Referências Bibliográficas

[1] ROSA, A. J.; DE SOUZA CARVALHO, R. ; XAVIER, J. A. D.. **Engenharia de Reservatórios de Petróleo**. Editora Interciência, Rio de Janeiro, 2006.

[2] MCAULIFFE, C. D.. **Oil-in-water emulsions and their flow properties in porous media**. SPE-AIME, Chevron Oil Field Research Co., 1973.

[3] OREN, P. E.; BAKKE, S.. **Reconstruction of berea sandstone and pore-scale modelling of wettability effects**. Journal of Petroleum Science and Engineering, 39:177 – 199, 2003.

[4] FATT, I.. **The network model of porous media:i**. Transactions of AIME 207, 207:144 – 159, 1956.

[5] NÚÑEZ, V. R. G.; CARVALHO, M. S. ; BASANTE, V. A.. **Oil displacement by oil-water emulsion injection in coreflooding experiments**. 20th International Congress of Mechanical Engineering, 2009.

[6] HO, B. P.; LEAL, L. G.. **The creeping motion of liquid drops through a circular tube of comparable diameter**. Journal of Fluid Mechanics, 71:361–383, 1975.

[7] MONTALVO, M. E. D. A.. **Escoamento de emulsões Óleo em Água através de microcapilares**. Dissertação de Mestrado, Departamento em Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 2008.

[8] COBOS, S.; CARVALHO, M. S. ; ALVARADO, V.. **Flow of oil-water emulsions through a constricted capillary**. International Journal of Multiphase Flow, 35:507–515, 2009.

[9] SHEN, E. I.; UDELL, K. S.. **A finite element study of low reynolds number two-phase flow in cylindrical tubes**. Journal of Applied Mathematics, 52:253–256, 1985.

[10] MARTINEZ, M. J.; UDELL, K. S.. **Axisymmetric creeping motion of drops through circular tubes**. Journal of Fluid Mechanics, 210:565–591, 1990.

[11] TSAI, T. M.; MIKSIS, M. J.. **Dynamics for a drop in a constricted capillary tube**. Journal of Fluid Mechanics, 274:197–217, 1994.

[12] SUSSMAN, M.. **A Level Set Approach for Computing Solutions to Incompressible Two-Phase Flow**. Tese de Doutorado, Department of Mathematics, University of California, Los Angeles, 1994.

[13] SUSSMAN, M.; FATEMI, E.; SMEREKA, P. ; OSHER, S.. **A level set approach for computing solutions to incompressible two-phase flow 2**. Proceedings of the 6th International Symposium on Computional Fluid Dynamics, 1995.

[14] TRYGGVASON, G.; BUNNER, B.; ESMAEELI, A.; JURIC, D.; AL-RAWAHI, N.; TAUBER, W.; HAN, J.; NAS, S. ; JAN, Y. J.. **A front-tracking method for the computations of multiphase flow**. Journal of Computational Physics, 169:708–759, 2001.

[15] SHEPEL, S. V.; SMITH, B. L.. **On surface tension modelling using the level set method**. International Journal for Numerical Methods in Fluids, 59:147–171, 2008.

[16] BRACKBILL, J. U.; KOTHE, D. B. ; ZEMACH, C.. **A continuum method for modeling surface tension**. Journal of Computational Physics, 100:335–354, 1992.

[17] OSHER, S.; FEDIW, R.. **Level Set Methods and Dynamic Implicit Surfaces**. Springer, New York, USA, 2003.

[18] DA SILVEIRA CARVALHO, M.; VALÉRIO, J. V.. **Introdução ao Método de Elementos Finitos**. Departamento de Mecânica da PUC-Rio, Rio de Janeiro, Brasil.

[19] SZADY, M. J.; SALAMON, T. R.; LIU, A. W.; BORNSIDE, D. E.; ARMSTRONG, R. C. ; BROWN, R. A.. **A new mixed finite element method for viscoelastic flows governed by differential constitutive equations**. Journal of Non-Newtonian Fluid Mechanics, 59(2-3):215 – 243, 1995.

[20] ROBLES, O.; CARVALHO, M. S.. **Oil-in-water emulsions flow through constricted micro-capillaries**. Proceedings of the 13th Brazilian Congress of Thermal Sciences and Engineering, 2010.

A
Primeiro Apêndice

A.1
Cálculo dos termos de tensor de tensões

Em coordenadas cilíndricas, as componentes do tensor de tensões $\bar{\bar{T}}$ são definidas como:

$$\begin{aligned} T_{zz} &= -p + 2\mu\frac{\partial v_z}{\partial z} \\ T_{zr} &= \mu(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}) \\ T_{rr} &= -p + 2\mu\frac{\partial v_r}{\partial r} \\ T_{\theta\theta} &= -p + 2\mu\frac{v_r}{r} \end{aligned} \tag{A-1}$$

Considerando a viscosidade como uma propriedade constante em cada uma das fases (óleo e água) tem-se,

$$\frac{\partial T_{rr}}{\partial V_{Rj}} = 2\mu\frac{\partial \phi_j}{\partial r}; j = 1, \cdots, 9 \tag{A-2}$$

$$\frac{\partial T_{zr}}{\partial V_{Rj}} = \mu\frac{\partial \phi_j}{\partial z}; j = 1, \cdots, 9 \tag{A-3}$$

$$\frac{\partial T_{\theta\theta}}{\partial V_{Rj}} = 2\mu\frac{\phi_j}{r}; j = 1, \cdots, 9 \tag{A-4}$$

$$\frac{\partial T_{zr}}{\partial V_{Zj}} = \mu\frac{\partial \phi_j}{\partial r}; j = 1, \cdots, 9 \tag{A-5}$$

$$\frac{\partial T_{zz}}{\partial V_{Zj}} = 2\mu\frac{\partial \phi_j}{\partial z}; j = 1, \cdots, 9 \tag{A-6}$$

$$\frac{\partial T_{rr}}{\partial C_j} = 2\frac{\partial \mu}{\partial C_j}\frac{\partial v_r}{\partial r}; j = 1, \cdots, 9 \tag{A-7}$$

$$\frac{\partial T_{zr}}{\partial C_j} = (\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z})\frac{\partial \mu}{\partial C_j}; j = 1, \cdots, 9 \tag{A-8}$$

$$\frac{\partial T_{zz}}{\partial C_j} = 2\frac{\partial \mu}{\partial C_j}\frac{\partial v_z}{\partial z}; j = 1, \cdots, 9 \tag{A-9}$$

$$\frac{\partial T_{\theta\theta}}{\partial C_j} = 2\frac{\partial \mu}{\partial C_j}\frac{v_r}{r}; j = 1, \cdots, 9 \tag{A-10}$$

$$\frac{\partial T_{rr}}{\partial P_j} = -\chi_j; j = 1, \cdots, 3 \tag{A-11}$$

$$\frac{\partial T_{zz}}{\partial P_j} = -\chi_j; j = 1, \cdots, 3 \tag{A-12}$$

$$\frac{\partial T_{\theta\theta}}{\partial P_j} = -\chi_j; j = 1, \cdots, 3 \tag{A-13}$$

B
Segundo Apêndice

Devido aos novos campos c_r e c_z o vetor $\mathbf{S_V}$ tem mudado,

$$\mathbf{S_V}^{**} = \begin{pmatrix} V_{Rj} \\ V_{Zj} \\ C_j \\ P_j \\ C_{Rj} \\ C_{Zj} \end{pmatrix} \tag{B-1}$$

Além disso, o vetor de resíduos $\mathbf{R}$ também é redefinido como,

$$\mathbf{R}^{**} = \begin{pmatrix} R^i_{mr} \\ R^i_{mz} \\ R^i_{c} \\ R^i_{mc} \\ R^i_{c_r} \\ R^i_{c_z} \end{pmatrix} \tag{B-2}$$

Consequentemente a matriz $\mathbf{J_{RP}}$ é dada por,

$$\mathbf{J^{**}_{RP}} = \begin{pmatrix}
\frac{\partial R^{i*}_{mr}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{mr}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{mr}}{\partial C_j} & \frac{\partial R^{i*}_{mr}}{\partial P_j} & \frac{\partial R^{i*}_{mr}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{mr}}{\partial C_{Zj}} \\
\frac{\partial R^{i*}_{mz}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{mz}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{mz}}{\partial C_j} & \frac{\partial R^{i*}_{mz}}{\partial P_j} & \frac{\partial R^{i*}_{mz}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{mz}}{\partial C_{Zj}} \\
\frac{\partial R^{i*}_{c}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{c}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{c}}{\partial C_j} & \frac{\partial R^{i*}_{c}}{\partial P_j} & \frac{\partial R^{i*}_{c}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{c}}{\partial C_{Zj}} \\
\frac{\partial R^{i*}_{mc}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{mc}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{mc}}{\partial C_j} & \frac{\partial R^{i*}_{mc}}{\partial P_j} & \frac{\partial R^{i*}_{mc}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{mc}}{\partial C_{Zj}} \\
\frac{\partial R^{i}_{c_r}}{\partial V_{Rj}} & \frac{\partial R^{i}_{c_r}}{\partial V_{Zj}} & \frac{\partial R^{i}_{c_r}}{\partial C_j} & \frac{\partial R^{i}_{c_r}}{\partial P_j} & \frac{\partial R^{i}_{c_r}}{\partial C_{Rj}} & \frac{\partial R^{i}_{c_r}}{\partial C_{Zj}} \\
\frac{\partial R^{i}_{c_z}}{\partial V_{Rj}} & \frac{\partial R^{i}_{c_z}}{\partial V_{Zj}} & \frac{\partial R^{i}_{c_z}}{\partial C_j} & \frac{\partial R^{i}_{c_z}}{\partial P_j} & \frac{\partial R^{i}_{c_z}}{\partial C_{Rj}} & \frac{\partial R^{i}_{c_z}}{\partial C_{Zj}}
\end{pmatrix} \tag{B-3}$$

B.1
Cálculo dos termos adicionais da matriz $\mathbf{J_{RP}}$

– Termo adicionado a $\frac{\partial R^{i*}_{mr}}{\partial C_{Rj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r, c_z)}{\partial C_{Rj}} \frac{\partial c}{\partial r} \phi_i \delta(c)\, d\Omega; i = 1, \cdots, 9; j = 1, \cdots, 4 \tag{B-4}$$

– Termo adicionado a $\frac{\partial R^{i*}_{mr}}{\partial C_{Zj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r, c_z)}{\partial C_{Zj}} \frac{\partial c}{\partial r} \phi_i \delta(c)\, d\Omega; i = 1, \cdots, 9; j = 1, \cdots, 4 \quad \text{(B-5)}$$

– Termo adicionado a $\frac{\partial R^{i*}_{mz}}{\partial C_{Rj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r, c_z)}{\partial C_{Rj}} \frac{\partial c}{\partial z} \phi_i \delta(c)\, d\Omega; i = 1, \cdots, 9; j = 1, \cdots, 4 \quad \text{(B-6)}$$

– Termo adicionado a $\frac{\partial R^{i*}_{mz}}{\partial C_{Zj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r, c_z)}{\partial C_{Zj}} \frac{\partial c}{\partial z} \phi_i \delta(c)\, d\Omega; i = 1, \cdots, 9; j = 1, \cdots, 4 \quad \text{(B-7)}$$

B.2 Cálculo das derivadas da curvatura k em função de C_{Rj} e C_{Zj}

$$\begin{aligned}\frac{\partial k}{\partial C_{Rj}} &= \{(c_z^2 \frac{\partial \varphi_j}{\partial r} - 2c_z \varphi_j c_{zr} + 2c_r \varphi_j c_{zz})(c_r^2 + c_z^2)^{\frac{3}{2}} \\ &\quad -3(c_z^2 c_{rr} - 2c_r c_z c_{zr} + c_r^2 c_{zz})(c_r^2 + c_z^2)^{\frac{1}{2}}(c_r \varphi_j)\} \\ &\quad /(c_r^2 + c_z^2)^3 + \frac{\varphi_j}{r}[\frac{1}{(c_r^2 + c_z^2)^{1/2}} \\ &\quad - \frac{c_r^2}{(c_r^2 + c_z^2)^{3/2}}]; j = 1, \cdots, 4 \qquad \text{(B-8)}\end{aligned}$$

$$\begin{aligned}\frac{\partial k}{\partial C_{Zj}} &= \{(2c_z \varphi_j c_{rr} - 2c_r[\varphi_j c_{zr} + c_r \frac{\partial \varphi_j}{\partial r}] + c_r^2 \frac{\partial \varphi_j}{\partial z})(c_r^2 + c_z^2)^{\frac{3}{2}} \\ &\quad -3(c_z^2 c_{rr} - 2c_r c_z c_{zr} + c_r^2 c_{zz})(c_r^2 + c_z^2)^{\frac{1}{2}}(c_z \varphi_j)\} \\ &\quad /(c_r^2 + c_z^2)^3 - \frac{\varphi_j}{r} \frac{c_r c_z}{(c_r^2 + c_z^2)^{3/2}}; j = 1, \cdots, 4 \qquad \text{(B-9)}\end{aligned}$$

B.3 Cálculo dos novos termos da matriz $\mathbf{J}^{**}_{\mathbf{RP}}$

$$\frac{\partial R^i_{c_r}}{\partial C_j} = \int_\Omega \frac{\partial \phi_j}{\partial r} \varphi_i \, d\Omega; i = 1, \cdots, 4; j = 1, \cdots, 9 \quad \text{(B-10)}$$

$$\frac{\partial R^i_{c_r}}{\partial C_{Rj}} = -\int_\Omega \varphi_j \varphi_i \, d\Omega; i, j = 1, \cdots, 4 \quad \text{(B-11)}$$

$$\frac{\partial R^i_{c_z}}{\partial C_j} = \int_\Omega \frac{\partial \phi_j}{\partial z} \varphi_i \, d\Omega; i = 1, \cdots, 4; j = 1, \cdots, 9 \quad \text{(B-12)}$$

$$\frac{\partial R^i_{c_z}}{\partial C_{Zj}} = -\int_\Omega \varphi_j \varphi_i \, d\Omega; i, j = 1, \cdots, 4 \quad \text{(B-13)}$$

O resto dos termos não definidos da matriz $\mathbf{J}^{**}_{\mathbf{RP}}$ consideram-se zeros.