3 The General Model

The four-period model of section 2 weighs the short-term gain of walking away from the mortgage obligations with the long-term expected cost of losing their homeowners status. The simple model shows that low-income families are more likely to avoid strategic default due to their higher chance of having credit denied in case they become delinquent. However, this simple model is not easily matched with observable mortgage delinquency variables.

To measure the impact of a large price drop on the incidence of strategic default, we generalize the model in a multi-period setting in which risk-neutral households decide whether to buy or rent a home. The advantage of this multi-period model is to allow a more direct connection with standard mortgage contracts observed in the US credit market. Accounting for differences in income distributions of mortgage holders, the model can be used to simulate households strategic default behavior among US regions.

Like in the simple model, we assume that households cannot commit to pay their debt obligations whenever possible and that this moral hazard problem is taken into account by the representative bank when setting the mortgage interest rate. Again, the lack of commitment is combined with uncertainty about a delinquent household having a mortgage request accepted by the bank. This combination implies that strategic default behavior is affected by households' income profile.

3.1 Agents and Time Horizon

We consider a discrete infinite-horizon model that starts at $t = 0$, where families earn income $y_t \in [\underline{y}, \bar{y}]$ at each time t. Households differ with respect to the initial wealth, w_0, and to the distribution of income in each date. In our economy, there are two distributions for the one-period income of households. The distribution of the high-income households, type H, is F^H, which stochastically dominates the distribution of the low-income households, F^L. The two distributions are common knowledge.

Households' preferences are represented by:

$$U_t(h_t, c_t) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau} (c_{\tau} + 1_{owner_{\tau}} \cdot \bar{u}) \tag{3-1}$$

where β is the discount factor and 1_{owner_τ} is the indicator function that assumes one only when the family owns a house in period τ.

In equation (3-1), we assume that households consume in every period. Besides, households that are homeowners receive an instantaneous private benefit of $\bar{u}$.

As before, we assume that the competitive banking sector is summarized by a representative bank that lends if the expected return is at least equal to the risk free interest rate, R.

3.2 Housing Supply

We focus on the default decision given a large price drop, so we assume that there is a continuum of houses that can be rented at time t for the value r_t. Rent is a random variable in the support $[0, \bar{r}]$ with CDF $G(.)$ and $E_t[r_{t+1}] = r_t$.

We define the fundamental house price, f_t, as the expected present value of the rent cash flow that could be earned by the homeowner, so it is given by $f_t = \beta \frac{r_t}{1-\beta}$. We also assume that there is a bubble in house prices, B_t, such that houses are sold for the price $S_t = f_t + B_t$. The bubble can either explode, $B_t = 0$, with probability $1 - q$ or grow, $B_t = \xi \cdot B_{t-1}$, with probability q.

3.3 Mortgage Contract

At $t = 0$, families choose whether to buy or rent a house. If they decide to buy the house, they need to borrow money from the representative bank. The loan takes the form of a fixed payment mortgage contract with length T^*. Differently from the last section's simple model, families only give a fraction d of their initial wealth as down payment. This modification allows families to keep $(1 - d) \cdot w_0$, so they may be able to avoid foreclosure when they receive a low income in some period.

Our main goal is to evaluate the strategic default decision when borrowers face a large price drop that can be perceived as the bubble's burst. Hence, we assume that borrowers are not allowed to refinance their mortgages. This is not an issue because it is unlikely to be profitable for banks to refinance an underwater mortgage contract.

At each period, families decide whether or not to walk away from their mortgages. If they decide to pay the mortgage, they gain the right to live in the house as homeowner until the next period. If a family decides to walk away from the mortgage, the bank gets the house with the recovery rate γ.

The bank takes into account the borrower's decision to default when evaluating the expected present value of the mortgage's cash flow. Denote by $V_t(x, w_t, S_t; i)$ the lender's value of a mortgage to a type i family at time t. The lender's mortgage value depends on the periodic payment, x, on the borrower's wealth level, w_t, and on the house price. The bank accepts a loan request if the present value of the mortgage exceeds the amount lent. Therefore, the periodic payment, x, of a mortgage at $t = 0$ must satisfy condition (3-2).

$$V_0(x, w_0, S_0; i) \geq S_0 - d \cdot w_0 \tag{3-2}$$

3.4 Returning to the mortgage market

The novelty of our paper is the concern of households about their capacity to access the mortgage market once they default strategically. To introduce this feature, the model departs from the literature by allowing delinquent families to apply for a new mortgage. To simplify the analysis, we assume that households default at time t are only able to request for a new mortgage at $t + 1$.

A delinquent household has its new loan request accepted only if there is a payment level, x, such that the value of mortgage's cash flow exceeds the loan amount. Define the maximum value of a mortgage contract as $V_t^{MAX}(w_t, S_t; i) \equiv \max_x V_t(x, w_t, S_t; i)$. Hence, a family that walks away from a mortgage at period t is offered a new mortgage in the following period if condition (3-3) is satisfied.

$$V_{t+1}^{MAX}(w_{t+1}, S_{t+1}; i) \geq S_{t+1} - d^D \cdot w_{t+1} \tag{3-3}$$

Condition (3-3) states that the maximum value of the mortgage contract is higher than the amount lent. If it holds, the bank lends the amount required for house purchase. In turn, households may choose to buy a house or rent one to maximize their expected utility.

3.5 The borrower's problem

The problem of the borrower at period t is to choose whether or not to walk away from his mortgage. The household computes both the gain from paying the mortgage and the gain from walking away and then chooses the action with the highest payoff. In doing so, the household solves the following problem:

$$U_t(x, w_t, S_t; i) \equiv \max_{\{h_\tau, c_\tau, w_\tau\}} E_t \sum_\tau \beta^\tau (c_\tau + 1_{owner_\tau} \cdot \bar{u}) \tag{3-4}$$

subject to

$$h_\tau \in \{tenant_\tau; \quad owner_\tau\} \qquad \forall \tau > t \tag{3-5}$$

$$c_\tau + w_\tau \leq R w_{\tau-1} + y_\tau - 1_{owner_\tau} \cdot x \qquad \forall \tau > t \tag{3-6}$$

Constraint (3-5) states that the mortgage holder either pays the mortgage ($h_t = owner_t$) or walks away ($h_t = tenant_t$). Constraint (3-6) states that the household's choice of consumption and savings must be lower than the household's available resources. These resources correspond to the sum of the family's current income and the last period savings. If the family decides to remain homeowners, they are required to make the mortgage payment, x. In equilibrium, this constraint holds with equality.

From constraint (3-6), we can see that the household can only pay the mortgage if their resources are high enough to cover the mortgage payment, that is $Rw_{t-1} + y_t \geq x$. Otherwise, the family has to default and become a tenant. Alternatively, the household that can afford the mortgage payment defaults strategically whenever the gain from paying the mortgage is lower than the gain from becoming a tenant.

In this context, the lender takes into account the family's decision in each state to evaluate the mortgage's cash flow for each payment level. In equilibrium, the competitive mortgage payment at $t = 0$, x_i^*, is the minimum value satisfying condition (3-2). That is,

$$x_i^* = \min\{x / \quad V_0(x, w_0, S_0; i) \geq S_0 - d \cdot w_0\} \tag{3-7}$$

From expression (3-7), it is possible to conclude that x_i^* depends both on the initial wealth value and on the house value. Because the income distribution affects default decisions, the mortgage payment is contingent on the borrower's type as well.

Just like in the simple version of the model, the price distribution also affects the payment level because the house price affects the collateral value in case of default. The probability of a price drop is related to the chance of strategic default, so the chance of a bubble burst is important to determine the mortgage payment.