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Introduction

Instances of repeated decision taking are pervasive in Economics. Members of partnerships, employers and employees, and couples in a household, among others, are subject to the problem of taking a common action repeatedly over time. In all these examples, the agents involved have some outside option. An employee can quit his job and look for an alternative one, whereas an employer can fire a worker and search for replacement. Partnerships are often dissolved by members that decide to implement individual business strategies. More prosaically, couples can always divorce and look for new matches.

In this paper, we introduce outside options in a setting in which, repeatedly, two agents have to take a joint action, cannot resort to side payments, and each period are privately informed about their favorite actions. We are specially interested in understanding how the outside options affect the dynamics of the actions taken by the agents and their relative bargain power in the partnership.

Outside options take a very simple form in the model: for any given contingency (current and past realization of private information), and any period of time, agent i can collect life-time utility of $\underline{w}_i$ outside the partnership. As a consequence, any feasible mechanism must satisfy ex-post participation constraints: for every contingency and period of time, it must provide to each agent expected lifetime utility of no less than what they can get exercising their outside options.

We derive a number of results. First off, in contrast to settings in which agents are forced to participate, we show that a mechanism satisfying ex-post participation constraints cannot approximate an (ex-ante) efficient outcome irrespective of how patient the agents are. The reason is simple. As shown by Jackson and Sonnenschein [8] and Carrasco and Fuchs [3] (we discuss these papers in detail when we review the literature), an efficient incentive compatible mechanism links current decisions to future ones: an agent who is given relatively more weight on a current decision has to relinquish future decision power. The way through which the mechanism grants an agent a lower future decision power is by promising him lower continuation values. The

outside options place a lower bound on what a mechanism can promise to any single agent, impeding the mechanism to implement the efficient intertemporal trade of decision power.

A mechanism (or contract) determines, for each period of time and every contingency, the whole sequence of actions to be taken. In any given period of time, the current action taken depends on whether or not participation constraints are binding. When the participation constraints slack, the relative weights on current actions are determined by two effects. The first one is the realization of the agents' current private information. The second one, summarized by a time varying weight the agents are given on decisions, is related to past actions: the agent who had more weight on past actions will have less weight on the current one. Both of these effects are already known from previous work on repeated action taking under forced participation. When a participation constraint binds, however, a third, new, effect comes into play. The player whose participation constraint binds is given, relatively to a forced participation setting, *less* weight on the current action. Although this may seem surprising, the reason why this is the case is simple. In states for which a player's participation constraint binds, it would be optimal to give that player more weight on current decisions in exchange for less weight on future ones. Since one cannot promise values that are lower than the agent's outside option value, this intertemporal exchange of decision rights cannot be implemented. The agent is then given less weight relatively to what would happen in a forced participation environment.

Future actions and decision power are fully embedded in promised continuation values. In an optimal mechanism, the dynamics of continuation values has two main features. First, whenever current values are higher than outside option values, there is positive probability of next period's promised values being higher or smaller than current values. Put differently, "off-corners", continuation values are continually spread to provide incentives for truthful reporting by the agents. In fact, an agent who is given more (less) weight on current decisions is promised lower (higher) continuation values. The spreading of values is a force toward promising extreme values for the agents. Second, whenever current values equal outside option values for an agent, the optimal mechanism assigns positive probability of next period's promised value for that agent being higher than his outside option value. This is a mean reversing force for promised values, which allows the members of the partnership to continue trading decisions in the future when one of the participation constraints binds. Both of these features imply that values must continually vary over time: there are no absorbing states.

When combined, the two features discussed above lead to our main results. Whenever both agents have outside options, the stochastic process that governs the agents' promised values converge to a unique invariant limiting distribution. The limiting distribution is non-degenerate, and assigns positive probability to all feasible values. We derive a close link between promised continuation values and the time varying weights on decisions the optimal mechanism assigns to the agents. Agent two's time varying weight on actions – which is the relevant measure of decision power in our setting – is, at an optimal, given by the derivative of agent one's value (given the promised value for agent two). Therefore, the convergence of promised values implies that (i) there will be a unique limiting distribution for those weights, (ii) this distribution will be itself non-degenerate, and will assign positive probability to all weights that are compatible with the participation constraints.

Two important properties of the limiting distribution of power are the following. First, it is memoryless: even if the partnership starts with, say, agent one having all the bargain power (meaning, the initial promised value to agent two is $\underline{w}_2$), in the far future the relative bargain power will have no dependence whatsoever on this fact. Second, power continually changes of hands in the limit, meaning that the weight agents have in decisions will continually vary.

Unfortunately, it is not possible to derive analytically the limiting distribution of power. To get a grasp of how the agents' outside options affect the limiting distribution of power, we consider the situation in which only one of the agents has an outside option. This is a simple and tractable way to capture the effect of differences in outside options on the limiting distribution of power in a partnership. For this case, we show that the relative weight on decisions of the agent *who has* an outside option is, on average, increasing over time.[1] This dynamics leads to a dictatorship in the limit: the agent who has an outside option will eventually take all decisions. Therefore, as intuition suggests, a player who has an outside option will have more power in the partnership at some point. More surprising is the fact that, eventually, he will be the only one with power. This result also shows that both players having outside options is necessary (as well as sufficient) for the limiting distribution of power being non-degenerate.

There are few papers that show that, by linking unrelated decisions (i.e., common actions), efficiency gains can be attained. In an environment in which there is a binary choice each period and agents can have either have weak or strong preferences for either option, Casella [4] proposes a mechanism in

[1] More precisely, when, say, agent two is the one with an outside option, his weight on decisions is a sub-martingale.

which agents are given a vote every period which they can use over time. The possibility of shifting votes intertemporally allows agents to concentrate their votes when preferences are more intense, leading to efficiency gains. In a two-player decision problem setting with two binary issues, but a continuum of preference intensities, Hortala-Vallve [7] shows that if the players are allowed to freely distribute a given number of votes across the two issues, the ex-ante efficient decision can be attained.

Jackson and Sonnenschein [8] propose a simple "budgeting mechanism", in which each agent is allowed to report a possible type (they have a discrete type space) a fixed number of times. The number of times an agent can report certain type is chosen to replicate the frequency with which that type should be realized. They show that, if players are patient, their mechanism leads to approximate efficiency. In a setting similar to the one in this paper, Carrasco and Fuchs [3] show that the optimal mechanism involves the trading of decisions power over time, and that such trading will arbitrarily approximate (but never attain) efficiency when participation is forced.

In our setting, and in contrast to those results, we show that the imposition of ex-post participation constraints bound the attainable payoffs away from the efficient ones irrespective of how patient the agents are. Therefore, much as in static mechanism design theory (e.g., Myerson and Satterwhaite [12]), there is a trade-off between efficiency and ex-post participation in a repeated common action setting: repetition does not slack ex-post participation constraints.[2]

The optimal mechanism in Carrasco and Fuchs [3] will eventually lead to a dictatorship. Hence, an ex-post consequence of an optimal mechanism that approximates efficiency ex-ante is that the limiting distribution of power is degenerate, with all decisions being taken by a single agent from a certain period of time onwards. Our paper shows that, while preventing approximate efficiency ex-ante, the possibility that agents have to exercise ex-post their outside options allows for a non-degenerate limiting distribution of power. Moreover, the weights the agents have on decisions will continually vary over time. Therefore, not only decisions will necessarily be shared but, also, the stakes the agents have on decisions will continually vary.

Lastly, our work also relates to the dynamic insurance literature, specially Atkenson and Lucas [2]. In a general equilibrium setting in which a planner has to provide insurance for agents whose binary effort toward finding jobs are unobservable, they show that, when those agents have outside options, a unique

[2]A related point is made by Hortala-Vallve [6], who shows that there is a trade-off between efficiency and ex-post Incentive Compatibility constraints in a voting environment in which decisions can be linked.

invariant distribution of wealth exists. Moreover, under certain conditions related to the interest rate of the economy, this distribution may be non-degenerate.[3] In our paper, an agent who reports to have an extreme type in a given period is like an agent that reports to have not found a job. The optimal mechanism will respond by giving that agent more weight in the current allocation decision (similarly a higher transfer today). Incentive compatibility then calls for the agent to 'pay' for this by forgoing future weight in the allocation decision (future consumption). In their setting, a Principal designing an optimal insurance policy trades-off risk-sharing (which calls for a constant consumption stream) and the provision of incentives – through varying continuation values. In both papers, the outside options put a bound on the provision of incentives, which – in some cases for them and always for us – leads to non-degenerate distributions of wealth and power.

The paper is organized as follows. We introduce the model in section 2. The optimal mechanism and some of its basic properties are characterized in Section 3. In section 4, we discuss the dynamics of decisions taking. Section 5 deals with the case in which only one of the agents has an outside option. Section 6 draws the concluding remarks. All proofs are relegated to the Appendix.

[3]In contrast to what happens in our model, in their paper, for certain values of the interest rate, the dynamics of wealth in their model has absorbing states, and the limiting distribution of wealth is degenerate.