4 Solving The Model

To characterize the default decision of households, first it is necessary to compute the payoff of the two possible actions available. In each period, the mortgage holder has an option to default in the future and the value of this option must be considered when evaluating the gain from paying the mortgage. Allowing delinquent families to apply for a new mortgage changes the default option value. As in the Option Approach to Mortgage Valuation literature, changing the default payoff imply that the value of making the mortgage payment changes as well.

The possibility of returning to the mortgage market makes the default payoff sensitive to the borrower's income distribution and initial wealth. In order to be eligible for a new loan, the household's income profile must be good enough to compensate for the probability of the costly foreclosure state. Delinquent households with bad income outlook will be less likely to have their new loan request accepted. Let us now turn to a formal characterization of the payoff of both actions.

4.1 The Gain from Walking Away

A household that decides to walk away from its mortgage contract chooses either to default on the mortgage or to sell the house and prepay the mortgage. Besides the traditional walking away payoff, a household must also take into account the value of the option of possibly buying a home again in the future with a new mortgage. To get an understanding of the value of this option, first consider a household that becomes a tenant permanently after defaulting on the mortgage. In such case the household's expected utility is given by the following expression:

$$U_t^{i,D} = Rw_{t-1} + y_{i,t} + \beta\frac{E[y_i]}{1-\beta} - \beta\frac{r_t}{1-\beta} \tag{4-1}$$

In equation (4-1), the household that rents a home from period $t+1$ onwards receives income and pays rent at each of the following periods. The gain from renting a home permanently, $U_t^{i,D}$, is the family's life-time expected consumption. Because we assume that $R \cdot \beta = 1$, risk-neutral households are indifferent between consuming at t or at any other future periods and tenants' consumption correspond to their wealth at time t, $Rw_t + y_{i,t}$, plus the discounted present value of the their expected future income, $\beta \frac{E[y_i]}{1-\beta}$ minus the expected rent expenses, $\beta \frac{r_t}{1-\beta}$.

The novelty of our paper is to introduce the possibility of delinquent households applying for a new mortgage. The value of possibly having a new mortgage request accepted must be considered by the household when deciding whether to default on the mortgage. Proposition 5 presents the gain from walking away when delinquent households are allowed to return to the credit market.

Proposition 5 *Consider a household that decides to walk away from a mortgage at time t.*

1. *The utility of walking away from the mortgage at time t is given by the expression:*

$$U_t(tenant_t; i) = U_t^{i,D} + \bar{u} + \max\{\gamma S_t - D_t; 0\} + M_t^i \tag{4-2}$$

where M_t^i is the expected utility of returning to the housing credit market at $t+1$.

2. *Define $Ref_t^{i,*}$ as the set of states in which a new mortgage is available to a type i family.*

$$Ref_t^{i,*} = \left\{ V_{t+1}^{MAX}(w_{t+1}, S_{t+1}; i) \geq S_{t+1} - d^D \cdot (Rw_t + y_{t+1}) \right\}$$

The value of M_t^i is given by

$$\begin{aligned} M_t^i = & \beta \cdot Prob_t[Ref_{t+1}^{i,*}] \cdot E_t \Big[\max\big\{ A_0(x_i^{**}, w_{t+1}, S_{t+1}, r_{t+1}; i) + \\ & + \bar{u} + x_i^{**} + r_{t+1} - d^D(Rw_t + y_{t+1}); 0 \big\} \Big| Ref_{t+1}^{i,*} \Big] \end{aligned} \tag{4-3}$$

where $A_0(.)$ is the additional gain of beginning a new mortgage that entails the periodic payment x_i^{**}.

In equation (4-2), we describe the gain from walking away from a mortgage at time t which is the sum of three terms. The first term is the household's outside option of becoming a tenant permanently. After abandoning their mortgage contracts, delinquent households may choose to never apply for a new mortgage which yields the expected utility $U_t^{i,D}$.

The second term is the value of the possibility of leaving a negative value portfolio by default. A non-recourse mortgage contract entitles the borrower to default on a mortgage contract anytime. On the one hand, if the value of the house is smaller than the debt, the borrower that decides to walk away defaults on the mortgage. On the other hand, if the selling value of the house is greater than the debt, the house can be sold, the mortgage prepaid and the difference $\gamma S_t - D_t$ kept by the household. Therefore, households gain $\max\{\gamma S_t - D_t; 0\}$ in case they decide to walk away from their mortgages.

The last term is the expected utility of returning to the housing credit market at $t + 1$, M_t^i. Equation (4-3) states that when a loan is available in the next period, the household selects the highest payoff action between continuing to rent a house or financing a house purchase with a new mortgage. A loan request is only available when condition (3-3) is satisfied. That is, the representative bank only lends if there is a mortgage contract with expected present value cash flow higher than the loan amount. If the household decides to finance the purchase of new house its gain is the additional utility $A_0(.)$ minus the new mortgage's down payment. We describe the gain $A_0(.)$ in Lemma 8 of the paper's appendix which corresponds to a mortgage without the possibility of returning to the credit market if the household walks away from the mortgage.

The mortgage value is sensible to the housing price distribution, so a family that was able to get a mortgage in the past may not be eligible for a new mortgage if the distribution of housing prices has changed since the initial period. A bubble increases the expected recovery rate of a mortgage in foreclosure, hence lenders in a competitive market reduce mortgage interest rates. After the bubble bursts, this favorable credit condition for low-income families no longer exists, therefore the expected utility of a new mortgage is approximately zero for them. The same is not true for High-income families because their income profile enables them to get a mortgage no matter what is the housing price distribution.

The existence of a bubble in the price of houses creates heterogeneity in default payoff among mortgage holders. On the one hand, the possibility of returning to the mortgage market in better terms available to High-income families constitutes an incentive for them to default strategically after a large price drop. On the other hand, Low-income families may be shut from the mortgage market if they default strategically, therefore they are more likely to pay an underwater mortgage. In order to analyze the strategic default behavior, we now describe the gain from paying the mortgage.

4.2 Gain from Paying the Mortgage

To properly analyze the strategic default decision, we evaluate the payoff of paying the mortgage. In paying the mortgage, the household receives the default option and this option's value must be taken into account. Proposition 6 presents the gain from paying the mortgage.

Proposition 6 *Consider a household that can afford to pay the mortgage at* t*,* $Rw_{t-1} + y_t \geq x$*.*

1. *If the borrower pays the mortgage in period* t*, his expected utility is*

$$U_t(owner_t; i) = U_t^{i,D} + \bar{u} + A_t^*(x, w_t, S_t, r_t; i) \tag{4-4}$$

where A_t^* *is the expected gain from paying the mortgage at* t *and* $w_t = Rw_{t-1} + y_t - x$*.*

2. *The expected gain from paying the mortgage is given by*

$$A_{T^*}^*(x, w_{T^*}, S_{T^*}, r_{T^*}; i) = \beta \frac{\bar{u}}{1-\beta} - x + \beta E_{T^*}[S_{T^*+1}] \tag{4-5}$$

$$\begin{aligned} A_t^*(x, w_t, S_t, r_t; i) &= \beta r_t + \beta \bar{u} - x + \beta Prob_t\big(Rw_t + y_{t+1} < x\big) \cdot \\ &\cdot E_t\Big[max\{\gamma S_{t+1} - D_{t+1}, 0\} + M_{t+1}^i \Big| Rw_t + y_{t+1} < x\Big] + \beta Prob_t\big(Rw_t + y_{t+1} \geq x\big) \cdot \\ &\cdot E_t\Big[\max\Big\{A_{t+1}^*; \max\{\gamma S_{t+1} - D_{t+1}, 0\} + M_{t+1}^i\Big\}\Big| Rw_t + y_{t+1} \geq x\Big] \end{aligned} \tag{4-6}$$

The gain from paying the mortgage in equation (4-4) includes the outside renting option, $U_t^{i,D}$ because the household may always choose to default and become a tenant forever. The second term in equation (4-4) is the private benefit of being a homeowner at time t, $\bar{u}$. Lastly, the third term is the additional gain from paying the mortgage, $A_t^*(.)$, that includes the value of the default option.

In the second part of Proposition 6, equation (4-6) states that the additional gain from paying the mortgage corresponds to the current benefit of home ownership, $\beta r_t + \beta \bar{u} - x$, plus the present value of the walking away option. In other words, the payoff of paying the mortgage at time t includes the value of choosing the best action available at $t+1$.

The general model with an infinite-horizon economy allows families to consider the value of the default option when deciding whether or not to walk away from their mortgages. A borrower that decides to pay the mortgage receives the option to default on the mortgage in the future and the value of this option is an important part of the payoff of paying the mortgage. Foote et al. (2008) show that the value of the default option is important to explain properly the behavior of mortgage holders facing a large price drop.

Note also that the set of possible actions depends on the realizations of S_{t+1} and y_{t+1}. In any state at $t+1$, the borrower can exercise his walking away option and become a tenant, in which case he gains $\max\{\gamma S_{t+1} - D_{t+1}; 0\} + M_{t+1}$. Alternatively, making the mortgage payment is only possible if the borrower's assets are greater than x, so the borrower may choose between paying the mortgage or walking away at $t+1$ if, and only if, $Rw_t + y_{t+1} \geq x$.

When deciding to default strategically on an underwater mortgage, the household must compare the additional gain from paying the mortgage with the expected value of having a new mortgage request accepted at $t+1$. Therefore, a household defaults strategically if the gain from walking away in equation (4-2) is higher than the gain from paying the mortgage in equation (4-4). That is,

$$M^i_{t+1} \geq A^*_t(x, w_t, S_t, r_t; i) \tag{4-7}$$

The default payoff is higher for families with better income distributions due to their higher chance of being eligible for a new loan. Given their wealth, families with higher income may default strategically more often because it is more likely that inequality (4-7) holds for them. Because A^*_t also depends on the income distribution in a nontrivial way, it is necessary to simulate the model to properly compare the default behavior of families with different income distribution.

At time $t = 0$, the household decides whether to buy or rent a house. If the household decides to get a mortgage, the periodic payment is such that equation (3-7) holds. Given the mortgage terms, a household of type i that buys a home at $t = 0$ has the following expected utility:

$$U_0(owner_0; i) = \Big[(1-d)\cdot w_0 + \beta\frac{E[y_i] - r_0}{1-\beta}\Big] + \bar{u} + x^*_i + A^*_0(x^*_i, (1-d)w_0, S_0, r_0; i) \tag{4-8}$$

In equation (4-8), the first term is the household's outside option of becoming a tenant from period $t = 1$ onwards in which the household has a wealth of $(1-d)\cdot w_0$ after making the mortgage down payment. The household also evaluates the present value of the difference between their life-time income minus rent expenses. The other terms correspond to the gains from the mortgage as described in Proposition 6.

The household also may decide to rent a house in which case its expected utility is given by:

$$U_0(tenant_0; i) = w_0 + \beta\frac{E[y_i] - r_0}{1-\beta}] - r_0 \tag{4-9}$$

In equation (4-9), we assume that the household that does buy a house at the initial period has to rent a home permanently. In this case the family keeps its initial wealth, w_0, and evaluates the expected present value of its future income discounted of rent costs. Furthermore, the household pays r_0 for renting a home at $t = 0$.

In deciding to buy a home at $t = 0$, the gain from the mortgage in equation (4-8) must be higher that the gain from renting in equation (4-9). Therefore, a household decides to finance the purchase of a house if the mortgage payment in equation (3-7) is such that

$$\bar{u} + x_i^* + A_0^*(x_i^*, (1-d)w_0, S_0, r_0; i) \geq d \cdot w_0 - r_0 \tag{4-10}$$

In equation (4-10), the right hand side is the mortgage benefit that equals the sum of the private benefit from home ownership at $t = 0$, $\bar{u}$, and the initial mortgage's gain, $x_i^* + A_0^*(x_i^*, (1-d)w_0, S_0, r_0; i)$. The left hand side is the cost of the mortgage that corresponds to the difference between the down payment and the rent paid at $t = 0$.

At this point, we still need to characterize the expected value of mortgage's cash flow in order to determine the equilibrium payment in equation (3-7). Proposition 7 writes this value in a recursive form.

Proposition 7

The mortgage contract's value at time t, $V_t(x, w_t, S_t; i)$, is given by

$$V_T = 0$$

$$V_t(x, S_t, w_t; i) = \beta E_{t+1}[h_t(x, w_t, S_{t+1}, y_{t+1}; i)] \tag{4-11}$$

where

$$h_{t+1}(x, w_t, S_{t+1}, y_{t+1}; i) = \begin{cases} x + V_{t+1}(x, S_{t+1}, w_{t+1}; i) & \textit{if the borrower pays at t+1} \\ \min\{\gamma S_{t+1}; D_{t+1}\} & \textit{if the borrower walks away at t+1} \end{cases} \tag{4-12}$$

Equation (4-11) assumes that the mortgage value is the discounted expected contract's cash flow. If the family decides to pay the mortgage at time t, the lender receives x plus the value of the contract at $t+1$. Alternatively, if the borrower decides to walk away from the mortgage, the lender receives the expected salvage value of the house.

A bubble in the price of houses has two effects on the lender's mortgage value. First, expected gains in house prices represent more valuable collateral in case of default. Second, a bubble raises the chance of strategic default due to a large price drop. The full consequences of a bubble over the lender's mortgage value are hard to evaluate analytically, so we simulate the model in the next section.