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# The Case of a Single Participation Constraint

It would be interesting to evaluate analytically how the agents' outside options shape the invariant distribution $\varphi^*$, and the resulting distribution of power implied by $-V'(\cdot)$. Unfortunately, we are not able to derive the distribution $\varphi^*$ analytically, so we cannot fully tackle such type of question.

We are nevertheless able to deal with a particular (and extreme) case that sheds some light on it. This section analyzes the case in which only one of the players has an outside option.

Assume at first that, while agent one is forced to participate in the Partnership, agent two has a potentially tempting outside option. Our first result for this case is

**Proposition 3** *When only agent two has an outside option, there exists a measure $\mathcal{Q}$ such that:*

$$E^{\mathcal{Q}}\left[V'\left(w\left(\theta,x\right)\right)\right] \leq V'\left(w\right).$$

*Hence, $V'(\cdot)$ is a supermartingale.*

Using $-V'(w) = \gamma$, the above result says that, on average, agent two's weight on decisions cannot decrease over time. Using slightly modified versions of Proposition 1 and 2, it can be shown that $w(\theta, x)$ must vary continually over time, so that agent two's weight on decisions must, in fact, *increase* on average over time. This will necessarily lead to agent 2 becoming a dictator in the long-run.

More formally, since $V'(\cdot)$ follows a supermartingale, it must converge almost surely to a random variable (Dobb [5]). As, in order to provide incentives, values must vary continually, we show in the Appendix that, in the limit, $V'(\cdot)$ cannot assign positive likelihood to any point in $(-\infty, V'(\underline{w}_2)]$. Therefore, $V'(\cdot)$ must converge almost surely to $-\infty$ , so that $w(\cdot)$ must converge to $\overline{v}$. Hence, agent two will eventually become a dictator.

The analysis for the case in which agent one has a tempting outside option, while agent two is forced to participate, is virtually the same. Indeed, we have:

**Proposition 4** *When only agent one has an outside option, there exists a measure $\mathcal{Q}$ such that:*

$$E^{\mathcal{Q}}\left[V'\left(w\left(\theta,x\right)\right)\right] \geq V'\left(w\right).$$

*Hence, $V'\left(w\left(\theta\right)\right)$ is a submartingale.*

Therefore, on average, agent two's weight on decisions cannot increase over time. Since in order to provide incentives, values must vary continually, agent two's weight must, in fact, decrease over time. This process will lead to agent one becoming a dictator.

The following result summarizes the above discussion.

**Theorem 4** *If only one of the agents has an outside option, dictatorship will ensue eventually. The dictator will be the agent who has the outside option.*

On top of illustrating how differences in outside options may affect the shape of the limiting distribution of power, the above theorem also shows that both players having outside options is necessary (as well as sufficient) for such distribution being non-degenerate.