# 7.

# Referência bibliográfica

*"Aquele que consegue fixar a mente, isto é, estabilizá-la, para que não sofra mais as contínuas alterações a que a ignorância a tem exposto, e faz resplandecer dentro dela seus melhores propósitos, terá enfocado sua vista para um destino melhor, o qual, inquestionavelmente, irá operando em sua vida as mudanças mais notáveis, que jamais poderia imaginar."*

Carlos Bernardo González Pecotche.


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# Apêndice 1

*"Conhecendo o homem qual é sua posição dentro do meio em que vive, saberá conduzir-se conforme a conduta que lhe determina essa posição, evitando os saltos que fazem a estabilidade correr perigo."*

Carlos Bernardo González Pecotche.

Coeficientes $\alpha_{vi}$, $\alpha_{wi}$ e $\alpha_{\gamma i}$ de Galerkin para $(i = 1,2,...)$, dados nas Equações (2.153), (2.154) e (2.155):

$$\alpha_{v1} = \int_0^1 \left[ -\beta_\gamma F_v \left(F'_\gamma F''_w\right)' + \left(1-\beta_y\right) F_v \left(F_\gamma F''_w\right)'' \right] ds \ , \tag{A1.1}$$

$$\alpha_{v2} = -\left(1-\beta_y\right) \int_0^1 \left[ F_v \left(F''_v F_\gamma^2\right)'' \right] ds \ , \tag{A1.2}$$

$$\alpha_{v3} = \int_0^1 \left(1-\beta_y\right) F_v \left[ F'''_w \int_0^s F'_v F''_w \, ds \right]' ds - \int_0^1 \beta_y \, F_v \left(F'_v F''^2_w\right)' ds - \int_0^1 F_v \left(F'_v F'_w F'''_w\right)' ds - \int_0^1 \left[ P_S + q_u \cos\left(\Omega_u t\right) \right] \left(F_v F'^2_w F''_v + 2 F_v F'_v F'_w F''_w\right) ds \ , \tag{A1.3}$$

$$\alpha_{v4} = -\beta_y \int_0^1 F_v \left[ F'_v \left(F'_v F''_v\right)' \right]' ds - 3 \int_0^1 \left[ P_S + q_u \cos\left(\Omega_u t\right) \right] F_v \, F'^2_v \, F''_v \, ds \ , \tag{A1.4}$$

$$\alpha_{v5} = -\frac{1}{2} \int_0^1 F_v \left( F'_v \int_L^s \int_0^s F'^2_v ds \, ds \right)' ds \ , \tag{A1.5}$$

$$\alpha_{v6} = -\frac{1}{2} \int_0^1 F_v \left( F'_v \int_L^s \int_0^s F'^2_w ds \, ds \right)' ds \ , \tag{A1.6}$$

$$\alpha_{v7} = J_\zeta \int_0^1 F_v \left[ \left(F'_v\right)^3 \right]' ds \ , \tag{A1.7}$$

$$\alpha_{v8} = \int_0^1 \left[ J_\eta \, F_v \left(F'_v F'^2_w\right)' + \left(J_\eta - J_\zeta\right) F_v \left( F'_w \int_0^S F'_v F''_w \right)' ds \right] ds \ , \tag{A1.8}$$

$$\alpha_{v9} = 2\,J_{\eta}\int_0^1 F_v\left(F_{\gamma}\,F_w'\right)'\,ds\ , \tag{A1.9}$$

$$\alpha_{v10} = 2\,J_{\eta}\int_0^1 F_v\left(F_w'\int_0^s F_v'\,F_w''\,ds\right)'\,ds\ , \tag{A1.10}$$

$$\alpha_{v11} = \int_0^1\left[2\,J_{\eta}\,F_v\left(F_w'\int_0^s F_v'\,F_w''\,ds\right)' - J_{\zeta}\int_0^1 F_v\left(F_v'\,F_w'^{\,2}\right)'\right]ds\ , \tag{A1.11}$$

$$\alpha_{v12} = \left(J_{\eta} - J_{\zeta}\right)\int_0^1 F_v\left(F_v'\,F_{\gamma}^{\,2}\right)'\,ds\ , \tag{A1.12}$$

$$\alpha_{v13} = \left(J_{\eta} - J_{\zeta}\right)\int_0^1 F_v\left(F_w'\,F_{\gamma}\right)'\,ds\ , \tag{A1.13}$$

$$\alpha_{v14} = \alpha_{v7} = J_{\zeta}\int_0^1 F_v\left[\left(F_v'\right)^3\right]'\ , \tag{A1.14}$$

$$\alpha_{v15} = -J_{\xi}\,c_{\gamma}\int_0^1 F_v\left(F_w'\,F_{\gamma}\right)'\,ds\ , \tag{A1.15}$$

$$\alpha_{w1} = \int_0^1\left[\beta_{\gamma}F_w\left(F_{\gamma}'\,F_v''\right)' + \left(1-\beta_y\right)F_w\left(F_{\gamma}\,F_v''\right)''\right]ds\ , \tag{A1.16}$$

$$\alpha_{w2} = \left(1-\beta_y\right)\int_0^1\left[F_w\left(F_w''\,F_{\gamma}^{\,2}\right)''\right]ds\ , \tag{A1.17}$$

$$\begin{aligned}\alpha_{w3} = &-\int_0^1\left(1-\beta_y\right)F_w\left[F_v'''\int_0^s F_w'\,F_v''\,ds\right]'\,ds - \int_0^1 F_w\left(F_w'\,F_v''^{\,2}\right)'ds - \\ &\int_0^1\beta_y\,F_w\left(F_v'\,F_w'\,F_v'''\right)'ds - \\ &\int_0^1\left[P_S + q_u\cos\left(\Omega_u\,t\right)\right]\left(F_w\,F_v'^{\,2}F_w'' + 2\,F_w\,F_w'\,F_v'F_v''\right)ds\ ,\end{aligned} \tag{A1.18}$$

$$\alpha_{w4} = -\int_0^1 F_w \left[ F_w' \left( F_w' \, F_w'' \right)' \right]' + 3\left[ P_S + q_u \cos\left(\Omega_u \, t\right) \right] F_w \, F_w'^2 \, F_w'' \;\; ds \;, \tag{A1.19}$$

$$\alpha_{w5} = -\frac{1}{2}\int_0^1 F_w \left( F_w' \int_L^s \int_0^s F_w'^2 ds\, ds \right)' ds \;, \tag{A1.20}$$

$$\alpha_{w6} = -\frac{1}{2}\int_0^1 F_w \left( F_w' \int_L^s \int_0^s F_v'^2 ds\, ds \right)' ds \;, \tag{A1.21}$$

$$\alpha_{w7} = J_\eta \int_0^1 F_w \left[ \left( F_w' \right)^3 \right]' ds \;, \tag{A1.22}$$

$$\alpha_{w8} = \int_0^1 \left[ J_\zeta \, F_w \left( F_w' \, F_v'^2 \right)' + \left( J_\eta - J_\zeta \right) F_w \left( F_v' \int_0^S F_w' \, F_v'' \right)' ds \right] ds \;, \tag{A1.23}$$

$$\alpha_{w9} = -2 \, J_\eta \int_0^1 F_w \left( F_\gamma \, F_v' \right)' ds \;, \tag{A1.24}$$

$$\begin{aligned} \alpha_{w10} = \int_0^1 \Bigg[ & - J_\xi \, F_w \left( F_v' \int_0^s F_v' \; F_w'' ds \right)' + J_\xi \int_0^1 F_w \left( F_w' \, F_v'^2 \right)' + \\ & \left( J_\eta - J_\zeta \right) F_w \left( F_v' \int_0^s F_w' \; F_v'' ds \right)' \Bigg] ds \;, \end{aligned} \tag{A1.25}$$

$$\begin{aligned} \alpha_{w11} = \int_0^1 \Bigg[ & - J_\xi \, F_w \left( F_v' \int_0^s F_v' \; F_w'' ds \right)' + J_\eta \int_0^1 F_w \left( F_w' \, F_v'^2 \right)' + \\ & \left( J_\eta - J_\zeta \right) F_w \left( F_v' \int_0^s F_w' \; F_v'' ds \right)' \Bigg] ds \;, \end{aligned} \tag{A1.26}$$

$$\alpha_{w12} = -\left( J_\eta - J_\zeta \right) \int_0^1 F_w \left( F_w' \, F_\gamma^{\,2} \right)' ds \;, \tag{A1.27}$$

$$\alpha_{w13} = -\left(J_\eta - J_\zeta\right)\int_0^1 F_w \left(F_v' F_\gamma\right)' ds \ , \tag{A1.28}$$

$$\alpha_{w14} = \alpha_{w7} = J_\eta \int_0^1 F_w \left[\left(F_w'\right)^3\right]' \ , \tag{A1.29}$$

$$\alpha_{\gamma 1} = -\frac{1-\beta_y}{J_\xi} \int_0^1 F_\gamma^2 \, F_v''^2 \, ds \ , \tag{A1.30}$$

$$\alpha_{\gamma 2} = \frac{1-\beta_y}{J_\xi} \int_0^1 F_\gamma^2 \, F_w''^2 \, ds \ , \tag{A1.31}$$

$$\alpha_{\gamma 3} = \frac{1-\beta_y}{J_\xi} \int_0^1 F_\gamma \, F_w'' \, F_v'' ds \ , \tag{A1.32}$$

$$\alpha_{\gamma 4} = -\int_0^1 F_\gamma \int_0^s F_v' \, F_w'' \, ds \, ds \ , \tag{A1.33}$$

$$\alpha_{\gamma 5} = \int_0^1 F_\gamma \, F_v' \, F_w' \, ds \ , \tag{A1.34}$$

$$\alpha_{\gamma 6} = \frac{J_\eta - J_\zeta}{J_\xi} \int_0^1 F_v'^2 \, F_\gamma^2 \, ds \ , \tag{A1.35}$$

$$\alpha_{\gamma 7} = \frac{-\left(J_\eta - J_\zeta\right)}{J_\xi} \int_0^1 F_w'^2 \, F_\gamma^2 \, ds \ , \tag{A1.36}$$

$$\alpha_{\gamma 8} = \frac{-\left(J_\eta - J_\zeta\right)}{J_\xi} \int_0^1 F_\gamma \, F_v' \, F_w' \, ds \ . \tag{A1.37}$$

# Apêndice 2

*"Os seres tem, pois, diante de si, dois caminhos a seguir: o do aperfeiçoamento individual ou o da inconsciência, no qual se vive a vida porque sim. A inteligência de cada um é suficientemente capaz para decidir entre um ou outro caminho."*

Carlos Bernardo González Pecotche.

Coeficientes $\alpha_{vi}$, $\alpha_{wi}$ e $\alpha_{\gamma i}$ de Galerkin, para $(i=1,2,...)$, dados nas Equações (2.156), (2.157) e (2.158):

$$\alpha_{v1} = -\beta_\gamma \int_0^1 F_v \left(F_\gamma' \, F_w''\right)' ds + \left(1-\beta_y\right) \int_0^1 F_v \left(F_\gamma \, F_w''\right)'' ds \ , \tag{A2.1}$$

$$\alpha_{v2} = -\left(1-\beta_y\right) \int_0^1 F_v \left(F_v'' \, F_\gamma^{\,2}\right)'' ds \ , \tag{A2.2}$$

$$\begin{aligned} \alpha_{v3} = \left(1-\beta_y\right) \int_0^1 F_v \left( F_w''' \int_0^s F_v' \, F_w'' \, ds \right)' ds - \beta_y \int_0^1 F_v \left(F_v' \, F_w''^{\,2}\right)' ds - \\ \int_0^1 F_v \left(F_v' \, F_w' \, F_w'''\right)' ds - \int_0^1 \left[ P_S + q_u \cos\left(\Omega_u \, t\right)\right] \left(F_v \, F_w'^{\,2} F_v'' + 2 F_v \, F_v' \, F_w' F_w''\right) ds \ , \end{aligned} \tag{A2.3}$$

$$\alpha_{v4} = -\beta_y \int_0^1 F_v \left[ F_v' \left(F_v' \, F_v''\right)' \right]' ds - \int_0^1 \left[ P_S + q_u \cos\left(\Omega_u \, t\right)\right] F_v \, F_v'^{\,2} \, F_v'' \, ds \ , \tag{A2.4}$$

$$\alpha_{v5} = -\left(\frac{1}{2}\right) \int_0^1 F_v \left( F_v' \int_1^s \int_0^s F_v'^{\,2} ds \, ds \right)' ds \ , \tag{A2.5}$$

$$\alpha_{v6} = -\left(\frac{1}{2}\right) \int_0^1 F_v \left( F_v' \int_1^s \int_0^s F_w'^{\,2} ds \, ds \right)' ds \ , \tag{A2.6}$$

$$\alpha_{v7} = J_\zeta \int_0^1 F_v \left(F_v'^{\,3}\right)' ds \ , \tag{A2.7}$$

$$\alpha_{v8} = J_\eta \int_0^1 F_v \left(F_v' \, F_w'^{\,2}\right)' ds - \left(J_\eta - J_\zeta\right) \int_0^1 F_v \left( F_w' \int_0^s F_v' \, F_w'' \, ds \right)' ds \ , \tag{A2.8}$$

$$\alpha_{v9} = 2\, J_{\eta} \int_0^1 F_v \left( F_{\gamma}\, F_w' \right)' ds \ , \tag{A2.9}$$

$$\alpha_{v10} = 2\, J_{\zeta} \int_0^1 F_v \left( F_w' \int_0^s F_v'\, F_w'' ds \right)' ds \ , \tag{A2.10}$$

$$\alpha_{v11} = 2\, J_{\zeta} \int_0^1 F_v \left( F_w' \int_0^s F_v' F_w'' ds \right)' ds - J_{\zeta} \int_0^1 F_v \left( F_v' F_w'^{\,2} \right)' ds \ , \tag{A2.11}$$

$$\alpha_{v12} = \left( J_{\eta} - J_{\zeta} \right) \int_0^1 F_v \left( F_v' F_{\gamma}^{\,2} \right)' ds \ , \tag{A2.12}$$

$$\alpha_{v13} = - \left( J_{\eta} - J_{\zeta} \right) \int_0^1 F_v \left( F_w' F_{\gamma} \right)' ds \ , \tag{A2.13}$$

$$\alpha_{v14} = \alpha_{v7} = J_{\zeta} \int_0^1 F_v \left( F_v'^{\,3} \right)' \ , \tag{A2.14}$$

$$\alpha_{v15} = - \beta_{\gamma} \int_0^1 F_v \left( F_{\gamma}' F_w'' \right)' ds \left( w_0 \right) + \beta_y \int_0^1 F_v \left( F_{\gamma} F_w'' \right)'' ds \left( w_0 \right) , \tag{A2.15}$$

$$\begin{aligned} \alpha_{v16} = {} & \left( 1 - \beta_y \right) \int_0^1 F_v \left( F_w''' \int_0^s F_v' F_w'' ds \right)' ds \left( w_0 \right) - \\ & \int_0^1 F_v \left( F_w' F_v'' F_w'' \right)' ds \left( w_0 \right) - \beta_y \int_0^1 F_v \left[ F_v' \left( F_w'^{\,2} \right)'' \right]' ds \left( w_0 \right) - \\ & \beta_y \int_0^1 F_v \left( F_w''' \int_0^s F_w' F_v'' ds \right)' ds \left( w_0 \right) - \\ & \int_0^1 \left[ P_S + q_u \cos \left( \Omega_u t \right) \right] \left[ 2 F_v F_v'' F_w'^{\,2} + 4 F_v F_v' F_w' F_w'' \right] ds \left( w_0 \right) , \end{aligned} \tag{A2.16}$$

$$\alpha_{v17} = \left(1-\beta_y\right)\int_0^1 F_v\left(F_w'''\int_0^s F_v' F_w'' ds\right)' ds\ (v_0) - \beta_y \int_0^1 F_v\left(F_v' F_w' F_w''\right)'' ds\ (v_0) - \int_0^1 \left[P_S + q_u \cos\left(\Omega_u t\right)\right]\left[F_v F_v'' F_w'^2 + 2 F_v F_v' F_w' F_w''\right] ds(v_0), \tag{A2.17}$$

$$\alpha_{v18} = 2 J_\zeta \int_0^1 F_v\left(F_w'\int_0^s F_v' F_w'' ds\right)' ds\ (w_0) + \left(J_\eta - J_\zeta\right)\int_0^1 F_v\left(F_v' F_w'^2\right)' ds\ (w_0), \tag{A2.18}$$

$$\alpha_{v19} = 2 J_\zeta \int_0^1 F_v\left(F_w'\int_0^s F_v' F_w'' ds\right)' ds(v_0) - 2 J_\zeta \int_0^1 F_v\left(F_v' F_w'^2\right)' ds(v_0), \tag{A2.19}$$

$$\alpha_{v20} = \left(J_\eta - J_\zeta\right)\int_0^1 F_v\left(F_v' F_w'^2\right)' ds\ (w_0) - \left(J_\eta - J_\zeta\right)\int_0^1 F_v\left(F_w'\int_0^s F_v' F_w'' ds\right)' ds(w_0) - \int_0^1 F_v\left(F_v'\int_1^s\int_0^s F_w'^2 ds\, ds\right)' ds(w_0), \tag{A2.20}$$

$$\alpha_{v21} = \left(J_\eta - J_\zeta\right)\int_0^1 F_v\left(F_v' F_w'^2\right)' ds\ (v_0) - \left(J_\eta - J_\zeta\right)\int_0^1 F_v\left(F_w'\int_0^s F_v' F_w'' ds\right)' ds(v_0), \tag{A2.21}$$

$$\alpha_{v22} = -\beta_y \int_0^1 F_v\left(F_v' F_v''^2\right)' ds\ (v_0) - \left(\frac{1}{2}\right)\beta_y \int_0^1 F_v\left(F_v''' F_v'^2\right)' ds(v_0) - 2\beta_y \int_0^1 F_v\left[F_v'\left(F_v' F_v''\right)'\right]' ds(v_0) - \int_0^1 \left[P_S + q_u \cos\left(\Omega_u t\right)\right]\left[9 F_v F_v'^2 F_v''\right] ds(v_0), \tag{A2.22}$$

$$\alpha_{v23} = -\beta_y \int_0^1 F_v \left[ F_v'' \left( F_w'^2 \right)' \right]' ds(v_0\, w_0) -$$
$$\beta_y \int_0^1 F_v \left[ F_v' \left( F_w'^2 \right)'' \right]' ds(v_0\, w_0) + (1-\beta_y) \int_0^1 F_v \left( F_\gamma\, F_w'' \right)'' ds(\gamma_0) -$$
$$\beta_y \int_0^1 F_v \left( F_w''' \int_0^s F_w'\, F_v'' ds \right)' ds(v_0\, w_0) - \beta_y \int_0^1 F_v \left( F_w'\, F_v''\, F_w'' \right)' ds(v_0\, w_0) -$$
$$\int_0^1 \left[ P_S + q_u \cos(\Omega_u\, t) \right] \left[ 2 F_v\, F_v'' F_w'^2 + 4 F_v\, F_v'\, F_w'\, F_w'' \right] ds(v_0\, w_0), \tag{A2.23}$$

$$\alpha_{v24} = -\left[ 1 - \left( \frac{1}{2} \right) \beta_y \right] \int_0^1 F_v \left( F_v'' F_\gamma^2 \right)'' ds\ (v_0), \tag{A2.24}$$

$$\alpha_{v25} = -\left[ 6 - (6) \beta_y \right] \int_0^1 F_v \left( F_\gamma\, F_\gamma'\, F_v'' \right)' ds\ (\gamma_0), \tag{A2.25}$$

$$\alpha_{v26} = -\int_0^1 F_v \left( F_v' \int_1^s \int_0^s F_v'^2 ds\, ds \right)' ds\ (v_0), \tag{A2.26}$$

$$\alpha_{v27} = -\left( \frac{1}{2} \right) \int_0^1 F_v \left( F_v' \int_1^s \int_0^s F_w'^2 ds\, ds \right)' ds\ (v_0), \tag{A2.27}$$

$$\alpha_{v28} = -\left( \frac{1}{2} \right) \int_0^1 F_v \left( F_v' \int_1^s \int_0^s F_v'^2 ds\, ds \right)' ds\ (v_0), \tag{A2.28}$$

$$\alpha_{v29} = -\int_0^1 F_v \left( F_v' \int_1^s \int_0^s F_w'^2 ds\, ds \right)' ds\ (v_0\, w_0), \tag{A2.29}$$

$$\alpha_{v30} = -J_\xi (c_\gamma) \int_0^1 F_v \left( F_w'\, F_\gamma \right)' ds\ , \tag{A2.30}$$

$$\alpha_{w1} = \beta_\gamma \int_0^1 F_w \left( F_\gamma'\, F_v'' \right)' ds + (1-\beta_y) \int_0^1 F_w \left( F_\gamma\, F_v'' \right)'' ds\ , \tag{A2.31}$$

$$\alpha_{w2} = (1-\beta_y)\int_0^1 F_w \left(F_w'' F_\gamma^2\right)'' ds \ , \tag{A2.32}$$

$$\begin{aligned}\alpha_{w3} = &-(1-\beta_y)\int_0^1 F_w \left(F_v''' \int_0^s F_w' F_v'' ds\right)' ds - \int_0^1 F_w \left(F_w' F_v''^2\right)' ds - \\ &\beta_y \int_0^1 F_w \left(F_v' F_w' F_v'''\right)' ds - \\ &\int_0^1 \left[P_S + q_u \cos(\Omega_u t)\right] \left(F_w F_v'^2 F_w'' + 2 F_w F_w' F_v' F_v''\right) ds \ ,\end{aligned} \tag{A2.33}$$

$$\alpha_{w4} = -\int_0^1 F_w \left[F_w' \left(F_w' F_w''\right)'\right]' + 3\left[P_S + q_u \cos(\Omega_u t)\right] F_w F_w'^2 F_w'' ds \ , \tag{A2.34}$$

$$\alpha_{w5} = -\left(\frac{1}{2}\right)\int_0^1 F_w \left(F_w' \int_1^s \int_0^s F_w'^2 ds\, ds\right)' ds \ , \tag{A2.35}$$

$$\alpha_{w6} = -\left(\frac{1}{2}\right)\int_0^1 F_w \left(F_w' \int_1^s \int_0^s F_v'^2 ds\, ds\right)' ds \ , \tag{A2.36}$$

$$\alpha_{w7} = J_\eta \int_0^1 F_w \left(F_w'^3\right)' ds \ , \tag{A2.37}$$

$$\alpha_{w8} = J_\zeta \int_0^1 F_w \left(F_w' F_v'^2\right)' ds + \left(J_\eta - J_\zeta\right)\int_0^1 F_w \left(F_v' \int_0^s F_w' F_v'' ds\right)' ds \ , \tag{A2.38}$$

$$\alpha_{w9} = -2 J_\eta \int_0^1 F_w \left(F_\gamma F_v'\right)' ds \ , \tag{A2.39}$$

$$\begin{aligned}\alpha_{w10} = &-J_\xi \int_0^1 F_w \left(F_v' \int_0^s F_v' F_w'' ds\right)' ds + J_\xi \int_0^1 F_w \left(F_w' F_v'^2\right)' ds + \\ &(J_\eta - J_\zeta)\int_0^1 F_w \left(F_v' \int_0^s F_w' F_v'' ds\right)' ds \ ,\end{aligned} \tag{A2.40}$$

$$\alpha_{w11} = -J_{\xi}\int_0^1 F_w\left(F_v'\int_0^s F_v'\ F_w''\,ds\right)'ds + J_{\zeta}\int_0^1 F_w\left(F_w'\,F_v'^{2}\right)'ds + (J_{\eta}-J_{\zeta})\int_0^1 F_w\left(F_v'\int_0^s F_w'\ F_v''ds\right)'ds\,, \tag{A2.41}$$

$$\alpha_{w12} = -\left(J_{\eta}-J_{\zeta}\right)\int_0^1 F_w\left(F_w'\,F_{\gamma}^{\,2}\right)'ds\,, \tag{A2.42}$$

$$\alpha_{w13} = -\left(J_{\eta}-J_{\zeta}\right)\int_0^1 F_w\left(F_v'\,F_{\gamma}\right)'ds\,, \tag{A2.43}$$

$$\alpha_{w14} = \alpha_{w7} = J_{\eta}\int_0^1 F_w\left(F_w'^{\,3}\right)'\,, \tag{A2.44}$$

$$\alpha_{w15} = \beta_{\gamma}\int_0^1 F_w\left(F_{\gamma}'\,F_v''\right)'ds(v_0) + \int_0^1 F_w\left(F_{\gamma}\,F_v''\right)''ds(v_0)\,, \tag{A2.45}$$

$$\alpha_{w16} = -\left(1-\beta_y\right)\int_0^1 F_w\left(F_v''\int_0^s F_w'\,F_v''ds\right)''ds(v_0) + \int_0^1 F_w\left[F_v'\left(F_v''F_w'\right)'\right]'ds(v_0) - \beta_y\int_0^1 F_w\left[F_w'\left(F_v''F_w'\right)'\right]'ds(v_0) - \int_0^1 F_w\left(F_v''\int_0^s F_w'\,F_v''ds\right)''ds(v_0) - \int_0^1\left[P_S+q_u\cos\left(\Omega_u\,t\right)\right]\left[2F_w\,F_w''\,F_v'^{2}+4F_w\,F_w'\,F_v'\,F_v''\right]ds(v_0)\,, \tag{A2.46}$$

$$\alpha_{w17} = -\left(1-\beta_y\right)\int_0^1 F_w\left(F_v''\int_0^s F_w'\,F_v''ds\right)''ds\,(w_0) - \beta_y\int_0^1 F_w\left[F_w'\left(F_v''F_v'\right)'\right]'ds\,(w_0) - \int_0^1\left[P_S+q_u\cos\left(\Omega_u\,t\right)\right]\left[F_w\,F_w''\,F_v'^{2}+2F_w\,F_w'\,F_v'\,F_v''\right]ds(w_0)\,, \tag{A2.47}$$

$$\alpha_{w18} = J_\xi \int_0^1 F_w \left(F_w' \, F_v'^2\right)' ds\,(v_0) - J_\xi \int_0^1 F_w \left( F_v' \int_0^s F_v' \, F_w'' \, ds \right)' ds\,(v_0) + \left(J_\eta - J_\zeta\right) \int_0^1 F_w \left( F_v' \int_0^s F_w' \, F_v'' ds \right)' ds\,(v_0), \tag{A2.48}$$

$$\alpha_{w19} = - J_\xi \int_0^1 F_w \left( F_v' \int_0^s F_v' \, F_w'' \, ds \right)' ds(w_0) + \left(J_\eta - J_\zeta\right) \int_0^1 F_w \left( F_v' \int_0^s F_w' \, F_v'' ds \right)' ds(w_0), \tag{A2.49}$$

$$\alpha_{w20} = \left(J_\eta - J_\zeta\right) \int_0^1 F_w \left( F_v' \int_0^s F_v'' F_w' ds \right)' ds\,(v_0) - \int_0^1 F_w \left( F_w' \int_1^s \int_0^s F_v'^2 ds\, ds \right)' ds(v_0), \tag{A2.50}$$

$$\alpha_{w21} = \left(J_\eta - J_\zeta\right) \int_0^1 F_w \left( F_v' \int_0^s F_w' \, F_v'' ds \right)' ds(w_0), \tag{A2.51}$$

$$\alpha_{w22} = - \int_0^1 F_w \left(F_w' \, F_w''^2\right)' ds\,(w_0) - \left(\frac{1}{2}\right) \int_0^1 F_w \left(F_w''' F_w'^2\right)' ds\,(w_0) - \int_0^1 F_w \left[ F_w'' \left(F_w'^2\right)' \right]' ds(w_0) - (2) \int_0^1 F_w \left(F_w''' F_w'^2\right)' ds(w_0) - \int_0^1 \left[ P_S + q_u \cos\left(\Omega_u \, t\right) \right] \left[ 9 F_w \, F_w'^2 \, F_w'' \right] ds(w_0), \tag{A2.52}$$

$$\alpha_{w23} = - \beta_y \int_0^1 F_w \left[ F_w' \left(F_v'' \, F_v'\right)' \right]' ds\,(v_0 \, w_0) - \int_0^1 F_w \left( F_v'' \int_0^s F_w' \, F_v'' ds \right)'' ds(v_0 \, w_0) + \left(1 - \beta_y\right) \int_0^1 F_w \left(F_\gamma \, F_v''\right)'' ds(\gamma_0) - \int_0^1 \left[ P_S + q_u \cos\left(\Omega_u \, t\right) \right] \left[ 2 F_w \, F_w'' \, F_v'^2 + 4 F_w \, F_w' \, F_v' \, F_v'' \right] ds(w_0 \, v_0), \tag{A2.53}$$

$$\alpha_{w24} = \left[\left(\frac{1}{2}\right) - \beta_y\right]\int_0^1 F_w \left(F_w'' F_\gamma^2\right)'' ds\ (w_0), \tag{A2.54}$$

$$\alpha_{w25} = \left[6 - (6)\beta_y\right]\int_0^1 F_w \left(F_\gamma\ F_\gamma'\ F_w''\right)' ds\ (\gamma_0), \tag{A2.55}$$

$$\alpha_{w26} = -\int_0^1 F_w \left(F_w' \int_1^s \int_0^s F_w'^2 ds\, ds\right)' ds\ (w_0), \tag{A2.56}$$

$$\alpha_{w27} = -\left(\frac{1}{2}\right)\int_0^1 F_w \left(F_w' \int_1^s \int_0^s F_v'^2 ds\, ds\right)' ds\ (w_0), \tag{A2.57}$$

$$\alpha_{v28} = -\left(\frac{1}{2}\right)\int_0^1 F_w \left(F_w' \int_1^s \int_0^s F_w'^2 ds\, ds\right)' ds\ (w_0), \tag{A2.58}$$

$$\alpha_{w29} = -\int_0^1 F_w \left(F_w' \int_1^s \int_0^s F_v'^2 ds\, ds\right)' ds\ (v_0\ w_0), \tag{A2.59}$$

$$\alpha_{\gamma 1} = -\frac{\left(1 - \beta_y\right)}{J_\xi}\int_0^1 F_\gamma^2\ F_v''^2\ ds\ , \tag{A2.60}$$

$$\alpha_{\gamma 2} = \frac{\left(1 - \beta_y\right)}{J_\xi}\int_0^1 F_\gamma^2\ F_w''^2\ ds\ , \tag{A2.61}$$

$$\alpha_{\gamma 3} = \frac{\left(1 - \beta_y\right)}{J_\xi}\int_0^1 F_\gamma\ F_w''\ F_v'' ds\ , \tag{A2.62}$$

$$\alpha_{\gamma 4} = -\int_0^1 F_\gamma \int_0^s F_v'\ F_w''\ ds\, ds\ , \tag{A2.63}$$

$$\alpha_{\gamma 5} = \int_0^1 F_\gamma\ F_v'\ F_w'\ ds\ . \tag{A2.64}$$

$$\alpha_{\gamma 6} = \frac{\left(J_\eta - J_\zeta\right)}{J_\xi} \int_0^1 F_v'^{\,2}\, F_\gamma^2\, ds\ , \tag{A2.65}$$

$$\alpha_{\gamma 7} = -\frac{\left(J_\eta - J_\zeta\right)}{J_\xi} \int_0^1 F_w'^{\,2}\, F_\gamma^2\, ds\ , \tag{A2.66}$$

$$\alpha_{\gamma 8} = -\frac{\left(J_\eta - J_\zeta\right)}{J_\xi} \int_0^1 F_\gamma\, F_v'\, F_w'\, ds\ . \tag{A2.67}$$

$$\alpha_{\gamma 9} = -\frac{\left(2 - \beta_y\right)}{J_\xi} \int_0^1 F_\gamma^2 F_v''^{\,2}\, ds\ \left(v_0\right). \tag{A2.68}$$

$$\alpha_{\gamma 10} = \frac{\left(1 - 2\,\beta_y\right)}{J_\xi} \int_0^1 F_\gamma^2 F_w''^{\,2}\, ds\ \left(w_0\right). \tag{A2.69}$$

$$\alpha_{\gamma 11} = -\frac{\left(1 - \beta_y\right)}{J_\xi} \int_0^1 F_\gamma^2\, F_v''^{\,2}\, ds\ \left(\gamma_0\right). \tag{A2.70}$$

$$\alpha_{\gamma 12} = -\frac{\left(1 - \beta_y\right)}{J_\xi} \int_0^1 F_\gamma^2\, F_w''^{\,2}\, ds\ \left(\gamma_0\right). \tag{A2.71}$$

$$\begin{aligned}\alpha_{\gamma 13} = &-\frac{\left(1 - \beta_y\right)}{J_\xi} \int_0^1 F_\gamma^2\, F_v''^{\,2}\, ds\left(\gamma_0\, v_0\right) - \frac{\left(\beta_\gamma + \beta_y\right)}{J_\xi} \int_0^1 F_\gamma\, F_v''\, F_w''\, ds\left(w_0\right) \\ &- \frac{\beta_\gamma}{J_\xi}\left[\int_0^1 F_\gamma\, F_v'''\, F_w'\, ds - \int_0^1 \left(F_\gamma\, F_v''\, F_w'\right) ds\right]\left(w_0\right).\end{aligned} \tag{A2.72}$$

$$\begin{aligned}\alpha_{\gamma 14} = &\frac{\left(1 - \beta_y\right)}{J_\xi} \int_0^1 F_\gamma^2\, F_w''^{\,2}\, ds\left(\gamma_0\, w_0\right) - \frac{\left(\beta_\gamma - 1\right)}{J_\xi} \int_0^1 F_\gamma\, F_v''\, F_w''\, ds\left(v_0\right) \\ &- \frac{\beta_\gamma}{J_\xi}\left[\int_0^1 F_\gamma\, F_v'''\, F_w'\, ds - \int_0^1 \left(F_\gamma\, F_v''\, F_w'\right) ds\right]\left(v_0\right).\end{aligned} \tag{A2.73}$$

$$\alpha_{\gamma 15} = -\int_0^1 F_\gamma \int_0^s F_v'\, F_w''\, ds\, ds\ \left(w_0\right). \tag{A2.74}$$

$$\alpha_{\gamma 16} = \left(\int_0^1 F_\gamma\, F_v'\, F_w'\, ds - \int_0^1 F_\gamma \int_0^s F_v'\, F_w''\, ds\, ds\ \right)\left(v_0\right) \tag{A2.75}$$