these airports presented in Table 4.25 do not have a significant amount of traffic nor a good infrastructure in terms of passenger terminals and/or runways. These facts enable the application of this new methodology, which will be described in the following sections.

This proposed methodology consists of solving the problem in two phases. In the first phase, the total number of nodes in the network is 135, which makes unlikely the resolution through the use of mathematical programming techniques. The idea is to reduce the network size to a reasonable number of nodes in order to achieve an optimal solution through the use of the optimization software AIMMS 3.9.

Taking into consideration the fact that some airports of the entire network (135 nodes) do not have a large traffic and/or are located in a sparse region, a p-median model will be applied to identify 33 medians (out of 135). The medians will be regional centroid points, and the areas covered by each of them will play the role of clusters for the traffic flow in the country. These 33 medians will integrate the final network for the second phase of the model. With this phase done, a new model will be proposed in the second phase, and just flow interactions amongst these 33 nodes will be considered to determine optimal location for hubs and allocation for spoke nodes.

4.3.1.1 Case Study B – Phase One

In this phase, issues such as passenger movements in all of the 135 airports/aerodromes and their locations in the network are going to be taken into consideration. As already explained in the previous section, the aim here is to reduce the final network to a reasonable amount of nodes, in order to achieve an optimal solution using mathematical programming techniques.

The first step on Phase one is to collect weighted network for the 135 nodes. For each site, the weight is defined by the sum of the passenger inflow and outflow. Thereafter, with all the location information (longitude and latitude) of the 135 sites available, the software TRANSCAD was used to determine the distance matrix (135 x 135).

In the second step, a p-median model will be applied to the location of 33 medians and every median will have a set of nodes assigned to it. The set of medians and nodes assigned to each median will be called regional clusters. Therefore, the interactions will be amongst clusters and then the network size will be reduced to 33 nodes.

Figure 4.12 exemplifies the second step of Phase One. The rectangles with an "M" inside are the medians, with the smaller circular shapes representing the nodes assigned to the medians, with the clusters being represented by the set of the medians and the nodes assigned to them. In total, there will be a set of 33 clusters.

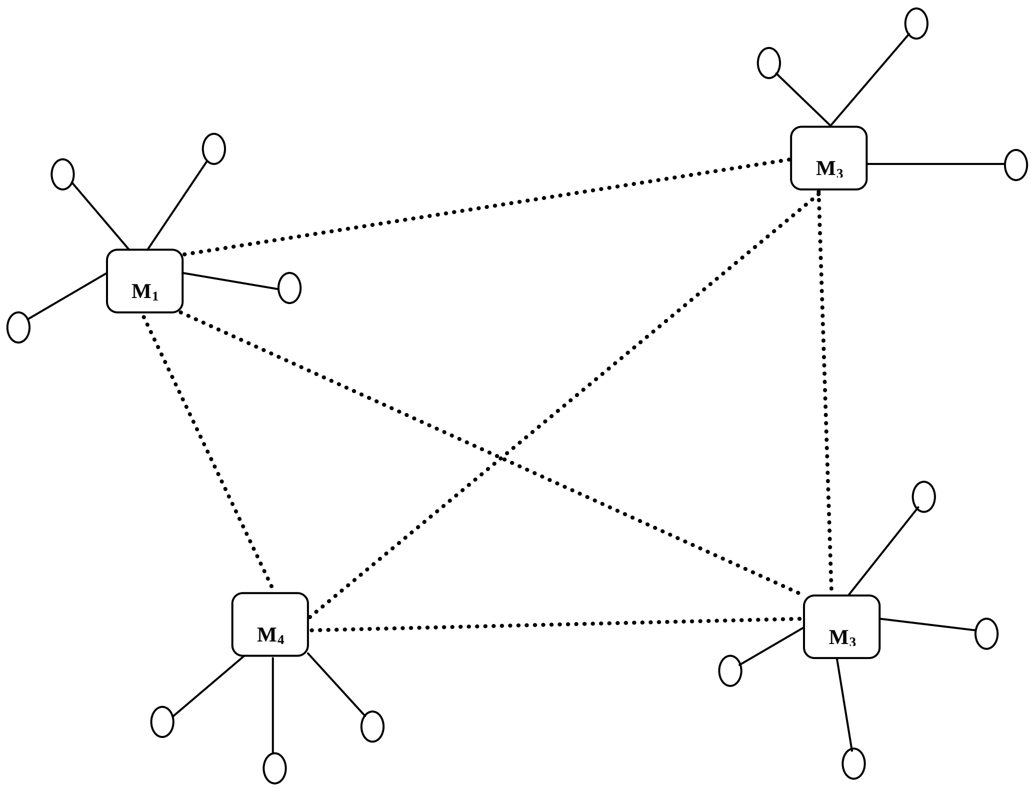

Figure 4.12: The Pattern of Flows Amongst Clusters

The idea is that a median represents a geographical area and every node that belongs to its geographical area must be assigned to this median. There is no restriction in terms of the size of each area nor the number of nodes to be assigned to a median. Table 4.26 shows the set of 33 clusters, with their respective medians and nodes assigned for each cluster.

Table 4.26: List of the 33 Clusters and the Nodes Assigned to Each Cluster

Cluster #	Median	Nodes Assigned		Cluster #	Median	Nodes Assigned
1	SBRB	SWCA.		18	SBTE	-
2	SBMO	-		19	SBFI	SBCA, SSZR, SBNM, SBUG, SSER, SSHZ and SBCH.
3	SBEG	SWOB, SWMW, SWPI, SWTF, SNRB, SNOX and SBTB.		20	SBCT	SSJA and SBJV.
4	SBMQ	SBHT, SNVS, SNMA, SBMD and SNMZ.		21	SBLO	SBMG, SBAU, SBXG and SBML.
5	SBSV	-		22	SBGL	SBJF, SDAG and SBRJ.
6	SBFZ	-		23	SBNT	SBFN.
7	SBBR	SJJL, SBMC, SJGX and SWGI.		24	SBPV	SWKO, SWHT, SJQH, SWUY and SWJI.
8	SBVT	SNMU, SBGV and SBCP		25	SBWQ	SWBC, SWQE and SWTP.
9	SBGO	SWLC, SBCN and SBBW.		26	SBPA	SSDM, SJRG, SBPK, SBPF, SBCX and SBSM.
10	SBSL	-		27	SBFL	SBCM and SBNF.
11	SBCF	SIDV, SBIP, SBMK, SBBH and SNJR.		28	SBSP	-
12	SBCG	SBCR and SSDO		29	SBGR	SBVG and SBSJ.
13	SBCY	SBAT, SJGP, SWFF and SBVH.		30	SBKP	SBRP, SBBU and SBAQ.
14	SBBE	SBSN.		31	SBIL	SBLP, SNGI, SBTC, SBQV and SBPS.
15	SBJP	SBKG		32	SBMA	SWGN, SINM, SBIZ, SJHG, SBCJ, SBAA, SDOW, SNKE, SBTU, SJRA and SNDC.
16	SBRF	-		33	SBME	SBBZ and SBCB.
17	SBPL	SBUF				

For instance, the cluster number three, represented by the median SBEG – Eduardo Gomes Airport, located in the city of Manaus, Amazonas State, has seven airports allocated to it. They are: SWOB, SWMW, SWPI, SWTF, SNRB, SNOX and SBTB. This means that every flow originated or destined to every one of these airports must connect at SBEG. In the passenger's point of view, a single trip to a similar node (with the same characteristics – remote node, allocated to a median) may be very stressful and demand more than two stops – four, in this case. This fact will not be considered as an operational constraint, because just a minority of the passengers in the system will face it.

4.3.1.2 –Case Study B – Phase Two

At this phase, the network size was reduced to 33 nodes. The interactions to be considered now will be amongst clusters. A distance matrix amongst these clusters (33 x 33) was determined using the software TRANSCAD, while the flow matrix (Wij) was extracted from ANAC Annual Statistics - year of 2007. An important feature that was considered in the previous modeling (to locate major and mini hubs) will also be considered here: the direct linkages (showed by the dashed lines in Figure 4.13), whenever it exceeds a minimum threshold. Figure 4.13 shows the network configuration for the Two Phases Model.

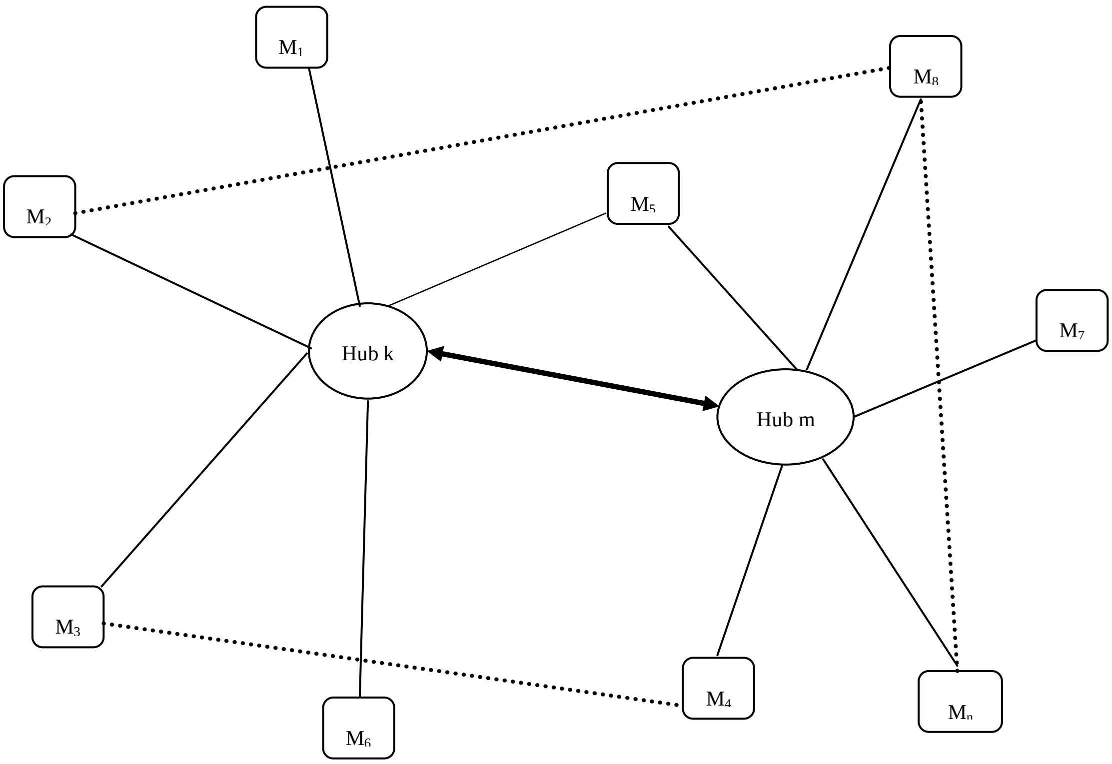

Figure 4.13: Final Configuration of the Network

The mathematical model is shown next. This formulation is strongly based on the multiple allocation p-hub median problem, where *p* hubs are located and the spoke nodes are allowed to be assigned to more than one hub. In terms of passengers, this is the best configuration for this type of networks, mainly because a path between an *i*,*j* pair is determined taking into consideration only shortest path issues. The sets of parameters and variables are defined as follows.

P: set of clusters in the network, totaling 33;

p: number of hubs to be located;

α: Discount factor for the linkages between hubs;

W_{ij}: flow between a clusters *i* and *j*;

C_{ij}: Unit cost of transportation between clusters *i* and *j*;

Y_k: Decision variable for the location of hubs: 1, if a site k is hub; 0, otherwise;

Z_{ij}: Decision variable for direct linkages: 1, if a flow between a pair *i*,*j* is made directly (non-stop); 0, otherwise;

X_{ijkm}: Flow decision variable: 1, if a flow between a pair *i*,*j* is made through hubs *k* and *m*; 0, otherwise;

Γ_s: Minimum threshold for a linkage to be made directly.

$$Min \sum_{i\in P}\sum_{j\in P}\sum_{k\in P}\sum_{m\in P} W_{ij}\left(C_{ik} + \alpha C_{km} + C_{mj}\right)X_{ijkm} + \sum_{i\in P}\sum_{j\in P} W_{ij}Z_{ij} \qquad (4.18)$$

s.t.:

$$\sum_{k\in P} Y_k = p, \qquad (4.19)$$

$$\sum_{k\in P}\sum_{m\in P} X_{ijkm} + Z_{ij} = 1, \qquad \forall i,j \in P, \qquad (4.20)$$

$$\sum_{m\in P} X_{ijkm} - Y_k \leq 0, \qquad \forall i,j,k \in P, \qquad (4.21)$$

$$\sum_{k\in P} X_{ijkm} - Y_m \leq 0, \qquad \forall i,j,m \in P, \qquad (4.22)$$

$$W_{ij} - \Gamma_s Z_{ij} \geq 0, \qquad \forall i,j \in P, \qquad (4.23)$$

$$Y_k \in \{0,1\}, \qquad \forall k \in P, \qquad (4.24)$$

The objective function aims the minimization of the total costs and has two terms. The former represents a flow that is routed using hubs k and/or m and the latter regards the flows that are made directly. Constraint (4.19) implies that a *p* number of hubs must be located. Constraint (4.20) imposes that a flow between two nodes *i* and *j* can only be made using hubs or directly. The sets of constraints (4.21) and (4.22) say that paths that use hubs are valid only if a hub is already established for those sites. Constraint (4.23) says that if a flow between *i* and *j* is greater or equal to a pre-specified threshold, this flow must be made directly.

The optimization software AIMMS version 3.9 was used for the solving of the problem. Table 4.27 shows the results for p = 3 hubs, Γ_s = 300,000 PAX, and α = 0.6.

Table 4.27: Results Achieved for p=3

Hubs Chosen	Number of Variables	Number of Constraints	Solving Time	Solver Used
SBMO, SBBR and SBGR	1,187,044	74,054	381.06 Sec	CPLEX 11.2

Figure 4.13 shows the pattern of flows for the hub locations chosen. As a characteristic of the multiple allocation flows, the linkages between hubs are not as dense as they would be in a single allocation model. Another feature that contributed for the splitting of flows is the allowance of direct linkages at a minimum threshold. The Voronoi Diagram was used to illustrate the medians catchment areas.

Two of the three airports chosen to be hubs by the model are already hubs in the Brazilian air transportation environment. The SBGR airport, located in the city of Guarulhos, Sao Paulo state, is an important generation and destination pole for both domestic and international passengers. With a strategic location in the middle of the country and a considerable amount of domestic traffic, SBBR, located in Brasilia, the federal capital of Brazil, was another choice for a hub chosen by the model.

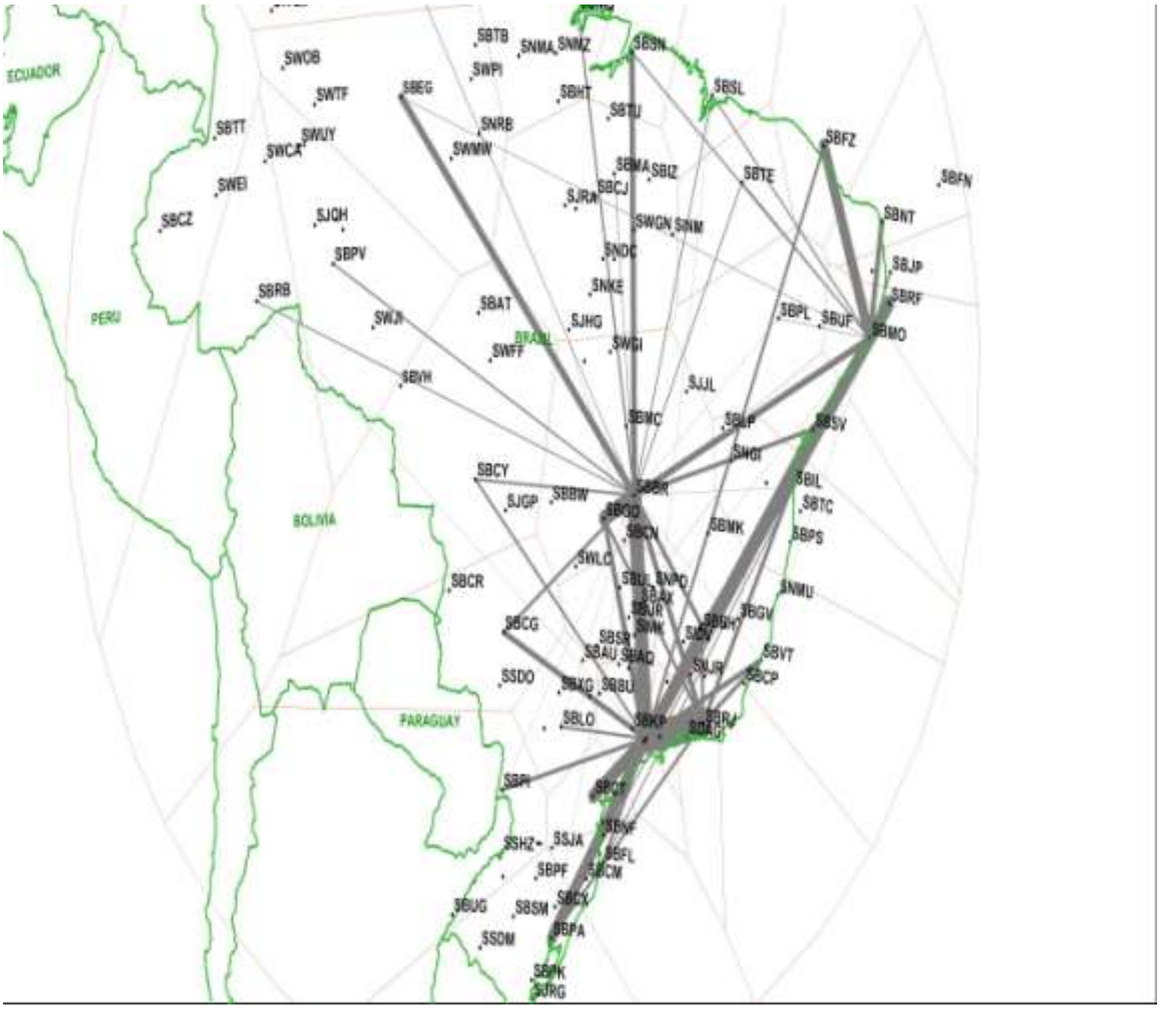

Figure 4.13: The Results of the Application of the Two Phases Model

An unexpected choice made by the model was the SBMO airport, located in the city of Maceio, Alagoas state. The fourth airport in the northeast region in terms of passenger movements in the year 2007, behind the airports of Recife-PE (SBRF), Salvador-BA (SBSV) and Natal-RN (SBNT), this choice can be partially explained by the geographical location of this city (Maceio) for serving the other airports of the Northeast region.